

J. W. Walls.

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A SERIES OF TEXTBOOKS PREPARED FOR THE STUDENTS OF THE
INTERNATIONAL CORRESPONDENCE SCHOOLS AND CONTAINING
IN PERMANENT FORM THE INSTRUCTION PAPERS,
EXAMINATION QUESTIONS, AND KEYS USED
IN THEIR VARIOUS COURSES

ARITHMETIC
APPLICATION OF FORMULÆ
GEOMETRY AND MENSURATION
PRELIMINARY BUILDING OPERATIONS
FOUNDATIONS AND FOOTINGS
AREAS, VAULTS, AND RETAINING WALLS
LIMES, CEMENTS, AND MORTARS
CONCRETE CONSTRUCTION
FIRE-RESISTING CONSTRUCTION

4 2202

LONDON
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PREFACE

The series of textbooks of which this volume forms a part is primarily intended for teaching by correspondence. A moment's consideration will show the reader that textbooks for this purpose must be prepared on an entirely different plan from those intended to be used in the classroom. The ordinary textbook used in technical schools is prepared on the assumption that a teacher will be at hand to explain any difficult points. Furthermore, such a textbook presupposes that students already possess a sufficient knowledge of preliminary subjects to enable them to understand what they are studying. In the preparation of I.C.S. textbooks, however, it is assumed that students must be given instruction in everything necessary for a thorough understanding of the technical subjects. The textbooks take the place of the teacher, and must therefore supply, besides the ordinary instruction, the help which the teacher gives personally. The instruction, moreover, being intended for persons with limited time at their disposal, must be clear and concise, and must avoid taking up any parts of subjects that have no direct bearing upon the end sought.

A fundamental point embodied in these textbooks is the freedom from advanced mathematics, and the inclusion of only so much simple mathematics as has been considered absolutely necessary. The demonstrations of formulæ have not, in general, been given, but students are thoroughly instructed how to apply the formulæ, by the aid of illustrative examples, founded, as far as possible, on conditions occurring in actual practice. We have endeavoured to foresee every difficulty that students are likely to encounter, and to make the explanations so clear and simple as to be readily understandable by any person, however limited his previous education may have been.

The illustrations, which have all been specially made for these textbooks, are worthy of note as indicating the care taken to obtain fullness and clearness of detail. The free use of reference letters on the illustrations is another feature which promotes accuracy and thoroughness of description.

These textbooks are issued under the general title of I.C.S. Reference Library. Each student entitled to them is furnished with the volumes of the Library which contain the subjects belonging to his Course. Inasmuch, however, as such volumes may be used for a number of Courses varying in some respects, a volume may contain one or more subjects that are not included in a student's Course. These subjects, although not necessary to the Course itself, are usually of advantage, as they are generally so closely related thereto as to constitute additional information. There is no Course in which any of the instruction belonging to that Course is omitted from the volumes.

In order to permit the frequent revision deemed necessary to keep the Papers up to date, the pages in each volume are not numbered consecutively throughout, but by a very simple method any reference can be readily found. Each Paper included in the volume has the pages numbered consecutively. On the inner side of each head-line, opposite the page number, is given a number preceded by the section mark (§), thus, §5. These section numbers are not always in sequence; that is, there may be some missing numbers, but a larger section number in no case precedes a smaller one. Each volume contains a very complete Index, so that any item may be found usually under several headings. A reference, such as section 22, page 15, may be found by turning over the pages until section 22 is noted, and then continuing to turn over the leaves until page 15 is reached. The Table of Contents is similarly arranged.

The Examination Questions are placed at the end of the volume, together with the Answers to the Questions, whenever such are issued.

We desire to express our appreciation of the courtesies extended to us, and the information and drawings furnished by the designers, manufacturers, and users of the machines, appliances, structures, processes, and methods which we have illustrated and described in these textbooks as being representative of the best British, Colonial, and American practice.

INTERNATIONAL CORRESPONDENCE SCHOOLS, LTD.

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ARITHMETIC

(PART 1)

DEFINITIONS

1. **Arithmetic** is the science of numbers and the art of reaching results by their use. The four fundamental processes of arithmetic are **addition, subtraction, multiplication, and division**. These processes will be described in detail farther on.

2. A **unit** is *one*, or a single thing, or the standard quantity used in measuring the magnitude or extent of any given quantity, as *one*, one *boy*, one *yard*, one *dozen*, one *pint*. Here the units are one, boy, yard, dozen, pint.

3. A **number** is a unit or a collection of units, as *three* apples, *five* boys, *seven*. Here the numbers are three, five, seven.

4. An **abstract number** is a number not applied to any object or quantity, as *three, five, ten*.

5. A **concrete expression** or **concrete term** is formed of a number and unit together, as *three horses, five pounds, ten yards*.

NOTATION AND NUMERATION

6. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

7. **Notation** is the system of expressing numbers by figures or letters.

8. **Numeration** is the art of reading numbers expressed by figures or letters.

ARABIC NOTATION

9. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten symbols, called **figures**, to represent numbers, viz. :

Figures	0	1	2	3	4	5	6	7	8	9
Names	nought, cipher, or zero	one	two	three	four	five	six	seven	eight	nine

The first figure (0) is called **nought**, **cipher**, or **zero**, and, when standing alone, has no value.

The other nine figures are called **digits**, and each one has a value of its own.

An **integer** is any whole number.

10. Since there are only ten *figures* used in expressing numbers, each *figure* must have different *values*, determined by the way in which it is used.

11. The value of a figure depends upon its *position* in relation to others.

12. Figures have **simple values**, and **local**, or **relative**, values.

13. The **simple value** of a figure is the value it expresses when standing alone.

14. The **local**, or **relative**, value of a figure is its value as determined by its position in a number.

Thus, 6 standing alone means *six ones, or six units* . . . 6

In the second place it denotes *six tens, or sixty units* . . . 60

In the third place it denotes *six hundreds, or six hundred units* 600

In the fourth place it denotes *six thousands, or six thousand units* 6,000

In the fifth place it denotes *six ten-thousands, or sixty thousand units* 60,000

In the sixth place it denotes *six hundred-thousands, or six hundred thousand units* 600,000

In the seventh place it denotes *six millions, or six million units* 6,000,000

15. The **value** of a figure increases tenfold with each remove to the left.

16. The **cipher** has no value in itself, but it is useful in determining the place of other figures. To represent the number *four hundred and five*, only two digits are necessary, one to denote *four hundred*, and the other to denote *five*; but if these two figures are placed together, as 45, the 4 being in the second place, will mean 4 *tens*. To denote 4 *hundreds*, the 4 should be in the third place. A cipher, therefore, must be inserted in the *tens* place to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred and five* is therefore written 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4,005. If it were *four hundred and fifty*, the cipher would be in the units place to show that there were no *units*, but only *hundreds* and *tens*; thus, 450. *Four thousand and fifty* is written 4,050, the ciphers indicating that there are no hundreds and no units.

17. In reading numbers, it is usual to divide them by commas into groups of three figures each, called **periods**, beginning at the right. The first figure, beginning at the right, is said to belong to the *first order*, the second to the *second order*, etc. Each **period** contains three orders, named as shown in the following table:

Name of period	Billions			Thousands of Millions			Millions			Thousands			Units		
Name of order	hundred-billions			hundred-thousand-millions			hundred-millions			hundred-thousands			hundreds		
	ten-billions			ten-thousand-millions			ten-millions			ten-thousands			tens		
Number	9	8	7,	4	3	2,	1	9	8,	7	6	5,	4	3	2
	billions			thousand-millions			millions			thousands			units		

The first period, beginning at the right, contains *units, tens, hundreds*; the second, *thousands, ten-thousands, hundred-thousands*; the third, *millions, ten-millions, hundred-millions*, etc.

The number in the table is read, *nine hundred and eighty-seven billion, four hundred and thirty-two thousand one hundred and ninety-eight million, seven hundred and sixty-five thousand, four hundred and thirty-two*.

The definition of a billión, shown in the table as being one thousand thousand millions, is not universally adopted. For instance, in the United States and France, one billion is considered to be one thousand millions.

18. The writing of numbers is called **notation**, and the reading of numbers is called **numeration**. It will be noticed that in reading and writing numbers the *s* at the end of thousands, millions, etc., is omitted.

ROMAN NOTATION

19. Another way of expressing numbers is by means of the **Roman notation**, a method much used in the past, but used now in a few special cases, chiefly for expressing high numbers. In the Roman notation one or more of the seven capital letters I, V, X, L, C, D and M are used. Their values, when standing alone, are as follows: I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1,000. By combinations of these letters, all other numbers are expressed. Their combinations are in accordance with the following

PRINCIPLES

1. *Repeating a letter repeats its value.*

Thus, II = 2, XX = 20, XXX = 30, CC = 200, CCC = 300.

V, D, and L are never repeated, and only I, X, C, and M are ever used more than once in any combination.

2. *If a letter precedes one of greater value, their difference is denoted; if it follows, their sum is denoted.*

Thus, IV = 4, VI = 6, IX = 9, XI = 11, XL = 40, LX = 60.

3. *A line placed over a letter multiplies its value by one thousand.*

Thus, $\overline{X}$ = 10,000, $\overline{L}$ = 50,000, $\overline{XCDXVII}$ = 90,517

20. The following table illustrates more fully the foregoing principles :

VII	= 7	LX	= 60	CLXIX	= 169	$\overline{\text{IXLX}}$	= 9,060
XIII	= 13	LXIX	= 69	CLXXX	= 180	$\overline{\text{LCXC}}$	= 50,190
XIV	= 14	LXX	= 70	CCXL	= 240	$\overline{\text{XIX}}$	= 19,000
XV	= 15	LXXX	= 80	CCCLIX	= 359	$\overline{\text{XLDX}}$	= 40,510
XX	= 20	XC	= 90	CCCCL	= 450	$\overline{\text{XCVII}}$	= 97,000
XXV	= 25	XCIX	= 99	CCCCXC	= 490	$\overline{\text{DCC}}$	= 700,000
XXVIII	= 28	CIX	= 109	DCCXL	= 740	$\overline{\text{XCIX}}$	= 99,000
XXIX	= 29	CXI	= 111	DCCCXI	= 811	$\overline{\text{M}}$	= 1,000,000
XXX	= 30	CXIX	= 119	DCCCC	= 900	$\overline{\text{MM}}$	= 2,000,000
XL	= 40	CLVI	= 156	DCCCCXL	= 940	$\overline{\text{MDCC}}$	= 1,700,000

21. Sometimes there are two ways of expressing numbers in Roman notation. For example, 1,909 may be indicated either by MDCCCXCIX or MCMIX; 459 may be expressed CDLIX or CCCCLIX; etc. A convenient use of Roman notation is in writing the day and the month, the latter being indicated by Roman figures; thus, 5th November would be expressed 5 XI. This plan prevents any misunderstanding of dates, as might happen in expressing them by Arabic figures; for instance, 5/8 might be read as 5th of August or 8th of May.

ADDITION

22. Addition is the *process of finding* a number that is equal to two or more numbers taken together. The sign of addition is +. It is read *plus*, and means *more*. Thus, 5 + 6 is read 5 *plus* 6, and means that 5 and 6 are to be added together.

Only the numbers of like things can be added. Thus, 6 pounds can be added to 7 pounds, and the sum will be 13 pounds, but 6 *pounds* cannot be added to 7 *feet*.

23. The sign of equality is =. It is read *equals*, or *is equal to*. Thus, 5 + 6 = 11 may be read, 5 *plus* 6 *equals* 11.

24. Table I gives the sum of any two numbers from 1 to 12 :

TABLE I

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 is 17.

25. Rule.—Place the numbers to be added directly under one another, taking care to place units under units, tens under tens, hundreds under hundreds, and so on.

26. EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

$$\begin{array}{r}
 \text{SOLUTION.} \quad \quad \quad \begin{array}{r}
 131 \\
 222 \\
 21 \\
 2 \\
 \hline
 413 \\
 \text{sum } 789 \text{ Ans.}
 \end{array}
 \end{array}$$

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the units column and add, mentally finding the different sums. Thus, three, five, six, eight, nine, the sum of the numbers in the units column. Place the 9 directly beneath, as the first or units figure in the sum.

The sum of the numbers in the tens column is 8, which is the second or tens figure in the sum.

The sum of the numbers in the hundreds column is 7, which is the third or hundreds figure in the sum.

27. EXAMPLE.—What is the sum of 61,803 + 43,429 + 47,712 + 62,138?

$$\begin{array}{r}
 \text{SOLUTION.} \quad \quad \quad \begin{array}{r}
 61803 \\
 43429 \\
 47712 \\
 62138 \\
 \hline
 22 \\
 60 \\
 2000 \\
 13000 \\
 200000 \\
 \text{sum } 215082 \text{ Ans.}
 \end{array}
 \end{array}$$

EXPLANATION.—The sum of the units column is 22 units; that is, 2 tens and 2 units. Write the unit 2 under the unit column and the other 2 under the tens column, as shown. The sum of the numbers in the tens column is 6 tens, or 60. Write 60 underneath 22, as shown. The sum of the numbers in the hundreds column is 20 hundreds, or 2,000. Write 2,000 under the two preceding sums, as shown. The sum of the numbers in the thousands column is 13 thousands, or 13,000, and of the numbers in the ten-thousands column 20 ten-thousands, or 200,000. Write the two latter sums, as shown. Adding these five results, the sum is 215,082.

inspection to be 24. Reducing each fraction to a fraction having a denominator of 24 (Art. 16), then

$$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}, \frac{3}{4} \times \frac{6}{6} = \frac{18}{24}, \frac{7}{8} \times \frac{3}{3} = \frac{21}{24}. \text{ Ans.}$$

34. Rule.—*Divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

COMPARISON OF FRACTIONS

35. Generally it is impossible to compare fractions without reducing them to a common denominator.

EXAMPLE.—Which is the greater, $\frac{4}{9}$ or $\frac{7}{11}$?

SOLUTION.—Reduce the fractions to the least common denominator, which is 99; then $\frac{4}{9} = \frac{44}{99}$, and $\frac{7}{11} = \frac{63}{99}$. And as $\frac{63}{99}$ is greater than $\frac{44}{99}$, $\frac{7}{11}$ is greater than $\frac{4}{9}$.

36. If the denominators of a series of fractions to be compared have a common factor, the fractions may be reduced so that the denominators of each shall be that common factor.

EXAMPLE.—Compare $\frac{4}{9}$, $\frac{17}{18}$, $\frac{22}{27}$.

SOLUTION.— $\frac{4}{9}$, $\frac{8\frac{1}{2}}{9}$, $\frac{7\frac{1}{3}}{9}$

EXPLANATION.—The factor common to 9, 18, 27 is obviously 9. Divide each denominator by 9 and 1, 2, 3 are the respective quotients. Now divide each fraction (that is, both numerator and denominator) by the respective quotient just obtained. Thus, $\frac{4}{9}$ divided by 1 remains $\frac{4}{9}$; $\frac{17}{18}$ divided by 2 becomes $\frac{8\frac{1}{2}}{9}$; $\frac{22}{27}$ divided by 3 becomes $\frac{7\frac{1}{3}}{9}$. By inspection it is seen that the order of magnitude of these fractions is $\frac{17}{18}$, $\frac{22}{27}$, $\frac{4}{9}$.

37. It is sometimes simpler to compare fractions by *converting the numerators to unity*.

EXAMPLE.—Compare $\frac{7}{34}$, $\frac{9}{64}$, $\frac{11}{92}$.

SOLUTION.—Divide the numerators and denominators of the fractions by their respective numerators, and the results are

$$\frac{1}{4\frac{1}{2}}, \frac{1}{7\frac{1}{3}}, \frac{1}{8\frac{1}{4}}$$

By inspection it is seen that the first fraction is greater than the second, and the second is greater than the third,

EXAMPLES FOR PRACTICE

38. Reduce to fractions having a common denominator :

$$\begin{array}{l}
 (a) \quad \frac{3}{4}, \frac{5}{8}, \frac{7}{8} \\
 (b) \quad \frac{3}{10}, \frac{3}{4}, \frac{7}{32} \\
 (c) \quad \frac{7}{8}, \frac{7}{88}, \frac{10}{11} \\
 (d) \quad \frac{3}{5}, \frac{5}{8}, \frac{11}{40} \\
 (e) \quad \frac{4}{10}, \frac{6}{40}, \frac{9}{20} \\
 (f) \quad \frac{7}{15}, \frac{17}{30}, \frac{21}{30}
 \end{array}
 \quad \text{Ans.} \quad \left\{ \begin{array}{l}
 (a) \quad \frac{3}{8}, \frac{5}{8}, \frac{7}{8} \\
 (b) \quad \frac{6}{32}, \frac{24}{32}, \frac{7}{32} \\
 (c) \quad \frac{7}{88}, \frac{7}{88}, \frac{80}{88} \\
 (d) \quad \frac{24}{40}, \frac{25}{40}, \frac{11}{40} \\
 (e) \quad \frac{16}{40}, \frac{6}{40}, \frac{18}{40} \\
 (f) \quad \frac{14}{30}, \frac{17}{30}, \frac{21}{30}
 \end{array} \right.$$

Reduce to fractions having as common denominator the greatest common factor :

$$\begin{array}{l}
 (g) \quad \frac{41}{80}, \frac{13}{20}, \frac{17}{40} \\
 (h) \quad \frac{17}{78}, \frac{11}{20}, \frac{1}{52}, \frac{7}{13}
 \end{array}
 \quad \text{Ans.} \quad \left\{ \begin{array}{l}
 (g) \quad \frac{10\frac{1}{4}}{20}, \frac{13}{20}, \frac{8\frac{1}{2}}{20} \\
 (h) \quad \frac{2\frac{5}{8}}{13}, \frac{5\frac{1}{2}}{13}, \frac{1}{13}, \frac{7}{13}
 \end{array} \right.$$

Reduce to fractions having unity as the numerator :

$$(i) \quad \frac{3}{104}, \frac{7}{200}, \frac{9}{312}, \frac{5}{97}
 \quad \text{Ans.} \quad \frac{1}{34\frac{2}{3}}, \frac{1}{28\frac{4}{7}}, \frac{1}{34\frac{2}{3}}, \frac{1}{19\frac{2}{5}}$$

ADDITION OF FRACTIONS

39. *Fractions cannot be added unless they have a common denominator.* One cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent different parts of a unit. Fourths can be added to fourths, but not to eighths.

If an apple is divided into 4 equal parts, and then 2 of these parts are divided into 2 equal parts, it is evident that there will be 2 one-fourths and 4 one-eighths. Now if these parts are added the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and there was only one apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the fourths to eighths, $\frac{2}{4} = \frac{4}{8}$; and, adding the other 4 eighths, $4 \text{ eighths} + 4 \text{ eighths} = 8 \text{ eighths}$. The result is correct, since $\frac{8}{8} = 1$. Or, in this case, the eighths may be reduced to fourths. Thus, $\frac{4}{8} = \frac{2}{4}$; whence, adding, $2 \text{ quarters} + 2 \text{ quarters} = 4 \text{ quarters}$, or fourths, which is a correct result, since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

40. EXAMPLE 1.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* that will exactly contain all the *denominators*, is 8.

$$\frac{1}{2} = \frac{4}{8}, \frac{3}{4} = \frac{6}{8}, \text{ and } \frac{4}{8} + \frac{6}{8} + \frac{5}{8} = \frac{4+6+5}{8} = \frac{15}{8} = 1\frac{7}{8}. \quad \text{Ans.}$$

EXPLANATION.—As the *denominator* indicates the names of the *parts*, only the *numerators* are added to obtain the total number of *parts* indicated by the *denominator*. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths = 15 one-eighths.

EXAMPLE 2.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{array}{r} 12\frac{3}{4} = 12\frac{12}{16} \\ 14\frac{5}{8} = 14\frac{10}{16} \\ 7\frac{5}{16} = 7\frac{5}{16} \\ \hline \text{sum } 33 + \frac{27}{16} = 33 + 1\frac{11}{16} = 34\frac{11}{16}. \quad \text{Ans.} \end{array}$$

The sum of the fractions = $\frac{27}{16}$, or $1\frac{11}{16}$, which, when added to the sum of the whole numbers, gives $34\frac{11}{16}$.

EXAMPLE 3.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{r} 17 \\ 13\frac{6}{32} \\ \frac{9}{32} \\ 3\frac{8}{32} \\ \hline \text{sum } 33\frac{23}{32}. \quad \text{Ans.} \end{array}$$

41. Rule.—I. Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.

II. When there are mixed numbers and whole numbers, add the integral parts and the fractional parts separately, and combine the results.

NOTE.—An *integral* part or number is one which does not contain a fraction. A whole number is an integer, or an integral number.

EXAMPLES FOR PRACTICE

42. Find the sum of:

- (a) $\frac{4}{9}$, $\frac{7}{24}$, $\frac{5}{8}$.
 (b) $\frac{2}{3}$, $\frac{15}{16}$, $\frac{24}{45}$.
 (c) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
 (d) $\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \quad 1\frac{7}{12} \\ (b) \quad 1\frac{8}{5} \\ (c) \quad 1\frac{3}{8} \\ (d) \quad 1\frac{7}{4} \end{array} \right.$$

Solve:

(e) $\frac{10}{11} + \frac{6}{33} + \frac{23}{36}$.

(f) $\frac{23}{45} + \frac{11}{15} + \frac{14}{45}$.

(g) $6\frac{4}{9} + 5\frac{5}{27} + 8\frac{7}{18}$.

(h) $\frac{3}{7} + \frac{14}{49} + \frac{2}{7}$.

$$\text{Ans. } \begin{cases} (e) & 1\frac{29}{36} \\ (f) & 1\frac{5}{9} \\ (g) & 20\frac{1}{54} \\ (h) & 1 \end{cases}$$

SUBTRACTION OF FRACTIONS

43. *Fractions cannot be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions. (Art. 39.)

EXAMPLE 1.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The least common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

EXAMPLE 2.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6 + 1$, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$, and $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

EXAMPLE 3.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The least common denominator of the fractions is 32.
 $17\frac{9}{16} = 17\frac{18}{32}$.

$$\begin{array}{r} \text{minuend} \quad 17\frac{18}{32} \\ \text{subtrahend} \quad 9\frac{15}{32} \\ \hline \text{difference} \quad 8\frac{3}{32} \quad \text{Ans.} \end{array}$$

EXAMPLE 4.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The least common denominator of the fractions is 16.
 $9\frac{1}{4} = 9\frac{4}{16}$.

$$\begin{array}{r} \text{minuend} \quad 9\frac{4}{16} \text{ or } 8\frac{20}{16} \\ \text{subtrahend} \quad 4\frac{7}{16} \\ \hline \text{remainder} \quad 4\frac{13}{16} \quad \text{Ans.} \end{array}$$

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore take 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was taken from 9, 8 remains; 4 from 8 = 4; $4 + \frac{13}{16} = 4\frac{13}{16}$.

EXAMPLE 5.—From 9 take $8\frac{3}{16}$.

$$\begin{array}{r} \text{minuend} \quad 9 \quad \text{or } 8\frac{16}{16} \\ \text{subtrahend} \quad 8\frac{3}{16} \\ \hline \text{remainder} \quad \frac{13}{16} \quad \text{Ans.} \end{array}$$

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, take 1, or $\frac{16}{16}$, from 9. $\frac{16}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was taken from 9, only 8 is left. 8 from 8 = 0.

44. Rule.—I. Reduce the given fractions to fractions having the least common denominator. Subtract one numerator from the other and place the remainder over the common denominator.

II. When there are mixed numbers, subtract the fractions and whole numbers separately.

III. When the fraction in the subtrahend is greater than the fraction in the minuend, take 1 from the whole number in the minuend, and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.

IV. When the minuend is a whole number, take 1 from it; reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and then subtract.

EXAMPLES FOR PRACTICE

45. Subtract:

$$\begin{array}{ll} (a) \frac{10}{24} \text{ from } \frac{11}{12}. & \text{Ans. } \left\{ \begin{array}{l} (a) \frac{1}{2} \\ (b) \frac{3}{8} \\ (c) \frac{11}{30} \\ (d) \frac{8}{14} \end{array} \right. \\ (b) \frac{7}{14} \text{ from } \frac{17}{28}. & \\ (c) \frac{4}{30} \text{ from } \frac{5}{10}. & \\ (d) \frac{15}{35} \text{ from } \frac{45}{70}. & \end{array}$$

Solve:

$$\begin{array}{ll} (e) \frac{57}{48} - \frac{15}{16}. & \text{Ans. } \left\{ \begin{array}{l} (e) \frac{1}{4} \\ (f) 17\frac{1}{4} \\ (g) 14\frac{7}{8} \\ (h) 24\frac{3}{4} \end{array} \right. \\ (f) 30\frac{1}{2} - 13\frac{1}{4}. & \\ (g) 27 - 12\frac{1}{8}. & \\ (h) 30 - 5\frac{1}{4}. & \end{array}$$

MULTIPLICATION OF FRACTIONS

46. In multiplication of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

47. Multiplying the numerator or dividing the denominator multiplies the fraction.

EXAMPLE 1.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3}{4} \times \frac{4}{1} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word “of” in multiplication of fractions means the same as $\times$, or times. Thus,

$$\begin{aligned} \frac{3}{4} \text{ of } 4 &= \frac{3}{4} \times 4 = 3 \\ \frac{1}{8} \text{ of } \frac{5}{16} &= \frac{1}{8} \times \frac{5}{16} = \frac{5}{128} \end{aligned}$$

EXAMPLE 2.—Multiply 2 by $\frac{3}{8}$.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. Ans.

Or, $2 \times \frac{3}{8} = \frac{3}{8} \div 2 = \frac{3}{4}$. Ans.

EXAMPLE 3.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancellation, $\frac{4 \times 7}{16 \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

EXAMPLE 4.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{4}{8} \times \frac{3}{4} \times \frac{16}{32} = \frac{3}{8 \times 4 \times 2} = \frac{3}{64}$. Ans.

EXAMPLE 5.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \text{ Ans.}$$

EXAMPLE 6.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—
$$\begin{array}{r} 15\frac{7}{8} \\ 3 \\ \hline 47\frac{5}{8} \end{array} \quad \begin{array}{r} 15\frac{7}{8} \\ 3 \\ \hline 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \end{array} \text{ Ans.}$$

48. Rule.—I. Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancellation.

II. To multiply one mixed number by another, reduce them both to improper fractions.

III. To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number, and add the whole-number part to the product of the multiplier and the whole number.

EXAMPLES FOR PRACTICE

49. Find the product of :

(a) $7 \times \frac{3}{19}$.

(b) $14 \times \frac{5}{16}$.

(c) $\frac{21}{32} \times \frac{5}{14}$.

(d) $\frac{16}{27} \times 4$.

(e) $\frac{19}{18} \times 7$.

(f) $17\frac{18}{21} \times 7$.

(g) $\frac{105}{224} \times 32$.

(h) $\frac{15}{28} \times 14$.

Ans. $\left\{ \begin{array}{l} (a) 1\frac{2}{19} \\ (b) 4\frac{3}{8} \\ (c) \frac{15}{64} \\ (d) 2\frac{10}{27} \\ (e) 7\frac{7}{18} \\ (f) 125 \\ (g) 15 \\ (h) 7\frac{1}{2} \end{array} \right.$

DIVISION OF FRACTIONS

50. In division of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

51. *Dividing the numerator or multiplying the denominator of a fraction, divides the fraction.*

EXAMPLE 1.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, then

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \text{ Ans.}$$

When *multiplying* the *denominator*, then

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}. \text{ Ans.}$$

EXAMPLE 2.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}. \text{ Ans.}$

EXAMPLE 3.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}. \text{ Ans.}$

52. To invert a fraction is to *turn it upside down*, that is, make the numerator and denominator change places.

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

53. EXAMPLE 1.—Divide $\frac{9}{16}$ by $\frac{3}{8}$.

SOLUTION.—1. The fraction $\frac{3}{8}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and the first numerator is contained in the second 3 times. 2. If now the divisor, $\frac{3}{8}$, is inverted and multiplication performed, the solution is

$$\frac{9}{16} \times \frac{8}{3} = \frac{3 \times \cancel{16}}{\cancel{16} \times 3} = 3. \text{ Ans.}$$

This gives the same quotient as in the first case.

EXAMPLE 2.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—As the denominators are not the same, it is not possible to divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above; therefore, the example must be solved as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}}{\cancel{8} \times 1} = \frac{3}{2} \text{ or } 1\frac{1}{2}. \text{ Ans.}$$

EXAMPLE 3.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{5 \times \cancel{16}}{\cancel{10} \times 2} = 8. \text{ Ans.}$$

EXAMPLE 4.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.—

$$3\frac{3}{4} = \frac{15}{4}; \quad 7\frac{7}{16} = \frac{119}{16}$$

$\frac{15}{4}$ inverted equals $\frac{4}{15}$

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}}{\cancel{15} \times 16} = \frac{119}{60} = 1\frac{59}{60}. \quad \text{Ans.}$$

54. Rule.—*Invert the divisor and proceed as in multiplication.*

EXAMPLES FOR PRACTICE

55. Divide :

(a) 15 by $6\frac{3}{7}$.

(b) 30 by $\frac{6}{8}$.

(c) 172 by $\frac{4}{5}$.

(d) $\frac{14}{18}$ by $1\frac{7}{16}$.

(e) $\frac{103}{0}$ by $14\frac{2}{3}$.

(f) $\frac{142}{27}$ by $17\frac{1}{5}$.

(g) $\frac{14}{18}$ by $1\frac{45}{72}$.

(h) $\frac{128}{18}$ by $72\frac{1}{3}$.

$$\text{Ans.} \left\{ \begin{array}{l} (a) \ 2\frac{1}{3} \\ (b) \ 40 \\ (c) \ 215 \\ (d) \ \frac{112}{207} \\ (e) \ \frac{115}{88} \\ (f) \ \frac{71}{281} \\ (g) \ \frac{56}{145} \\ (h) \ \frac{64}{651} \end{array} \right.$$

COMPLEX FRACTIONS

56. To reduce a complex fraction to its lowest terms, divide the numerator by the denominator, by inverting the denominator and multiplying, cancelling where possible.

EXAMPLE 1.—Reduce $\frac{\frac{3}{4}}{\frac{9}{16}}$ to its lowest terms.

$$\text{SOLUTION.—} \quad \frac{\frac{3}{4}}{\frac{9}{16}} = \frac{3}{4} \div \frac{9}{16} = \frac{3}{4} \times \frac{16}{9} = \frac{4}{3} = 1\frac{1}{3}. \quad \text{Ans.}$$

EXAMPLE 2.—Reduce $\frac{\frac{4}{5}}{7}$ to its lowest terms.

$$\text{SOLUTION.—} \quad \frac{\frac{4}{5}}{7} = \frac{4}{5} \div 7 = \frac{4}{5} \times \frac{1}{7} = \frac{4}{35}. \quad \text{Ans.}$$

57. In example 1 in Art. 56, the effect of inverting the fractional denominator and multiplying is to produce a simple fraction in which the denominators of the original fractional numerator and denominator are transposed, the one below the line, the other above it. Thus $\frac{\frac{3}{4}}{\frac{9}{16}}$ becomes $\frac{3 \times 16}{4 \times 9}$. Therefore, in reducing complex fractions to simple terms, transpose the denominators of the fractions contained in the complex fraction, bringing those above, to below the heavy line, and those below, to above it.

EXAMPLE.—Reduce $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$ to its lowest terms.

SOLUTION.—Reducing the mixed numbers to improper fractions, the fraction becomes $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Transposing denominators, and

cancelling, then $\frac{1 \times \overset{10}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{6}{\cancel{72}}}{\underset{2}{\cancel{40}} \times 9 \times \overset{31}{\cancel{31}} \times \underset{2}{\cancel{4}} \times \underset{2}{\cancel{4}}} = \frac{27}{2} = 13\frac{1}{2}$. Ans.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE

58. Simplify the following :

(a)	$\frac{4\frac{2}{3} \times 10\frac{2}{5} \times 26\frac{1}{4}}{8\frac{2}{5} \times 4\frac{7}{8} \times 12\frac{3}{5}}$	(b)	$\frac{10\frac{2}{3} \times 12\frac{4}{7} \times 15\frac{3}{4}}{4\frac{7}{8} \times \frac{24}{5} \times 8\frac{2}{5}}$	Ans. $\left\{ \begin{array}{l} (a) \ 2\frac{38}{81} \\ (b) \ 80 \\ (c) \ 6 \\ (d) \ \frac{5488}{8415} \end{array} \right.$
(c)	$\frac{20\frac{2}{5} \times 34\frac{2}{3} \times 8\frac{5}{8}}{14\frac{1}{6} \times 3\frac{9}{10} \times 18\frac{2}{5}}$	(d)	$\frac{15\frac{3}{4} \times 32\frac{2}{3} \times 25\frac{3}{5}}{28\frac{1}{5} \times 38\frac{1}{4} \times 17\frac{7}{9}}$	

ARITHMETIC

(PART 3)

DECIMALS

NOTATION AND NUMERATION

1. A decimal, or a decimal fraction, is a fraction whose denominator is 10, 100, 1,000, etc.

2. The denominator, which is always 10 or a power of 10 (as 100, 1,000, 10,000, etc.), is not expressed as in vulgar fractions, by writing it under the numerator, with a line between them; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure of the number. In order to distinguish the unit figure, a period (.), called the **decimal point**, is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, the decimal point serving merely to point out the unit figure.

3. The reading of a *decimal* depends upon the number of decimal places in it; i.e., upon the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths*.

The second figure to the right of the unit figure expresses *hundredths*.

The third figure to the right of the unit figure expresses *thousandths*.

The fourth figure to the right of the unit figure expresses *ten-thousandths*.

The fifth figure to the right of the unit figure expresses *hundred-thousandths*.

The sixth figure to the right of the unit figure expresses *millionths*.

Thus :

$$\begin{aligned}
 \cdot 3 &= \frac{3}{10} = 3 \text{ tenths.} \\
 \cdot 03 &= \frac{3}{100} = 3 \text{ hundredths.} \\
 \cdot 003 &= \frac{3}{1000} = 3 \text{ thousandths.} \\
 \cdot 0003 &= \frac{3}{10000} = 3 \text{ ten-thousandths.} \\
 \cdot 00003 &= \frac{3}{100000} = 3 \text{ hundred-thousandths.} \\
 \cdot 000003 &= \frac{3}{1000000} = 3 \text{ millionths.}
 \end{aligned}$$

The first figure to the right of the unit figure is called the *first decimal place*; the second figure, the *second decimal place*, etc. It is seen from the above that the number of decimal places in a decimal equals the *number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction*. This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. This part is called the *integral part*. The decimal point affects only the figures to its *right*.

When a whole number and decimal are written together, the expression is a *mixed number*. Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table :

9	hundreds of millions.	6	hundreds of thousands.	3	hundredths.	6	hundred-thousandths.
8	tens of millions.	5	tens of thousands.	2	tenths.	5	ten-thousandths.
7	millions.	4	thousands.	1	units.	4	thousandths.
6	hundreds of thousands.	3	hundreds.	0	decimal point.	3	ten-thousandths.
5	tens of thousands.	2	tens.	2	tenths.	2	hundred-thousandths.
4	thousands.	1	units.	1	units.	1	millionths.
3	hundreds.	0	decimal point.	0	decimal point.	0	ten-millionths.
2	tens.	9	hundreds of millions.	9	hundred-millionths.		

The figures to the left of the decimal point represent whole numbers ; those to the right are decimals.

In both the decimals and whole numbers the *units* place is made the starting point of notation and numeration. The *decimals decrease* on the scale of ten to the right, and the *whole numbers increase* on the scale of ten to the left. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, but the decimals have the ending *ths*, which distinguishes them from whole numbers. The following is the numeration of the number in the above table : nine hundred and eighty-seven million, six hundred and fifty-four thousand, three hundred and twenty-one, and twenty-three million, four hundred and fifty-six thousand, seven hundred and eighty-nine hundred-millionths.

The decimals increase to the left, on the scale of ten, the same as whole numbers ; for, beginning at, say, 4 thousandths, in the table, the next figure to the left is hundredths, which is ten times as great, and the next tenths, or ten times the hundredths, and so on through both decimals and whole numbers.

While the above numeration is correct, it is found laborious to read the decimals in that way, and in practice it is usual to read, for example, 23.3421, as twenty-three, decimal (or twenty-three, point) three, four, two, one.

PRINCIPLES OF DECIMALS

4. I. *The value of a decimal is not changed by annexing or rejecting a cipher at the right.*

Thus, $.5 = .50$. For $.5 = \frac{5}{10} = \frac{1}{2}$, and $.50 = \frac{50}{100} = \frac{1}{2}$.

II. *A decimal is divided by 10, 100, 1,000, . . . by moving the decimal point 1, 2, 3, . . . places respectively to the left. Ciphers are supposed to be at the left of the decimal point in case there are no digits.*

$$\text{Thus, } 4.2 \div 10 = 4\frac{2}{10} \div 10 = \frac{42}{10} \div 10 = \frac{42}{100} = .42$$

$$.5 \div 10 = \frac{5}{10} \div 10 = \frac{5}{100} = .05$$

$$.7 \div 100 = \frac{7}{10} \div 100 = \frac{7}{1000} = .007$$

III. *A decimal is multiplied by 10, 100, 1,000, . . . by moving the decimal point 1, 2, 3, . . . places respectively to the right.*

$$\text{Thus, } .05 \times 10 = \frac{5}{100} \times 10 = \frac{5}{10} = .5$$

$$.25 \times 10 = \frac{25}{100} \times 10 = 2\frac{5}{10} = 2.5$$

$$53.7 \times 1,000 = \frac{537}{10} \times 1,000 = 53,700$$

EXAMPLES FOR PRACTICE

5. Read the following numbers :

1. .00707.

5. .004001008.

2. .100707.

6. 75040.0742.

3. 114.75206.

7. 234.542000.

4. 40,032.890001.

8. 961,724.009.

Write the following numbers :

9. Five hundred and forty-six ten-millionths.

10. Three thousand and four, and three thousand four hundred and seventeen hundred-thousandths.

11. Five hundred and four, and three-tenths.

12. Nineteen thousand and thirteen, and one hundred and four thousand, five hundred and one millionths.

13. Six hundred and thirty-eight million, four hundred and twenty-five thousand, six hundred and seventy-two, and thirty-two million, six hundred and seventy-two thousand, five hundred and forty-five hundred-millionths.

ADDITION OF DECIMALS

6. To add decimals, they must be written so that the units of the same order are in the same column. That this may be, it is necessary only to see that the decimal points are in the same vertical column.

<i>whole numbers</i>		<i>decimals</i>		<i>mixed decimals</i>
342		.342		342.032
4234		.4234		4234.5
26		.26		26.6782
3		.03		3.06
sum 4605	Ans.	sum 1.0554	Ans.	sum 4606.2702
				Ans.

7. EXAMPLE.—What is the sum of 242, ·36, 118·725, 1·005, 6, and 100·1?

SOLUTION.—

$$\begin{array}{r}
 242 \\
 \cdot 36 \\
 118\cdot 725 \\
 1\cdot 005 \\
 6 \\
 \hline
 100\cdot 1 \\
 \hline
 \text{sum } 468\cdot 190 \quad \text{Ans.}
 \end{array}$$

8. Rule.—Place the numbers to be added so that the decimal points shall be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.

EXAMPLES FOR PRACTICE

9. Find the sum of :

- | | |
|--|--|
| (a) ·2143, ·105, 2·3042, and 1·1417. | Ans. { <ul style="list-style-type: none"> (a) 3·7652 (b) 805·9647 (c) 204·5078 (d) 225·9814 (e) 234·3285 (f) 1,437·7601 (g) 2,640·6143 (h) 4,539·02625 |
| (b) 783·5, 21·473, ·2101, and ·7816. | |
| (c) 21·781, 138·72, 41·8738, ·72, and 1·413. | |
| (d) ·3724, 104·15, 21·417, and 100·042. | |
| (e) 200·172, 14·105, 12·1465, ·705, and 7·2. | |
| (f) 1,427·16, ·244, ·32, ·032, and 10·0041. | |
| (g) 2,473·1, 41·65, ·7243, 104·067, and 21·073. | |
| (h) 4,107·2, ·00375, 21·716, 410·072, and ·0345. | |

SUBTRACTION OF DECIMALS

10. For the same reason as in addition of decimals, the numbers are placed so that the decimal points shall be in the same vertical column.

EXAMPLE 1.—Subtract ·132 from ·3063.

SOLUTION.—

$$\begin{array}{r}
 \text{minuend} \quad \cdot 3063 \\
 \text{subtrahend} \quad \cdot 132 \\
 \hline
 \text{difference} \quad \cdot 1743 \quad \text{Ans.}
 \end{array}$$

EXAMPLE 2.—What is the difference between 7·895 and ·725?

SOLUTION.—

$$\begin{array}{r}
 \text{minuend} \quad 7\cdot 895 \\
 \text{subtrahend} \quad \cdot 725 \\
 \hline
 \text{difference} \quad 7\cdot 170 \text{ or } 7\cdot 17 \quad \text{Ans.}
 \end{array}$$

EXAMPLE 3.—Subtract $\cdot 625$ from 11.

SOLUTION.—

<i>minuend</i>	11·000	
<i>subtrahend</i>	$\cdot 625$	
	$\underline{}$	
<i>difference</i>	10·375	Ans.

11. Rule.—Place the subtrahend under the minuend, so that the decimal points shall be in the same vertical column. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.

When there are more decimal places in the subtrahend than in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE

12. From :

(a) 407·385 take 235·0004.	Ans.	(a) 172·3846
(b) 22·718 take 1·7042.		(b) 21·0138
(c) 1,368·17 take 13·6817.		(c) 1,354·4883
(d) 70·00017 take 7·000017.		(d) 63·000153
(e) 630·630 take $\cdot 6304$.		(e) 629·9996
(f) 421·73 take 217·162.		(f) 204·568
(g) 1·000014 take $\cdot 00001$.		(g) 1·000004
(h) $\cdot 783652$ take $\cdot 542314$.		(h) $\cdot 241338$

MULTIPLICATION OF DECIMALS

13. In multiplication of decimals, no attention is paid for the time being to the decimal points. Write the multiplier under the multiplicand, so that the right-hand figure of the one is under the right-hand figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.

EXAMPLE.—Multiply $\cdot 825$ by 13.

SOLUTION.—

<i>multiplicand</i>	$\cdot 825$	
<i>multiplier</i>	13	
	$\underline{}$	
	2475	
	$\underline{}$	
	825	
<i>product</i>	10·725	Ans.

In this example there are three decimal places in the multiplicand and none in the multiplier; therefore, $3 + 0 = 3$ decimal places are pointed off in the product.

14. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

<i>multiplicand</i>	4 2 6
<i>multiplier</i>	.0 0 5
<i>product</i>	2.1 3 0 or 2.13, Ans.

In this example there are three decimal places in the multiplier and none in the multiplicand; therefore, three decimal places are pointed off in the product.

15. It is not necessary to multiply by the ciphers on the left of a decimal; they merely determine the number of decimal places. Ciphers to the right of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

EXAMPLE.—Multiply 1.205 by 1.15

SOLUTION.—

<i>multiplicand</i>	1.2 0 5
<i>multiplier</i>	1.1 5
	6 0 2 5
	1 2 0 5
<i>product</i>	1.3 8 5 7 5 Ans.

In this example there are three decimal places in the multiplicand, and two in the multiplier; therefore $3 + 2$, or five decimal places must be pointed off in the product.

16. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

<i>multiplicand</i>	.2 3 2
<i>multiplier</i>	.0 0 1
<i>product</i>	.0 0 0 2 3 2 Ans.

In this example the multiplicand is multiplied by the digit in the multiplier, which makes 232 in the product; but since there are three decimal places in the multiplier and three in the multiplicand, three ciphers must be prefixed to the 232 to make $3 + 3$, or six decimal places in the product.

17. Rule.—Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole

numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.

EXAMPLES FOR PRACTICE

18. Find the product of :

(a) $\cdot 000492 \times 4 \cdot 1418$.

(b) $4,003 \cdot 2 \times 1 \cdot 2$.

(c) $78 \cdot 6531 \times 1 \cdot 03$.

(d) $\cdot 3685 \times \cdot 042$.

(e) $178,352 \times \cdot 01$.

(f) $\cdot 00045 \times \cdot 0045$.

(g) $\cdot 714 \times \cdot 00002$.

(h) $\cdot 00004 \times \cdot 008$.

(i) $2 \cdot 43 \times 1,000$.

(j) 463×100 .

Ans. { (a) $\cdot 0020377656$
 (b) $4,803 \cdot 84$
 (c) $81 \cdot 012693$
 (d) $\cdot 015477$
 (e) $1,783 \cdot 52$
 (f) $\cdot 000002025$
 (g) $\cdot 00001428$
 (h) $\cdot 00000032$
 (i) $2,430$
 (j) $46,300$

DIVISION OF DECIMALS

19. In division of decimals, no attention is paid to the decimal point until after the division has been performed. The number of decimal places in the dividend must equal (or be made to equal by annexing ciphers) the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.

EXAMPLE 1.—Divide $\cdot 625$ by 25 .

	divisor	dividend	quotient	
SOLUTION.—	25	$\cdot 625$	$(\cdot 025$	Ans.
		50		
		$\underline{125}$		
		125		
		$\underline{0}$		
	remainder			

In this example there are no decimal places in the divisor, and three decimal places in the dividend ; therefore, there are $3 - 0 = 3$ decimal places in the quotient. One cipher has to be prefixed to the 25 to make the three decimal places.

EXAMPLE 2.—Divide $6 \cdot 035$ by $\cdot 05$.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
·0 5)	6·0 3 5	(1 2 0·7	Ans.
	<u>5</u>		
	1 0		
	<u>1 0</u>		
		3 5	
		<u>3 5</u>	
			<i>remainder</i> 0

In this example 5 is used as divisor, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor ; therefore, one decimal place is pointed off in the quotient.

EXAMPLE 3.—Divide ·125 by ·005.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
·0 0 5)	·1 2 5	(2 5	Ans.
	<u>1 0</u>		
		2 5	
	<u>2 5</u>		
			<i>remainder</i> 0

In this example there are the same number of decimal places in the dividend as in the divisor ; therefore, the quotient has no decimal places, and is a whole number.

EXAMPLE 4.—Divide 326 by ·25.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
·2 5)	3 2 6·0 0	(1 3 0 4	Ans.
	<u>2 5</u>		
		7 6	
	<u>7 5</u>		
		1 0 0	
	<u>1 0 0</u>		
			<i>remainder</i> 0

In this example two ciphers were annexed to the dividend to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

EXAMPLE 5.—Divide ·0025 by 1·25.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
1·2 5)	·0 0 2 5 0	(·0 0 2	Ans.
	<u>2 5 0</u>		
			<i>remainder</i> 0

EXPLANATION.—In this example, ·0025 is to be divided by 1·25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present) ; also, consider the divisor as a whole number, or 125. It is evident that the dividend 25 will not contain the divisor 125 ; one cipher must therefore be annexed to the 25, thus making the dividend 250. 125 is contained twice in 250, so the figure 2 is placed

in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making $4 + 1 = 5$ decimal places. Since there are five decimal places in the dividend and two decimal places in the divisor, there must be pointed off $5 - 2 = 3$ decimal places in the quotient. In order to point off three decimal places, two ciphers must be prefixed to the figure 2, thereby making $\cdot 002$ the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

20. Rule.—I. *Place the divisor to the left of the dividend and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder and continue dividing until there are three or four decimal places in the quotient, and then, if there still is a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried farther.*

EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.—

divisor	dividend	quotient	
15	199	$(13 + \frac{4}{15})$	Ans.

15

49

45

remainder 4

or 15) 199·000 (13·266 + Ans.

15

49

45

40

30

100

90

100

90

remainder 10

$13\frac{4}{15} = 13\cdot 266 +$

21. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide how many decimal places are desired in the quotient and then carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign ($-$), thus indicating that the quotient is not quite so great as indicated; if the figure thus obtained is less than 5, write the plus sign ($+$) after the quotient, thus indicating that the number is slightly greater than indicated. In dividing 199 by 15, were it desired to obtain the answer correct to four decimal places, the work would be carried to five places, obtaining 13.26666, and the answer would be given as 13.2667—. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it were desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it were desired to retain five decimal places, it would be expressed as .24713—. Both the + and — signs are frequently omitted; they are seldom used in this connection outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE

22. Divide :

(a) 101.6688 by 2.36.

(b) 187.12264 by 123.107.

(c) .08 by .008.

(d) .0003 by 3.75.

(e) .0144 by .024.

(f) .00375 by 1.25.

(g) .004 by 400.

(h) .4 by .008.

(i) 2.43 by 1000.

(j) 463 by 100.

Ans. { (a) 43.08
(b) 1.52
(c) 10
(d) .00008
(e) .6
(f) .003
(g) .00001
(h) 50
(i) .00243
(j) 4.63

TO REDUCE A VULGAR FRACTION TO A DECIMAL

23. EXAMPLE 1.—Reduce $\frac{3}{4}$ to a decimal.

SOLUTION.— $4 \overline{) 3.00}$ or $\frac{3}{4} = .75$. Ans.

EXAMPLE 2.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.— $8 \overline{) 7.000}$ (.875
 $\begin{array}{r} 64 \\ \underline{60} \\ 56 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$ or $\frac{7}{8} = .875$. Ans.

24. Rule.—Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.

EXAMPLES FOR PRACTICE

25. Reduce the following vulgar fractions to decimals :

(a) $\frac{15}{32}$.	Ans. {	(a) .46875
(b) $\frac{7}{8}$.		(b) .875
(c) $\frac{21}{32}$.		(c) .65625
(d) $\frac{51}{64}$.		(d) .796875
(e) $\frac{4}{25}$.		(e) .16
(f) $\frac{5}{8}$.		(f) .625
(g) $\frac{10}{200}$.		(g) .05
(h) $\frac{4}{1000}$.		(h) .004

26. To reduce inches to decimal parts of a foot.

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in 1 foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12} = \frac{9}{12}$ of a foot. This, reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is,

$12 \overline{) 9.00}$ (.75 foot. Ans.
 $\begin{array}{r} 84 \\ \underline{60} \\ 60 \\ \underline{60} \\ 0 \end{array}$

27. Rule.—I. To reduce inches to decimal parts of a foot, divide the number of inches by 12.

II. Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater increase the fourth figure by 1. Omit the signs + and -.

EXAMPLES FOR PRACTICE

28. Reduce to the decimal part of a foot :

- | | | |
|----------------------------|--------|----------------|
| (a) 3 inches. | Ans. { | (a) .25 foot |
| (b) $4\frac{1}{2}$ inches. | | (b) .375 foot |
| (c) 5 inches. | | (c) .4167 foot |
| (d) $6\frac{5}{8}$ inches. | | (d) .5521 foot |
| (e) 11 inches. | | (e) .9167 foot |

TO REDUCE A DECIMAL TO A VULGAR FRACTION

29. EXAMPLE 1.—Reduce .125 to a vulgar fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$.

EXAMPLE 2.—Reduce .875 to a vulgar fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

30. Rule.—Under the figures of the decimal, place 1 followed by as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms.

EXAMPLES FOR PRACTICE

31. Reduce the following to vulgar fractions :

- | | | |
|-------------|--------|--------------------|
| (a) .125. | Ans. { | (a) $\frac{1}{8}$ |
| (b) .625. | | (b) $\frac{5}{8}$ |
| (c) .3125. | | (c) $\frac{5}{16}$ |
| (d) .04. | | (d) $\frac{1}{25}$ |
| (e) .06. | | (e) $\frac{3}{50}$ |
| (f) .75. | | (f) $\frac{3}{4}$ |
| (g) .15625. | | (g) $\frac{5}{32}$ |
| (h) .875. | | (h) $\frac{7}{8}$ |

32. To express a decimal approximately as a vulgar fraction having a given denominator.

EXAMPLE 1.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE 2.—Express $\cdot 3917$ in 12ths.

SOLUTION.— $\cdot 3917 \times \frac{12}{12} = \frac{4 \cdot 7004}{12}$, say $\frac{5}{12}$.

Hence, $\cdot 3917 = \frac{5}{12}$, nearly. Ans.

33. Rule.—Write down a fraction with the given denominator and having the same number for its numerator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.

EXAMPLES FOR PRACTICE

34. Express :

- (a) $\cdot 625$ in 8ths.
- (b) $\cdot 3125$ in 16ths.
- (c) $\cdot 15625$ in 32nds.
- (d) $\cdot 77$ in 64ths.
- (e) $\cdot 81$ in 48ths.
- (f) $\cdot 923$ in 96ths.

$$\text{Ans.} \left\{ \begin{array}{l} (a) \frac{5}{8} \\ (b) \frac{5}{16} \\ (c) \frac{5}{32} \\ (d) \frac{49}{64} \\ (e) \frac{39}{48} \\ (f) \frac{89}{96} \end{array} \right.$$

SYMBOLS OF AGGREGATION

35. The vinculum —, and (), [], and { } are the various forms of brackets, called **symbols of aggregation**, and are used to include numbers that are to be considered together ; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$ shows that 13 is to be multiplied by the difference between 8 and 3.

$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$

$$13 \times 8 - 3 = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or any other form of bracket is not used, then

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

36. Rule.—In simplifying an expression containing a bracket, the operations indicated inside the bracket should be performed first ; then, when the quantity in the bracket has been simplified to a single term, the operations indicated outside the bracket may be performed.

37. In any series of numbers connected by the signs +, —, $\times$, and $\div$, the operations indicated by the signs must

be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number, located on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. When signs of multiplication and division follow each other, it is customary to perform the indicated operations in order from left to right before adding or subtracting.

EXAMPLE 1.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

EXAMPLE 2.—What is the value of the following expression: $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1,296 \div 12 = 108$; $108 + 160 = 268$; here 22 cannot be subtracted from 268 because the sign of multiplication *follows* 22; hence, multiplying 22 by $3\frac{1}{2}$, the product is 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to divide $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been found in the following manner: $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* of the multiplication or division included between the signs $+$ and $-$, or $-$ and $+$, before adding or subtracting.

It likewise follows that if a succession of multiplication and division signs occur, the indicated operations must be performed in order, from left to right. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = 20$. Ans.

EXAMPLES FOR PRACTICE

38. Find the values of the following expressions:

(a) $(8 + 5 - 1) \div 4$,

(b) $5 \times 24 - 32$,

(c) $5 \times 24 \div 15$.

(d) $144 - 5 \times 24$.

(e) $(1,691 - 540 + 559) \div 3 \times 57$.

(f) $2,080 + 120 - 80 \times 4 - 1,670$,

(g) $(90 + 60 \div 25) \times 5 - 29$.

(h) $9 + 6 \times 25 \div 5$.

Ans., $\left\{ \begin{array}{l} (a) \ 3 \\ (b) \ 88 \\ (c) \ 8 \\ (d) \ 24 \\ (e) \ 32,490 \\ (f) \ 210 \\ (g) \ 433 \\ (h) \ 75 \end{array} \right.$

APPROXIMATE VALUES

39. Significant Figures.—In any number, the figures beginning with the first digit at the left and ending with the last digit at the right, are called the **significant figures** of the number. A cipher is not a digit, but it is counted as such in case it is enclosed by digits. Thus, the number 405,800 has four significant figures 4, 0, 5, 8, the cipher between 4 and 5 being counted as a significant figure. The cipher or ciphers at the right side of the decimal point and preceding the first digit are not counted as significant figures. Thus, the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

That part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

40. The following rule will be found of value in multiplication and division, when it is desired to have the result limited to a certain number of significant figures :

In multiplying or dividing one number by another, it is unnecessary to use more significant figures in any of the numbers operated on than the desired number of significant figures in the result, plus 1.

For example, if only four significant figures are desired in the final result, all the numbers used in the various operations may be limited to $4 + 1 = 5$ significant figures, the fifth figure being increased by 1 in all cases, if the sixth figure is 5 or any greater digit.

The effect of having one more significant figure in the number operated on than the number of significant figures required in the result is seen from the following :

For example, $4,562,347 \times 6,421,849 = 29,298,703,519,603$, or 29,299,000,000,000 to five significant figures; 4,562,300

$\times 6,421,800 = 29,298,178,140,000 = 29,298,000,000,000$ to five significant figures, the fifth figure being 1 less than it should be but $4,562,350 \times 6,421,850 = 29,298,727,347,500 = 29,299,000,000,000$ to five significant figures.

The following is an example in division :

$6,421,849 \div 4,562,357 = 1.407572 + = 1.4076$ to five significant figures ; also, $642,185 \div 456,236 = 1.407571 + = 1.4076$ to five significant figures.

41. Accuracy of Calculations.—It should always be remembered that in order that the result of a calculation may be correct, it is absolutely necessary that the data forming the basis of the calculation should possess the same degree of accuracy as that of the result. That is to say, if a given dimension has three decimals, it is useless to give the result of the calculation in a number containing, say, nine decimals. No amount of calculation can change the given number into one of greater accuracy ; that is, into one equivalent to nine decimals.

This will be seen from the following example. The diameter of a roller is given as $.723$ inch and it is desirable to calculate its circumference. This is done by multiplying the diameter by a certain number, viz., 3.1415927 . The number of decimals retained in this number depends on the accuracy required ; ordinarily it is given as 3.14 . Using in this case all of the seven decimals, then $.723 \times 3.1415927 = 2.2713715221$ inches, which is the circumference of the roller. After a little consideration, it is evident that the number indicating the circumference cannot be more accurate than the number indicating the diameter. Hence, as the latter contains but three decimals, anything beyond three decimals in the answer will be useless ; the last seven decimals may therefore be omitted and the practical value of the circumference will be 2.271 .

42. Approximate Correctness of Values.—A value is *approximately* correct when it is as near the true value as is required for practical purposes. The purpose of an approximation is to express by means of a *given* number of figures a value that shall be nearer the true value than that expressed by a *similar*

number of other figures. Limiting the result of a calculation to a given number of significant figures produces a result that is only approximately correct.

It is in calculations that aim only at a given number of significant figures that certain arithmetical methods, such as contracted multiplication and division, are particularly applicable. They not only save a great amount of work, but tend also to check the indiscriminate use of decimals.

NOTE.—The study of Arts. 42 to 47, inclusive, is optional; that is, they may be omitted if desired.

CONTRACTED METHODS

MULTIPLICATION

43. In practical work it is often preferable to make abbreviated calculations that will give only a specified number of significant figures in a product or in a quotient.

In these cases, where only part of a product is to be used, it is a waste of labour to multiply the whole of the multiplicand by each figure in the multiplier. Methods, according to which only a partial multiplication is performed, are called **contracted multiplication**. There are several of these methods, but only the following will be given. The method is best explained by an example.

EXAMPLE.—Multiply 30.4076 by 21.314, giving the product correct to three decimal places.

SOLUTION.—

<i>Contracted Method</i>	<i>Ordinary Method</i>
30.4076	30.4076
41312	21.314
608152	1216304
30408	304076
9122	912228
304	304076
122	608152
648.108 Ans.	648.1075864 Ans.

EXPLANATION.—Write the units figure of the multiplier under the decimal place in the multiplicand to which it is desired to carry the

product (in this case the third decimal place, because it is required to carry the product to three decimal places). Then write the other figures of the multiplier on either side of the units figure in the *reverse order* to that in which they are given in the example. Thus, 21,314 is written 41312. Multiply by 2 in the usual way. Then in multiplying by 1, begin with the figure immediately above it, but bring forward the last carried figure from the part that has been dropped. If the multiplication of the last dropped figure gives a product that is equal to or greater than 5, count it as 10. Thus, $6 \times 1 = 6$, and as 6 is nearer to 10 than to 1, count it as 10, and so carry 1. $7 \times 1 = 7$, and $7 + 1 = 8$. Write 8 under the right-hand figure of the line above, and proceed as in ordinary multiplication. For the third line, the 7 and 6 in the multiplicand are ignored, except to see what figure to carry forward. $7 \times 3 = 21$; therefore, carry 2; $0 \times 3 = 0$, and $0 + 2 = 2$; write 2 and proceed. Similarly, in the fourth line $4 \times 1 = 4$, and as there is no figure to bring forward, write 4 and proceed. In the fifth line, $4 \times 4 = 16$, and as 16 is nearer to 20 than to 10, carry 2. $0 \times 4 = 0$, and $0 + 2 = 2$. Write 2 and proceed. Adding up, and placing the decimal point three places to the left, the product is correct to three decimal places.

44. If the multiplier is a decimal, a cipher must be written for the unit figure. If the right-hand figure of the multiplier, when written in the reverse order, extends beyond the right-hand figure of the multiplicand, extend the multiplicand to the right-hand figure of the multiplier by annexing ciphers. The two examples which follow illustrate these points.

EXAMPLE 1.—Multiply 53.64 by .7854, obtaining the result correct to three decimal places.

SOLUTION.—

$$\begin{array}{r}
 53.640 \\
 45870 \\
 \hline
 0 \\
 37548 \\
 4291 \\
 268 \\
 21 \\
 \hline
 42.128 \text{ Ans.}
 \end{array}$$

EXPLANATION.—Write a cipher for the unit figure of the multiplier. $53640 \times 0 = 0$. The rest of the solution should be readily understood.

EXAMPLE 2.—Multiply 4,530.62 by 1,724.53, obtaining the product correct to one decimal place.

SOLUTION.—

$$\begin{array}{r}
 4530\cdot62 \\
 \underline{354271} \quad \text{or} \quad \underline{45306200} \\
 354271 \\
 45306200 \\
 31714340 \\
 906124 \\
 181225 \\
 22653 \\
 \underline{1359} \\
 7813190\cdot1 \quad \text{Ans.}
 \end{array}$$

EXPLANATION.—As the right-hand figure of the reversed multiplier extends two places beyond the right-hand figure of the multiplicand, add two ciphers to the decimal part of the multiplicand. This does not affect the value of the quantity. Multiply as before explained and place the decimal point in the product one place to the left, as required by the terms of the example. Care must always be taken to write the units figure of the multiplier under the decimal place of the multiplicand to which it is desired to carry the product.

EXAMPLES FOR PRACTICE

45. Find the product of :

- | | |
|--|---|
| (a) $3\cdot105 \times \cdot241$ (correct to three decimal places). | $\left. \begin{array}{l} (a) \cdot748 \\ (b) 33\cdot540 \\ (c) \cdot0172 \\ (d) 23393\cdot4570 \\ (e) \cdot14881 \\ (f) \cdot001153 \end{array} \right\} \text{Ans.}$ |
| (b) $33\cdot47 \times 1\cdot0021$ (correct to three decimal places). | |
| (c) $\cdot05678 \times \cdot303$ (correct to four decimal places). | |
| (d) $63\cdot012 \times 371\cdot254$ | |
| (e) $\cdot667321 \times \cdot223$ (correct to five decimal places). | |
| (f) $\cdot534 \times \cdot00216$ (correct to six decimal places). | |

DIVISION

46. The method of contracted division will be best understood by considering the following examples :

EXAMPLE.—Find the quotient of $230,896\cdot735 \div 5,236\cdot5104$ to five significant figures.

SOLUTION.—In contracted division it is always desirable to exclude the decimal points in dividend and divisor by multiplying each by 10, 100, 1,000, ... as the case requires, and as explained in Art. 4. But, as shown in *Arithmetic*, Part 2, both terms must be multiplied by the same number, in this case 10,000. Hence $230,896\cdot735 \div 5,236\cdot5104 = 2,308,967,350$

$\div 52,365,104$. Next limit both dividend and divisor to $5 + 1 = 6$ significant figures, obtaining $230,897 \div 523,651$, and proceed as in long division.

$$\begin{array}{r}
 230897 \cdot 0 \quad (5'2'3'6'5'1 \\
 \underline{214366} \quad 440937 \\
 4906 \\
 \underline{193} \\
 36
 \end{array}$$

EXPLANATION.—The divisor being greater than the dividend, annex a cipher to the latter and divide. The first figure of the quotient is 4. Multiply the divisor by 4 and subtract the result from the dividend. In this example the subtrahends have been omitted. Instead of annexing a cipher to the remainder and dividing by the entire divisor, the right-hand figure of the divisor is cut off, and the divisor thus shortened is used. 52365 goes into 214366 four times. Multiply the shortened divisor by 4, the second figure of the quotient, as in contracted multiplication, taking care to add the figure that would have been brought forward had the complete divisor been multiplied. In this case there is nothing to bring forward, as 4×1 (the figure eliminated) = 4, less than 5. Completing the operation, the new remainder is 4906. The second figure from the right of the divisor is now cut off, leaving 5236, and since this is larger than 4906, the third figure of the quotient is 0. Another figure is cut off from the divisor, leaving 523, which is contained in 4906 nine times; hence, the fourth figure of the quotient is 9. While 6 (the last eliminated figure in the divisor) $\times 9 = 54$, it is seen that 6·5 (the last two eliminated figures) $\times 9 =$ a number greater than 55; hence, add 6 to the product of 3 by 9. Cutting off another figure the divisor reduces to 52, and the fifth figure of the quotient is 3. Cutting off one more figure the sixth figure of the quotient is 7. Therefore, the first five significant figures of the quotient are 44094. To locate the decimal point, write the divisor under the dividend with the left-hand digits in the same column; thus, $230896 \cdot 735$. Since, however, the first eight figures of the dividend

above the eight figures of the divisor constitute a smaller number than the divisor (disregarding the position of the decimal point), move the divisor one place to the right, obtaining $\begin{array}{r|l} 2308 & 96 \cdot 735 \\ 523 & 6 \cdot 5104 \end{array}$. Then draw

a vertical line between the units and tens figures of the *divisor*, and count the number of figures in the *dividend* from this vertical line to the decimal point. If the decimal point lies to the *right* of the line, the number of figures so counted will be the number of integral places in the quotient; if the decimal point lies to the *left* of the line, the number so counted will be the number of ciphers between the decimal point and the first digit of the quotient, which will be entirely decimal; if the line passes through

the decimal point the quotient will be entirely decimal, and the first digit will express *tenths*. In the present case there are two figures between the line and the decimal point, which lies to the *right*; hence, there are two integral places, and the quotient is 44·094—. Ans.

47. Had the dividend been 2308·96735 the line would have passed through the decimal point and the quotient would have been 44094—. Had the dividend been 23·0896735, the line would have passed two figures to the right of the decimal point, and the quotient would have been 4044094—.

EXAMPLE 1.—Divide 710·49 by 3·1416 and obtain the result correct to four decimal places.

$$\begin{array}{r}
 \text{SOLUTION.} \quad 710\cdot49 \div 3\cdot1416 = 7,104,900 \div 31,416. \\
 \begin{array}{r}
 710\cdot49 \quad 7104900\cdot0 \text{ (3'1'4'1'6'0'0'0)} \\
 3\cdot1416 \quad \underline{8217000} \quad 226\cdot15546 \text{ or } 226\cdot1555\text{—. Ans.} \\
 \quad \quad \quad 1933800 \\
 \quad \quad \quad \underline{48840} \\
 \quad \quad \quad 17424 \\
 \quad \quad \quad \underline{1716} \\
 \quad \quad \quad 145 \\
 \quad \quad \quad \underline{19}
 \end{array}
 \end{array}$$

EXPLANATION.—First determine how many significant figures there will be in the result. For this purpose it is necessary to ascertain the number of integral places in the quotient by means of the rule given in the preceding example, and as shown at the left of the solution in the present example. It is seen that the number of integral places in the quotient is 3. Since there are to be four decimal places in the quotient and the work should be carried one place farther, the quotient will contain $3 + 4 + 1 = 8$ significant figures. Make the divisor contain 8 figures, either by cutting off superfluous figures or by adding ciphers, as in this case. Add ciphers after the decimal point in the dividend, if more figures are required, as in the present instance. Then divide as in the last example.

EXAMPLE 2.— $710\cdot49 \div 31,416 = ?$ to four decimal places.

SOLUTION.—

$$\begin{array}{r}
 710\cdot4 \mid 9 \quad 7104\cdot9 \text{ (3'1'4'1'6)} \\
 3141 \mid 6 \quad \underline{822} \quad \cdot02262 \text{ or } \cdot0226\text{+}. \text{ Ans.} \\
 \quad \quad \quad 194 \\
 \quad \quad \quad \underline{6}
 \end{array}$$

EXPLANATION.—As seen from the arrangement of the numbers at the left of the solution, there will be one cipher between the decimal point and the first digit. The work should be carried to the fifth decimal place

to make the fourth correct; consequently, there will be $5 - 1 = 4$ significant figures in the quotient. Cutting off one figure each in the divisor and dividend to reduce them to four figures, proceed as shown.

EXAMPLE 3.—Divide $\cdot 07854$ by 237 and find the quotient correct to five decimal places.

SOLUTION.—

$$\begin{array}{r} \cdot 078 \overline{) 54} \\ 23 \overline{) 7} \end{array} \qquad \begin{array}{r} \cdot 07854 \overline{) 237} \\ \underline{74} \\ 3 \end{array} \quad \cdot 000331 \text{ or } \cdot 00033+. \text{ Ans.}$$

EXPLANATION.—There will be three ciphers between the decimal point and the first digit of the quotient. Since the work is to be carried to $5 + 1 = 6$ decimal places, there will be $6 - 3 = 3$ integral places in the quotient; hence the last digit in the dividend may be omitted, as shown.

EXAMPLE 4.—Divide $710\cdot 49$ by $\cdot 007854$ and find the quotient correct to two decimal places.

SOLUTION.—

$$\begin{array}{r} 0071049 \\ 0007854 \end{array} \qquad \begin{array}{r} 710490000 \overline{) 0078540000} \\ \underline{3630000} \\ 488400 \\ \underline{17160} \\ 1452 \\ \underline{667} \\ 39 \end{array} \quad \begin{array}{l} 90462\cdot 184 \\ \text{or } 90462\cdot 18+. \text{ Ans.} \end{array}$$

EXPLANATION.—Since there are five figures between the line and the decimal point, there are five integral places in the quotient. The number of significant figures in the result is then $5 + 2 = 7$, and the work is carried to $7 + 1 = 8$ figures. Annex four ciphers to the divisor so that it will contain eight figures, not counting the two ciphers on the left, and four ciphers to the dividend so that it will contain the divisor, and proceed as shown.

EXAMPLES FOR PRACTICE

48. Divide :

- (a) $364\cdot 271$ by $42\cdot 321$ to three places of decimals.
 (b) $42\cdot 001$ by $364\cdot 5$ to two places of decimals.
 (c) $571\cdot 32$ by $\cdot 0631$ to four places of decimals.
 (d) $\cdot 3415$ by 626 to six places of decimals.

$$\text{Ans.} \left\{ \begin{array}{l} (a) 8\cdot 607 + \\ (b) \cdot 12 - \\ (c) 9054\cdot 1997 - \\ (d) \cdot 000546 - \end{array} \right.$$

ARITHMETIC

(PART 4)

COMPOUND EXPRESSIONS

1. In expressing arithmetical quantities, whether of extension, capacity, weight, money, etc. (other than those which are expressed by the decimal system), use is made of various units. For instance, the distance from London to Manchester is expressed in *miles*, while the length of a room is expressed in *feet*; again, the weight of a locomotive is expressed in *tons*, while the weight of tobacco is expressed in *pounds*.

2. In each case these units have a distinct relation to one another. The exact number of each unit contained in one of the next higher units, as well as the exact dimensions of one of the units, have been defined by law or custom.

3. A **principal unit** is a unit which is defined without reference to other units.

4. A **secondary, or derived, unit** is a unit which is defined by reference to, or derived from, a principal unit.

5. Whenever one unit only is used in denoting a quantity, the expression is called a **simple expression**. When two or more units are used, the expression is called a **compound expression**. Thus, *8 men*, *12 yards*, *14 pounds* are simple expressions; while *3 miles 2 furlongs 1 pole 3 yards*, *£10 4s. 8d.*, *3 gallons 3 quarts 1 pint* are compound expressions.

6. Compound expressions form a very important section of arithmetic. It is therefore necessary to obtain a clear idea of their use, and to be able to add, subtract, multiply, and divide them. The only real difficulty that arises is the memorizing

of the tables. The best way to do this is to read them over very carefully several times, and then, after carefully studying reduction, addition, etc., work the examples (all of them), constantly referring to the tables for help. But, before leaving the subject to take up the next section, *all* of the tables should be thoroughly memorized.

7. The measurement of quantities may be divided into six classes, namely .

- | | |
|--|------------|
| 1. Money. | 4. Weight. |
| 2. Extension (length and area). | 5. Time. |
| 3. Volume (solid, fluid, and dry measure). | 6. Angles. |

They will be considered in the above order.

BRITISH MONEY

4 farthings (<i>far.</i>)	—	1 penny	<i>d.</i>
12 pence	=	1 shilling	<i>s.</i>
20 shillings	=	1 pound, or sovereign	<i>£</i>
far. d. s. £			
4 = 1			
48 = 12 = 1			
960 = 240 = 20 = 1			

8. The other coins in use in Great Britain are :

	s.	d.		s.	d.
Halfpenny (bronze)	0	0½	Half-crown (silver)	2	6
Threepenny piece (silver)	0	3	Double florin (silver)	4	0
Sixpenny piece (silver)	0	6	Crown (silver)	5	0
Florin (silver)	2	0	Half-sovereign (gold)	10	0

9. The unit of British money is the **pound**, frequently spoken of as a **pound sterling**, to distinguish it from the unit of weight. Another monetary unit frequently used is the **guinea** = 21 shillings, but it is no longer coined. The letters *£ s. d.* are the initials of the words *Libra*, *solidus*, and *denarius*, names of Roman coins corresponding respectively to the pound, shilling, and penny.

10. It is required by Act of Parliament that in British coinage the gold used shall be 22-carat gold (that is, 22 parts pure gold

in every 24 parts of alloy), or $\frac{11}{12}$ fine, and that 40 pounds troy shall be coined in 1,869 sovereigns or twice that number of half-sovereigns; that the silver used shall be $\frac{27}{10}$ fine, and that 1 pound troy shall be coined into 66 shillings, or a proportionate number of other silver coins; and that the bronze used shall consist of 95 per cent. copper, 4 per cent. tin, and 1 per cent. zinc, and that 3 pennies or 5 halfpennies or 10 farthings shall weigh 1 ounce avoirdupois.

MEASURES OF EXTENSION

11. Measures of extension are used in measuring lengths (distances) and surfaces (areas) and are divided, accordingly, into *linear measure* and *square measure*.

LINEAR MEASURE (OR MEASURES OF LENGTH)

12. The *principal unit*, or standard, to which all measures of extension are referred is the **yard**, which in Great Britain an Act of Parliament defines as the distance at 62 degrees Fahrenheit between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the Office of the Exchequer.

LINEAR MEASURE

12 inches (<i>in.</i>)		=	1 foot		ft.
3 feet		=	1 yard		yd.
$5\frac{1}{2}$ yards		=	1 rod (<i>rd.</i>), pole (<i>po.</i>), or perch (<i>per.</i>)		
40 poles		=	1 furlong		fur.
8 furlongs		=	1 mile		mi.

	in.	ft.	yd.	po.	fur.	mi.
12 =		1				
36 =		3	1			
198 =	16 $\frac{1}{2}$	=	5 $\frac{1}{2}$	=	1	
7,920 =	660	=	220	=	40	= 1
63,360 =	5,280	=	1,760	=	320	= 8 = 1

13. Other abbreviations frequently used for inches and feet are (") and ('). Thus, instead of writing 4 feet 6 inches as 4 ft. 6 in., it may be written 4' 6".

SURVEYORS' LINEAR MEASURE

7.92 inches (<i>in.</i>)		= 1 link		<i>li.</i>
25 links		= 1 rod		<i>rd.</i>
4 rods	}		= 1 chain	<i>ch.</i>
100 links				
80 chains		= 1 mile		<i>mi.</i>
	<i>in.</i>	<i>li.</i>	<i>ft.</i>	<i>rd.</i>
	<i>ch.</i>	<i>mi.</i>		
7.92 =	1			
198 =	25 =	16½ =	1	
792 =	100 =	66 =	4 =	1
63,360 =	8,000 =	5,280 =	320 =	80 = 1

14. Surveyors' linear measure is used in measuring land, roads, etc. The unit is a steel chain 66 feet long, and made of 100 links, all of equal length; therefore, the length of a link is $66 \times 12 \div 100 = 7.92$ inches. For railway surveying and other purposes, engineers use a steel tape 100 feet long, the feet being divided into tenths and hundredths. In computations, the links are written as so many hundredths of a chain.

15. Other units of length are the **nautical mile** = 6,080* feet; the **fathom** = 6 feet; the **cable's length** = 100 fathoms; the **hand** (for measuring height of horses) = 4 inches; the **nail** (for cloth) = $2\frac{1}{4}$ inches; the **ell** = $1\frac{1}{4}$ yards; the **span** = 9 inches; the **cubit** = 18 inches.

The ordinary mile of 5,280 feet is called the **statute mile**, as the length of it is laid down by Act of Parliament.

SQUARE MEASURE (OR MEASURES OF AREA)

144 square inches (<i>sq. in.</i>)		= 1 square foot		<i>sq. ft.</i>
9 square feet		= 1 square yard		<i>sq. yd.</i>
30¼ square yards		= 1 square rod (<i>sq. rd.</i>), square pole (<i>sq. po.</i>), or square perch (<i>sq. per.</i>)		
40 square poles		= 1 rood		<i>r.</i>
4 roods		= 1 acre		<i>ac.</i>
640 acres		= 1 square mile		<i>sq. mi.</i>
10,000 square links or 484 square yards	} = 1 square chain; 10 square chains = 1 acre			

* A geographical mile is the length of one minute of an arc measured on the earth's equator and equals 6,086.36 feet. The nautical mile is the length of one minute of an arc measured on a great circle of a sphere having the same area as the earth, and equals 6,080 feet.

sq. in.	sq. ft.	sq. yd.	sq. rd.	r.	ac.	sq. mi.
144 =	1					
1,296 =	9 =	1				
39,204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1			
1,568,160 =	10,890 =	1,210 =	40 =	1		
6,272,640 =	43,560 =	4,840 =	160 =	4 =	1	
4,014,489,600 =	27,878,400 =	3,097,600 =	102,400 =	2,560 =	640 =	1

16. Square measure is used in estimating the *area* of surfaces. In commercial use, the square yard is the largest unit

employed, the square rod or pole, rood, acre, and square mile being used for measuring land. The unit of square measure is a *square* whose sides are equal in length to the linear unit. The units of square measure are *derived units*; that is, they depend for their value upon the values of some other units, which, in this case, are the unit of linear measure. If the large

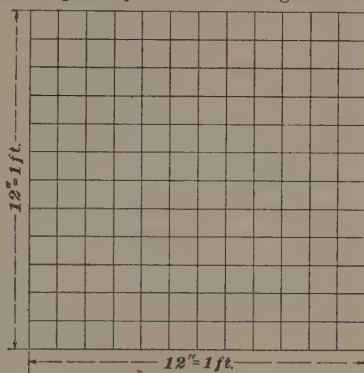


FIG. 1

square in Fig. 1 measures 1 foot on each side, as indicated by the dimension lines, then the space which the square covers, on a flat surface, is 1 *square foot*. If horizontal and vertical lines be drawn 1 inch apart, as shown, the small squares so formed will measure 1 inch on each side; and if these small squares be counted, they will be found to number 144. Hence, there are 144 square inches in 1 square foot. In the same way, it can be shown that 1 square yard contains 9 square feet. The *acre* is the only unit that forms an exception—it cannot be expressed as an exact square of any unit. A piece of land 208.71 feet square contains almost exactly an acre.

SURVEYORS' SQUARE MEASURE

625 square links (<i>sq. li.</i>)		= 1 square rod (<i>sq. rd.</i>), square pole (<i>sq. po.</i>), or square perch (<i>sq. per.</i>)
16 square poles	= 1 square chain	<i>sq. ch.</i>
10 square chains	= 1 acre	<i>ac.</i>
640 acres	= 1 square mile	<i>sq. mi.</i>

17. Surveyors' square measure is used only by civil engineers and surveyors. For this reason, no further remarks will be made concerning this measure.

MEASURES OF VOLUME

18. There are two classes of bodies whose *volume* may have to be measured: (1) Solids, such as stone, wood, etc. (2) Liquids and finely divided solids. For each of these two classes there is a table of measurement.

CUBIC MEASURE

1,728 cubic inches (<i>cu. in.</i>)		= 1 cubic foot	<i>cu. ft.</i>
27 cubic feet	= 1 cubic yard	<i>cu. yd.</i>	
	<i>cu. in.</i>	<i>cu. ft.</i>	<i>cu. yd.</i>
	1,728	= 1	
	46,656	= 27	= 1

19. Cubic measure is used in measuring the volumes of solids or bodies which have length, breadth, and thickness.

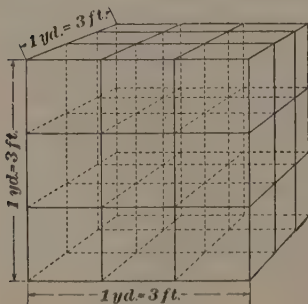


FIG. 2

The units of cubic measure are also derived units, since they depend upon linear measurements for their values. The unit of cubic measure is a cube whose edges are equal in length to the corresponding linear unit. Fig. 2 represents a cube whose sides are all 1 yard, or 3 feet, long. By dividing it into equal parts, as shown, it is readily seen that 27 small

cubes, measuring 1 foot on each edge, will be formed; hence,

1 cubic yard contains 27 cubic feet. In a similar manner, it can be shown that 1 cubic foot contains 1,728 cubic inches.

20. It should be noted that $1,728 \text{ cubic inches} = 12 \text{ inches} \times 12 \times 12$, and $27 \text{ cubic feet} = 3 \text{ feet} \times 3 \times 3$. Therefore, to obtain the cubic contents of any solid, find the continued product of the numbers of linear units in its (average) length, breadth, and thickness.

LIQUID MEASURE

4 gills (<i>gill</i>)	=	1 pint	<i>pt.</i>
2 pints	=	1 quart	<i>qt.</i>
4 quarts	=	1 gallon	<i>gal.</i>
		<i>gill</i> <i>pt.</i> <i>qt.</i> <i>gal.</i>	
		4 = 1	
		8 = 2 = 1	
		32 = 8 = 4 = 1	

21. There are special large units of measurement employed for different liquids, but the smaller ones, given above, are common to all.

In the wine trade, 42 gallons = 1 tierce; 63 gallons = 1 hogshead (*hhd.*); 84 gallons = 1 puncheon (*pun.*); 2 hogsheads = 1 pipe; 2 pipes = 1 tun. In measuring beer, 9 gallons = 1 firkin; 18 gallons = 1 kilderkin; 36 gallons = 1 barrel. The ordinary barrels and hogsheads used in commerce vary greatly in size, and their contents can be determined only by gauging or actual measurement.

22. The gallon is the principal unit. By an Act of Parliament the imperial gallon is required to contain "ten imperial standard pounds weight of distilled water (free from air), weighed in air against brass weights, with the water and air at 62 degrees Fahrenheit, and with the barometer at 30 inches." Until recent years the weight of a cubic inch of water under the above conditions was taken as 252.458 grains. More careful experiment has shown that the weight is only 252.286 grains, whereas 1 gallon contains 277.463 cubic inches, and this was made the legal capacity of a gallon in 1889. The legal value previous to that date was 277.274 cubic inches, usually taken as $277\frac{1}{4}$,

or more approximately 277. Inasmuch, however, as the legal value is the one that must be used in commercial transactions, the capacity of the imperial gallon will be here considered as 277.463 cubic inches, when this degree of accuracy is necessary.

APOTHECARIES' FLUID MEASURE

60 minims, or drops, (℥)	. . . = 1 fluid drachm	 f ℥
8 fluid drachms	 = 1 fluid ounce	 f ℥
20 fluid ounces	 = 1 pint	 O.
8 pints	 = 1 gallon	 Cong.

23. Apothecaries' fluid measure is used in prescribing and compounding medicines. When the *symbols* are used instead of the abbreviations, they are placed before the figures denoting the number of units. Thus, Cong. 2 O. 7 f ℥ 12 means 2 gallons 7 pints 12 fluid ounces. *Cong.* is the abbreviation of the Latin word *congius*, gallon; *O.* is the abbreviation of the Latin word *octavius*, one-eighth. A minim = $\frac{1}{60 \times 8} = \frac{1}{480}$ of an ounce.

DRY MEASURE

2 pints (<i>pt.</i>)	 = 1 quart	 qt.
4 quarts	 = 1 gallon	 gal.
2 gallons	 = 1 peck	 pk.
4 pecks	 = 1 bushel	 bush.
8 bushels or } 2 sacks	 = 1 quarter	 qr.

pt.	qt.	gal.	pk.	bush.	qr.
2 =	1				
8 =	4 =	1			
16 =	8 =	2 =	1		
64 =	32 =	8 =	4 =	1	
512 =	256 =	64 =	32 =	8 =	1

24. Dry measure, as its name implies, is used in measuring dry articles, such as fruits, grains, etc. The unit is the gallon, containing 277.463 cubic inches.

MEASURES OF WEIGHT

AVOIRDUPOIS WEIGHT

16 drams (<i>dr.</i>)	=	1 ounce	<i>oz.</i>
16 ounces	=	1 pound	<i>lb.</i>
28 pounds	=	1 quarter	<i>qr.</i>
4 quarters	=	1 hundredweight	<i>cwt.</i>
20 hundredweights	=	1 ton	<i>ton</i>
dr.	oz.	lb.	qr. cwt. ton
16 =	1		
256 =	16 =	1	
7,168 =	448 =	28 =	1
28,672 =	1,792 =	112 =	4 = 1
573,440 =	35,840 =	2,240 =	80 = 20 = 1

25. Other weights are 1 stone = 14 lb.; 1 quintal = 100 lb.; a fodder of lead = $19\frac{1}{2}$ cwt.

26. There are various standards of weight in use in Great Britain, namely Avoirdupois, Troy, and Apothecaries'. The avoirdupois weight is used in nearly all commercial transactions. In America, the ton of commerce for most purposes weighs 2,000 lb.

TROY WEIGHT

24 grains (<i>gr.</i>)	=	1 pennyweight	<i>dwt.</i>
20 pennyweights	=	1 ounce	<i>oz. tr.</i>
12 ounces	=	1 pound	<i>lb. tr.</i>
gr.	dwt.	oz.	lb.
24 =	1		
480 =	20 =	1	
5,760 =	240 =	12 =	1

27. Troy weight is used for gold, silver, and precious stones; the ounce is also divided into 24 parts called carats, used in measuring the fineness of gold. A carat used in weighing diamonds = 3.1683 grains.

28. The grain is common to avoirdupois, troy, and apothecaries' weights. The troy pound contains 5,760 grains, while the avoirdupois pound contains 7,000 grains. Hence the troy ounce contains $5,760 \div 12 = 480$ grains, and the avoirdupois ounce contains $7,000 \div 16 = 437\frac{1}{2}$ grains. Thus, 1 oz. troy is *heavier* than 1 oz. avoirdupois. On the other hand, 1 lb. avoirdupois is *heavier* than 1 lb. troy.

APOTHECARIES' WEIGHT

20 grains (<i>gr.</i>)	= 1 scruple	<i>sc.</i> or $\mathfrak{D}$
3 scruples	= 1 drachm	<i>dr.</i> or $\mathfrak{Z}$
8 drachms	= 1 ounce	<i>oz.</i> or $\mathfrak{Z}$
12 ounces	= 1 pound	<i>lb.</i> or $\mathfrak{lb}$

<i>gr.</i>	$\mathfrak{D}$	$\mathfrak{Z}$	$\mathfrak{Z}$	$\mathfrak{lb}$
20 =	1			
60 =	3 =	1		
480 =	24 =	8 =	1	
5,760 =	288 =	96 =	12 =	1

29. Apothecaries' weight is used by physicians and chemists in prescribing and compounding medicines not in the liquid state. As in apothecaries' fluid measure, the symbols precede the numbers to which they refer. For example, $\mathfrak{Z}\mathfrak{V}\mathfrak{D}2$ means 5 drachms 2 scruples. The pound, ounce, and grain are the same as in troy weight.

Drugs and medicines are sold, when in large quantities, by avoirdupois weight.

MEASURES OF TIME

60 seconds (<i>sec.</i>)	= 1 minute	<i>min.</i>
60 minutes	= 1 hour	<i>hr.</i>
24 hours	= 1 day	<i>day</i>
7 days	= 1 week	<i>wk.</i>
4 weeks	= 1 month	<i>mo.</i>
12 months	= 1 year	<i>yr.</i>
100 years	= 1 century	<i>C.</i>

sec.	min.	hr.	day	wk.	yr.
60 =	1				
3,600 =	60 =	1			
86,400 =	1,440 =	24 =	1		
604,800 =	10,080 =	168 =	7 =	1	
31,556,930 =	525,948 =	8,765 =	365 $\frac{1}{4}$ =	52 $\frac{5}{8}$ =	1

30. The divisions of time are peculiar. The only units that may be called *natural* are the day, the lunar month, and the year, the other divisions being artificial. The time in which the earth makes one complete revolution around the sun is called a **solar year**, and it equals 365 days, 5 hours, 48 minutes, 47.5 seconds, or $365\frac{1}{4}$ days, very nearly. A **solar day** is the interval between two consecutive returns of the sun to the meridian. On account of the earth moving in an elliptical path around the sun, the length of the solar day varies; hence, for civil purposes, the average of all the days in the year is taken as a unit. The months contain from 28 to 31 days.

In order to make the *calendar*, or *civil*, year agree as nearly as possible with the solar year, it is customary to add 1 day to the year (making 366 days) every fourth year. Years containing 366 days are called **leap years**. Every leap year is exactly divisible by 4. Thus 1896, which was a leap year, is divisible by 4 without a remainder. Any year which ends with two or more ciphers, as 1800, 1900, 2000, etc., is called a **secular year**. Since the year falls short of $365\frac{1}{4}$ days by 11 minutes and 12.5 seconds, the addition of 1 day every 4 years makes about $\frac{3}{4}$ of a day too much in a century; hence, to correct this, secular years are not leap years except when exactly divisible by 400. Thus, 1900 was not a leap year but 2000 will be a leap year. This last correction makes an error of about 1 day in 3,526 years.

31. The following is a list of months, in regular order, with the number of days which each contains:

	Days		Days
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.) . . .	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.) . . .	30
6. June	30	12. December (Dec.) . . .	31

In leap years, one day is added to February, giving it 29 days. The following lines will assist the student in remembering the number of days in each month :

“ Thirty days hath September,
 April, June, and November ;
 All the rest have thirty-one,
 Excepting February alone,
 Which hath twenty-eight days clear,
 And twenty-nine in each leap year.”

MEASURES OF ANGLES OR ARCS

CIRCULAR MEASURE

60 seconds (")		= 1 minute	'
60 minutes		= 1 degree	°
90 degrees		= 1 right angle	rt. $\angle$
360 degrees		= 1 circle	⊙
60"	=	1'	
3,600"	=	60' = 1°	
324,000"	=	5,400' = 90° = 1 rt. $\angle$	
1,296,000"	=	21,600' = 360° = 4 rt. $\angle$ = 1 ⊙	

32. A quadrant is one-fourth of a circle, or 90° ; a sextant is one-sixth of a circle, or 60° . The unit of measurement is the degree, or $\frac{1}{360}$ of the circumference of a circle.

33. Circular or angular measure is used principally by surveyors, navigators, astronomers, and by technical men generally, for measuring angles and arcs of circles.

MISCELLANEOUS TABLES

34. The following table is used in counting certain articles :

12 of anything		= 1 dozen	doz.
12 dozen		= 1 gross	gr.
20 of anything		= 1 score	
	units	doz.	gr.
	12	= 1	
	144	= 12 = 1	

35. The following table is used in the paper trade :

24 sheets	= 1 quire	qr.
20 quires	= 1 ream	rm.
	sheets	qr. rm.
	24 = 1	
	480 = 20 = 1	

REDUCTION

36. Reduction is the process of changing the denomination of quantities without changing their values. Reduction is divided into two cases. In one case a quantity already expressed in high units is to be expressed in terms of lower units, in the other case, a quantity already expressed in low units is to be expressed in terms of higher units. The first case is called *descending reduction*, and the second, *ascending reduction*.

DESCENDING REDUCTION

37. **Descending reduction**, or reducing units of a higher to those of a lower denomination, is performed by multiplication. Thus, to reduce 4 miles (4 mi.) to poles, yards, feet, and inches, successively, it is known that there are 320 po. in 1 mi. ; hence, in 4 mi. there are evidently $4 \times 320 = 1,280$ po. Again, since there are $5\frac{1}{2}$ yd. in 1 po., in 1,280 po. there are $1,280 \times 11 = 14,080$ half-yd. = 7,040 yd. In 1 yd. there are 3 ft. ; hence, in 7,040 yd. there are $7,040 \times 3 = 21,120$ ft. And, since in 1 ft. there are 12 in., in 21,120 ft. there are $21,120 \times 12 = 253,440$ in. All examples in descending reduction are performed in the same manner.

38. If more than one unit is given, reduce the highest unit to the next lower denomination mentioned, and then add the units of this next lower denomination to the result previously obtained. So proceed until the lowest denomination is reached. The following example will serve as a model :

EXAMPLE.—Reduce 5 mi. 47 po. 3 yd. $1\frac{1}{2}$ ft. to feet.

SOLUTION.—	mi.	po.	yd.	ft.
	5	47	3	$1\frac{1}{2}$
	320			
	1600	po.		
	47			
	1647	po.		
	11			
2	18117	half-yd.		
	9058	$\frac{1}{2}$ yd.		
	3			
	9061	$\frac{1}{2}$ yd.		
	3			
	1	$\frac{1}{2}$		
	27183			
	27184	$\frac{1}{2}$ ft.		
	1	$\frac{1}{2}$		
	27186	ft.	Ans.	

EXPLANATION.—Since there are 320 po. in 1 mi., multiplying 5 by 320 and adding 47 to the product will give the number of poles in 5 mi. 47 po., or 1,647 po. Since there are $5\frac{1}{2}$ yd., or 11 half-yd. in 1 po., multiplying 1,647 by 11 gives 18,117 half-yd., or dividing by 2 gives $9,058\frac{1}{2}$ yd., adding 3 to the product will give the number of yards in 5 mi. 47 po. 3 yd., or $9,061\frac{1}{2}$ yd. Since there are 3 ft. in 1 yd., multiplying $9,061\frac{1}{2}$ by 3, and adding the $1\frac{1}{2}$ to the product, will give the required result, or 27,186 ft. Ans.

39. Rule.—*Multiply the number of units of the highest denomination in the given compound expression by the number of units of the lower denomination required to make one unit of the higher denomination, and to the product add the given number of units of the lower denomination. Proceed in this manner until the given compound expression is reduced to the required denomination.*

40. In order to avoid mistakes, if any denomination is omitted between the highest and lowest denominations of the given number, represent it by a cipher.

EXAMPLE. — Reduce 40 ac. 21 sq. yd. 6 sq. ft. 93 sq. in. to square inches.

SOLUTION.—

ac.	r.	sq. po.	sq. yd.	sq. ft.	sq. in.
40	0	0	21	6	93
	4				
	160	r.			
	40				
	6400	sq. po.			
	30	$\frac{1}{4}$			
	1600				
	192000				
	193600	sq. yd.			
	21				
	193621	sq. yd.			
	9				
	1742589	sq. ft.			
	6				
	1742595	sq. ft.			
	144				
	6970380				
	6970380				
	1742595				
	250933680	sq. in.			
	93				
	250933773	sq. in.	Ans.		

EXPLANATION.—The absence of any roods and square poles is denoted by the cipher, as shown. Since there are 4 r. in 1 ac., multiplying 40 by 4 will give the number of roods in 40 ac. There being no roods to add, proceed to reduce the roods to square poles by multiplying by 40, the product 6,400 being the number of square poles in 40 ac. There being also no square poles to add, proceed to reduce the square poles to square yards by multiplying by $30\frac{1}{4}$. Multiplying 6,400 by $\frac{1}{4}$ gives a product of 1,600, and adding to this the product of 6,400 and 30, or 192,000, gives the sum 193,600. If to this product is added 21, the sum will give the number of square yards in 4 ac. 21 sq. yd., or 193,621 sq. yd. Since there are 9 sq. ft. in 1 sq. yd., multiplying 193,621 by 9 and adding 6 to the product will give the number of square feet in 4 ac. 21 sq. yd. 6 sq. ft., or 1,742,595 sq. ft. Since there are 144 sq. in. in 1 sq. ft., multiplying 1,742,595 by 144, and adding 93 to the product, will give the required result, or 250,933,773 sq. in. Ans.

EXAMPLES FOR PRACTICE

41. Reduce :

(a) 7 mi. 4 po. 2 yd. 2 ft. to feet.	(a) 37,034 ft.
(b) 4 bush. 2 pk. 2 qt. to quarts.	(b) 146 qt.
(c) 8 lb. 4 oz. 6 dwt. 12 gr. to grains.	(c) 48,156 gr.
(d) 52 hhd. 24 gal. 1 pt. to pints.	(d) 26,401 pt.
(e) $5^{\circ} 16' 20'' 46'''$ to seconds.	(e) 6,538,846"
(f) 14 bush. to quarts.	(f) 448 qt.
(g) 11 tons 9 cwt. 57 lb. to pounds.	(g) 25,705 lb.
(h) £9 13s. 10d. to pence.	(h) 2,326d.
(i) 10 st. 3 lb. to pounds.	(i) 143 lb.
(j) 97 sq. rd. to square feet.	(j) $26,408\frac{1}{4}$ sq. ft.
(k) 4 yr. 8 mo. 1 wk. 3 days to days. (Count 12 mo. to the year, and 4 wk. to the month.)	(k) 1,578 days
(l) 2 firkins, 3 gal. 2 qt. to quarts.	(l) 86 qt.
(m) 21 cu. yd. to cubic feet.	(m) 567 cu. ft.
(n) Cong. 2 O. 6 f $\bar{3}$ 10 f $\bar{3}$ 5 to fluid drachms.	(n) f $\bar{3}$ 3,605
(o) lb1 $\bar{3}$ 9 $\bar{3}$ 6 $\bar{2}$ to grains.	(o) 10,480 gr.
(p) Find the actual number of days in example (k). (Count 1 leap year, and 30 days to the month.)	(p) 1,711 days

ASCENDING REDUCTION

42. Ascending reduction, or reducing units of a lower to those of a higher denomination, is performed by division. Thus, to reduce 253,440 in. to feet, the number must evidently be divided by 12, since there are 12 in. in 1 ft. That is, $253,440 \text{ in.} \div 12 = 21,120 \text{ ft.}$ To reduce this last number to yards, divide by 3; or, $21,120 \text{ ft.} \div 3 = 7,040 \text{ yd.}$ Continuing the process, $7,040 \text{ yd.} \div 5\frac{1}{2} = 1,280 \text{ po.}$; $1,280 \text{ po.} \div 40 = 32 \text{ fur.}$; $32 \text{ fur.} \div 8 = 4 \text{ mi.}$ It will be noticed that this is the reverse of the reduction of 4 mi. to inches, as in Art. 37.

43. If, in reducing a number, as in the last article, to a higher denomination there is a remainder, reserve it and divide the quotient obtained by the division by the number of units required to make one unit of the next higher denomination. Or carry the division further by annexing ciphers to the remainder, thus obtaining a decimal part of the next higher unit.

EXAMPLE.—Reduce 27,186 feet to miles, furlongs, poles, etc.

SOLUTION.—Instead of giving all the work of division, the short method of division will be used to express the various steps. This will save space and make the work plainer.

$$\begin{array}{r}
 3 \overline{) 27186 \text{ ft.}} \\
 \underline{9062 \text{ yd.}} \\
 2 \\
 11 \overline{) 18124 \text{ half-yd.}} \\
 \underline{40 \overline{) 1647 \text{ po.} + 7 \text{ half-yd., or } 3\frac{1}{2} \text{ yd.}}} \quad 3\frac{1}{2} \text{ yd.} = 3 \text{ yd. } 1\frac{1}{2} \text{ ft.} \\
 \underline{8 \overline{) 41 \text{ fur.} + 7 \text{ po.}}} \\
 5 \text{ mi.} + 1 \text{ fur.} \\
 \text{Hence, } 27,186 \text{ ft.} = 5 \text{ mi. } 1 \text{ fur. } 7 \text{ po. } 3 \text{ yd. } 1\frac{1}{2} \text{ ft.} \quad \text{Ans.}
 \end{array}$$

EXPLANATION.—Since there are 3 ft. in 1 yd., divide 27,186 by 3 and obtain 9,062 yd. This quotient should be divided by $5\frac{1}{2}$ yd. to obtain the number of poles, but since it is easier to perform the division when the yards are reduced to half-yards, multiply 9,062 by 2, obtaining 18,124 half-yd. As $5\frac{1}{2}$ yd. = 11 half-yd., divide 18,124 by 11 and obtain 1,647 po. and 7 half-yd., or $3\frac{1}{2}$ yd., remaining. Write as shown. Dividing 1,647 po. by 40, the number of poles in a furlong, the result is 41 fur. and 7 po. remaining. Dividing 41 fur. by 8, the number of furlongs in a mile, the result is 5 mi. and 1 fur. remaining. Hence, 27,186 ft. = 5 mi. 1 fur. 7 po. $3\frac{1}{2}$ yd. But $\frac{1}{2}$ yd. = $\frac{1}{2} \times 3 = 1\frac{1}{2}$ ft.; consequently, instead of the above, 5 mi. 1 fur. 7 po. 3 yd. $1\frac{1}{2}$ ft. may be written.

44. Instead of retaining the lower units, the preceding number may be reduced to miles and decimals of a mile, as in the following example :

EXAMPLE.—Reduce 27,186 feet to miles and decimals of a mile.

SOLUTION.—From Table in Art. 12 there are 5,280 feet in a mile, hence, use 5,280 as a divisor :

$$\begin{array}{r}
 5280 \overline{) 27186} \\
 \underline{5.14886 \frac{4}{11} \text{ mi.}} \quad \text{Ans.}
 \end{array}$$

EXPLANATION.—Dividing until the given number of decimals is found, there is a remainder of 1,920, or $\frac{1920}{5280}$. Reducing this fraction to its lowest terms by dividing by 480, it is equal to $\frac{4}{11}$, which fraction is added to the decimals as shown.

45. Rule.—*Divide the number of units of the denomination given by the number of units of that denomination that are required to make one unit of the next higher denomination. The remainder (if any) will be of the same denomination, but the quotient will be of the next higher denomination. Divide this quotient by the*

number of units of its denomination that are required to make one unit of the next higher denomination. Continue thus until the required denomination is reached. If it is desired to obtain a decimal fraction of the required denomination, instead of a series of remainders of lower denomination, proceed as explained in Art. 44.

EXAMPLES FOR PRACTICE

46. Reduce all answers to the Examples for Practice in Art. 41 to units of higher denomination. Also solve the following, reducing as required, and expressing the result in decimals, as in Art. 44.

Reduce the following :

(a) 875 pt. to gallons.	Ans. {	(a) 109.375 gal.
(b) 10,000 pt. to bushels.		(b) 156.25 bush.
(c) 147,368 cu. in. to cubic yards.		(c) 3.1586 + cu. yd.
(d) 10,000 gr. to troy pounds.		(d) 1.7361 + lb.
(e) 8,375d. to pounds.		(e) £34.8958 +
(f) 28,140 sq. yd. to acres.		(f) 5.814 + ac.
(g) 49,175 in. to miles.		(g) 0.776 + mi.
(h) 380,421" to degrees.		(h) 105.6725°

COMPOUND ADDITION

47. Compound addition is the process by which to find the sum of a number of similar compound quantities. The method of working is similar to that of simple addition. An example will serve to show the process.

EXAMPLE.—Find the sum of 4 tons 3 cwt. 1 qr. 18 lb. 12 oz. ; 8 cwt. 12 lb. 13 oz. ; 2 tons 12 cwt. 1 qr. 22 lb. 13 oz. ; 1 ton 27 lb. 4 oz.

SOLUTION.—	tons	cwt.	qr.	lb.	oz.
	4	3	1	18	12
		8		12	13
	2	12	1	22	13
	1			27	4
	7	23	2	79	42
	or 8 tons 4 cwt. 0 qr. 25 lb. 10 oz. Ans.				

EXPLANATION.—Write the units of the same denomination in the same column, as shown, with the abbreviation of the name of the unit at the head of each column, and arranging the units in decreasing order from left to right. Beginning with the right-hand column, the sum of the numbers in that column is 42, i.e., 42 oz. The sum of the numbers in the

second column is 79 lb. ; in the third column, 2 qr. ; in the fourth column, 23 cwt. ; and in the fifth column, 7 tons. Now, since 42 oz. are more than 1 lb., reduce the 42 oz. to lb. and oz., obtaining 2 lb. 10 oz. Reserve the 10 oz., add the 2 lb. to the 79 lb. in the second column, obtaining 81 lb. ; this reduced to qr. and lb. gives 2 qr. 25 lb. Reserve the 25 lb. add the 2 qr. to the 2 qr. in the third column, obtaining 4 qr. ; this reduced to cwt. gives 1 cwt. without a remainder. The 1 cwt. added to the 23 cwt. in the next column gives 24 cwt. or 1 ton 4 cwt. Finally, 7 tons + 1 ton = 8 tons. Hence, the sum is 8 tons 4 cwt. 25 lb. 10 oz.

48. That the process just described is the same in all respects as the addition of simple numbers will be manifest after a little consideration. Thus, suppose that *thousands* is represented by *thds.*, *hundreds* by *h.*, *tens* by *t.*, and *units* by *u.* ; then the addition of 2,046, 812, 2,151, and 1,707 might be performed as follows :

thds.	h.	t.	u.
2		4	6
	8	1	2
2	1	5	1
1	7		7
5	16	10	16
or 6 thds. 7 h. 1 t. 6 u., which equals 6,716			

It is easily seen that the same principles govern both cases.

49. Instead of writing the sum of each column separately and then reducing, as in the preceding example (Art. 47), it is customary to add the right-hand column, and, if the sum contains more units than are required to make one unit of the next higher denomination, to reduce it to the next higher denomination, placing the remainder (if any) under the right-hand column and carrying the quotient to the next column. This will be recognized as corresponding exactly to the ordinary process of addition.

EXAMPLE.—What is the sum of £39 4s. 5½d. ; £329 15s. 2¾d. ; £15 7s. 4¼d. ; £23 13s. 7d. ?

SOLUTION.—

£	s.	d.
39	4	5½
329	15	2¾
45	7	4¼
23	13	7
438	0	7½

EXPLANATION.—The first column shows farthings and halfpennies, and as two farthings equal one halfpenny, the sum of this column is 6 farthings, or 1 penny and 1 halfpenny. Writing the $\frac{1}{2}$ d. under the right-hand column, and carrying the 1d. to the next column and adding, the result is 1 (carried) + 7 + 4 + 2 + 5 = 19d. = 1s. 7d. Writing the 7d. under the pence column, and carrying the 1s. to the shillings column, and adding, the result is 40s. = £2 0s. Carrying the £2 to the pounds column and adding, the sum is £438. The total addition is therefore £438 0s. $7\frac{1}{2}$ d.

50. Rule.—Place the numbers in vertical columns so that like denominations are under each other. Begin at the right-hand column, and add. If the sum contains more units than are necessary to make one unit of the next higher denomination, reduce the sum to the next higher denomination, placing the remainder under the column added, and carrying the unit (or units) of the next higher denomination so obtained to the second column. Continue in this manner until the highest denomination is reached. Should fractions of a unit be obtained, or should they occur in the numbers to be added, reduce them to units of lower denomination and add them to the sum first obtained.

EXAMPLES FOR PRACTICE

51. Find the sum of the following :

(a) 25 lb. 7 oz. 15 dwt. 23 gr. ; 17 lb. 16 dwt. ; 15 lb. 4 oz. 12 dwt. ; 18 lb. 16 gr. ; 10 lb. 2 oz. 11 dwt. 16 gr.

(b) 9 mi. 13 rd. 4 yd. 2 ft. ; 16 rd. 5 yd. 1 ft. 5 in. ; 16 mi. 2 rd. 3 in. ; 14 rd. 1 yd. 9 in. ; 7 fur. 33 rd. 3 yd. 2 ft.

(c) £16 5s. 4d. ; £12 8s. 9d. ; £13 14s. 8d. ; £42 0s. 7d. ; 18s. 6d.

(d) 13 cwt. 1 qr. 18 lb. 12 oz. ; 12 cwt. 9 lb. 8 oz. ; 2 cwt. 2 qr. 21 lb. 9 oz.

(e) 4 bush. 3 pk. 6 qt. 1 pt. ; 10 bush. 2 pk. 7 qt. 1 pt. ; 11 bush. 3 pk. 1 qt. 1 pt. ; 9 bush. 2 pk. 5 qt. 1 pt.

(f) 10 yr. 8 mo. 5 wk. 3 days ; 42 yr. 6 mo. 7 days ; 7 yr. 5 mo. 18 wk. 4 days ; 17 yr. 17 days (taking 7 days to the week, and 4 weeks to the month).

(g) 14 sq. yd. 8 sq. ft. 19 sq. in. ; 105 sq. yd. 16 sq. ft. 240 sq. in. ; 42 sq. yd. 28 sq. ft. 165 sq. in.

(h) 16 gal. 3 qt. 1 pt. ; 45 gal. 2 qt. ; 17 gal. 1 qt. 1 pt. ; 4 gal. 3 qt. ; 15 gal. 1 pt. ; 24 gal. 3 qt. 1 pt.

(2) 16 hr. 43 min. 48 sec.; 3 hr. 12 min. 40 sec.; 1 hr. 49 min. 13 sec.; 5 hr. 19 sec.

$$\text{Ans.} \left\{ \begin{array}{l} (a) \text{ 86 lb. 3 oz. 16 dwt. 7 gr.} \\ (b) \text{ 26 mi. 1 fur. 4 yd. 5 in.} \\ (c) \text{ £85 7s. 10d.} \\ (d) \text{ 1 ton 8 cwt. 21 lb. 13 oz.} \\ (e) \text{ 37 bush. 5 qt.} \\ (f) \text{ 78 yr. 1 mo. 3 wk. 3 day} \\ (g) \text{ 5 sq. rd. 15 sq. yd. 7 sq. ft. 100 sq. in.} \\ (h) \text{ 124 gal. 2 qt.} \\ (i) \text{ 26 hr. 46 min.} \end{array} \right.$$

COMPOUND SUBTRACTION

52. Compound subtraction is the process of finding the difference between two compound expressions of the same kind; or of finding what must be added to the smaller of two compound quantities to make it equal to the greater. The operation is the same in principle as in simple subtraction, the principles of reduction of compound expressions being brought into use as in compound addition.

EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—

rd.	yd.	ft.	in.
21	2	2	$6\frac{1}{2}$
9	4		$10\frac{1}{4}$
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$

or 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in. = 11 rd. 4 yd. $2\frac{1}{4}$ in. Ans.

EXPLANATION.—Writing the numbers as for addition, with units of like denomination under each other, begin with the right-hand column and subtract. Since $10\frac{1}{4}$ inches cannot be subtracted from $6\frac{1}{2}$ inches, 1 foot (= 12 inches) is taken from the 2 feet in the minuend, reduced to inches, and added to the inches in the minuend, giving $18\frac{1}{2}$ inches. Then, $18\frac{1}{2}$ in. — $10\frac{1}{4}$ in. = $8\frac{1}{4}$ in. Since 1 foot was taken from the 2 feet, only 1 foot remains, and, as there are no feet in the subtrahend, the 1 foot is brought down in the remainder. As 4 yards cannot be taken from 2 yards, 1 rod is taken from the 21 rods in the minuend, reduced to yards, and added to the 2 yards, giving $7\frac{1}{2}$ yards; then, $7\frac{1}{2}$ yd. — 4 yd. = $3\frac{1}{2}$ yd. Finally, 20 rd. — 9 rd. = 11 rd., and the remainder is 11 rd. $3\frac{1}{2}$ yd. 1 ft. $8\frac{1}{4}$ in. Reducing the $\frac{1}{2}$ yd., the remainder becomes 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in.; or, since $14\frac{1}{4}$ in. = 1 ft. $2\frac{1}{4}$ in., and 2 ft. + 1 ft. = 3 ft. = 1 yd. the remainder is 11 rd. 4 yd. $2\frac{1}{4}$ in.

53. Rule.—Place the smaller quantity under the greater quantity, with units of like denomination under each other. Beginning

at the right, subtract successively the units in the subtrahend in each denomination from those above, and place the several remainders beneath, as in simple subtraction. If the units of any denomination in the minuend are fewer than the units of the same denomination in the subtrahend, take one unit from the next higher denomination in the minuend, reduce it to the next lower denomination, and add it to the proper number in the minuend; then subtract as before. Reduce fractional results in the remainder if there are any.

54. EXAMPLE.—From £364 2s. $2\frac{1}{4}$ d. take £148 15s. $6\frac{1}{2}$ d.

SOLUTION.—

£	s.	d.	
364	2	$2\frac{1}{4}$	
148	15	$6\frac{1}{2}$	
215	6	$7\frac{3}{4}$	Ans.

EXPLANATION.—Since $\frac{1}{2}$ d. or 2 farthings cannot be taken from 1 farthing, take 1 penny (= 4 farthings) from the pence in the minuend and add it to the 1 farthing, obtaining 5 farthings. $5 - 2 = 3$ farthings, written $\frac{3}{4}$ d. Six pence cannot be taken from 1 penny, therefore take 1 shilling, or 12 pence, from the shillings, and add to the 1 penny. $12\text{d.} + 1\text{d.} = 13\text{d.}$, and $13\text{d.} - 6\text{d.} = 7\text{d.}$ Again, as 15 shillings cannot be taken from 1 shilling, take 1 pound, or 20 shillings, from the pounds. $20\text{s.} + 1\text{s.} = 21\text{s.}$, and $21\text{s.} - 15\text{s.} = 6\text{s.}$ Subtracting £148 from £363, the result is £215, and the total remainder is £215 6s. $7\frac{3}{4}$ d.

EXAMPLES FOR PRACTICE

55. Solve the following examples :

- From £10 6s. 4d. take £8 15s. 3d.
- From 125 lb. 8 oz. 14 dwt. 18 gr. take 96 lb. 9 oz. 10 dwt. 4 gr.
- From 16 yr. 8 mo. 10 days take 12 yr. 5 mo. 8 days.
- From 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
- From 65 tons 14 cwt. 2 qr. 6 lb. 10 oz. take 16 tons 11 cwt. 14 oz.
- From 148 sq. yd. 16 sq. ft. 102 sq. in. take 132 sq. yd. 136 sq. in.
- Subtract 28 bush. 2 pk. 5 qt. 1 pt. from 100 bush.
- Subtract 3 mi. 27 rd. 11 yd. 4 ft. 10 in. from 14 mi. 34 rd. 16 yd. 13 ft. 11 in.
- Subtract $27^{\circ} 19' 47''$ from $126^{\circ} 37' 23''$.

Answers.—(a) £1 11s. 1d.; (b) 28 lb. 11 oz. 4 dwt. 14 gr.; (c) 4 yr. 3 mo. 2 days; (d) 22 hhd. 12 gal. 2 qt. 1 pt.; (e) 49 tons 3 cwt. 2 qr. 5 lb. 12 oz.; (f) 16 sq. yd. 15 sq. ft. 110 sq. in.; (g) 71 bush. 1 pk. 2 qt. 1 pt.; (h) 11 mi. 8 rd. 2 yd. 1 ft. 7 in.; (i) $99^{\circ} 17' 36''$.

COMPOUND MULTIPLICATION

56. Compound multiplication is the process of multiplying a compound expression; in other words, it is a short way of obtaining the sum of a number of equal compound expressions.

EXAMPLE.—A merchant divided his syrup into 12 equal parts, each containing 2 gal. 2 qt. $1\frac{1}{2}$ pt.; how much did he have altogether?

SOLUTION.—He evidently had 12 times 2 gal. 2 qt. $1\frac{1}{2}$ pt., or

gal.	qt.	pt.	
2	2	$1\frac{1}{2}$	
		12	
24	24	18	= 32 gal. 1 qt. Ans.

EXPLANATION.—Multiplying the units of each denomination by 12, and reducing the units of each denomination of the products to higher denominations, the result is 32 gal. 1 qt.

57. Such examples are usually solved as follows:

gal.	qt.	pt.	
2	2	$1\frac{1}{2}$	
		12	
32	1	0	Ans.

EXPLANATION.— $1\frac{1}{2}$ pt. $\times$ 12 = 18 pt. = 9 qt.; reserve this, and add to the quarts product. 2 qt. $\times$ 12 + 9 qt. = 33 qt. = 8 gal. 1 qt. Write the 1 qt. and reserve the 8 gal. 2 gal. $\times$ 12 + 8 gal. = 32 gal. Hence, the answer is 32 gal. 1 qt.

58. When the multiplier contains a fraction, it is usually easier to reduce the multiplicand to the *lowest* denomination before multiplying, and then reduce the product to higher denominations. In any case, the method used in Art. 56 is to be preferred to that given in Art. 57, when the multiplier contains a fraction. All three methods will be applied to an example to illustrate the point.

EXAMPLE.—Multiply 7 lb. 5 oz. 13 dwt. 15 gr. by $5\frac{1}{2}$.

SOLUTION.—*First Method:*

	lb.	oz.	dwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	$38\frac{1}{2}$	$27\frac{1}{2}$	$71\frac{1}{2}$	$82\frac{1}{2}$	
or	38	33	81	$94\frac{1}{2}$	
or	41 lb.	1 oz.	4 dwt.	$22\frac{1}{2}$ gr.	Ans.

<i>Second Method :</i>	lb.	oz.	dwt.	gr.
	7	5	13	15
				$5\frac{1}{2}$
	$40\frac{1}{2}$	$6\frac{1}{2}$	$14\frac{1}{2}$	$10\frac{1}{2}$
or	40	12	24	$22\frac{1}{2}$
or	41 lb.	1 oz.	4 dwt.	$22\frac{1}{2}$ gr. Ans.

Third Method : 7 lb. 5 oz. 13 dwt. 15 gr. = 43,047 gr. (if the lowest denomination given had not been grains, the expression would still have been reduced to grains). Hence, $43,047 \text{ gr.} \times 5.5 = 236,758.5 \text{ gr.} = 41 \text{ lb. } 1 \text{ oz. } 4 \text{ dwt. } 22\frac{1}{2} \text{ gr.}$ Ans.

EXAMPLES FOR PRACTICE

59. Solve the following :

(a) Multiply £17 10s. 8d. by 7; by 9; by 15.

(b) Find the weight of 2 dozen silver spoons, each spoon weighing 1 oz. 13 dwt.

(c) If 15 men perform a certain piece of work in 3 days 16 hr. 52 min., how long would it take one man to perform the work?

(d) Multiply 3 tons 15 cwt. 3 qr. 6 lb. by 5.

(e) Multiply 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

(f) Multiply 6 ac. 2 r. 34 sq. po. 19 sq. yd. 53 sq. ft. by 13.

(g) Multiply £5 7s. $2\frac{1}{4}$ d. by 81.

Ans. $\left\{ \begin{array}{l} (a) \text{ £122 14s. 8d.; £157 16s.; £263} \\ (b) \text{ 3 lb. 3 oz. 12 dwt.} \\ (c) \text{ 55 days 13 hr.} \\ (d) \text{ 18 tons 19 cwt. 2 lb.} \\ (e) \text{ 48 hhd. 40 gal. 2 qt.} \\ (f) \text{ 87 ac. 1 r. 12 sq. po. 21 sq. yd. } \frac{1}{2} \text{ sq. ft.} \\ (g) \text{ £434 2s. } 2\frac{1}{4}\text{d.} \end{array} \right.$

COMPOUND DIVISION

60. Compound division is the process of dividing a compound expression into a given number of parts.

SHORT DIVISION

61. Compound division can be done by *short division* when the divisor is less than 13, or is a number that can be split into factors less than 13.

EXAMPLE 1.—Divide 48 lb. 11 oz. 6 dwt. by 8.

SOLUTION.—	lb.	oz.	dwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 dwt.	6 gr.

Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 oz. once with 3 oz. remaining. 3×20 dwt. = 60; 60 dwt. + 6 dwt. = 66 dwt.; 66 dwt. $\div$ 8 = 8 dwt. and 2 dwt. remaining; 2×24 gr. = 48 gr.; 48 gr. $\div$ 8 = 6 gr. Therefore, the entire quotient is 6 lb. 1 oz. 8 dwt. 6 gr. Ans.

EXAMPLE 2.—Divide 426 tons 4 cwt. 1 qr. 15 lb. by 15.

SOLUTION.—	ton	cwt.	qr.	lb.
3)	426	4	1	15
5)	142	1	1	23·7
	28	8	1	4·7

Ans.

EXPLANATION.—The factors of 15 being 3 and 5, divide first by 3, and then by 5, as explained in the preceding example.

LONG DIVISION

62. When the divisor is a number larger than 12, and cannot be split into factors less than 13, we divide as in *long division*.

EXAMPLE.—Divide £5,672 12s. 6d. by 79.

SOLUTION.—	£	s.	d.
79)	5672	12	6 (£71
	553		
	142		
	79		
	63		
	20		
79)	1272	(16s.	
	79		
	482		
	474		
	8		
	12		
79)	102	(1d.	
	79		
	23 d.	remainder	

£71 16s. $1\frac{2}{3}$ d. Ans.

EXPLANATION.—Divide £5,672 by 79 as in long division of numbers. The quotient is £71 with a remainder of £63, which is multiplied by 20 to reduce to shillings, and 12 shillings in the original amount added. The

result is 1,272s., which, divided by 79, gives 16s., with a remainder of 8s. Reducing this to pence and adding 6d., the result is 102d., which divided by 79 gives 1d., with a remainder of 23d., which divided by 79 gives $\frac{2}{7}$ d. The total quotient is therefore £71 16s. $1\frac{2}{7}$ d.

63. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE

64. Divide :

(a) 376 mi. 6 fur. 36 po. by 22 ; (b) 1,137 bush. 3 pk. 4 qt. 1 pt. by 10 ;
(c) 84 cwt. 1 qr. 23 lb. 4 oz. by 16 ; (d) 78 sq. yd. 18 sq. ft. 41 sq. in.
by 18 ; (e) £83 10s. 6d. by 12 ; (f) 100 tons 16 cwt. 18 lb. 11 oz. by 15 ;
(g) 36 lb. 18 oz. 18 dwt. 14 gr. by 8 ; (h) £156 11s. 7d. by 56.

Ans. { (a) 17 mi. 1 fur. 1 po. 3 yd. 1 ft. 6 in.
(b) 113 bush. 3 pk. 1 qt. 2 gills
(c) 5 cwt. 1 qr. 3 lb. 3 oz. 4 dr.
(d) 4 sq. yd. 4 sq. ft. $2\frac{5}{8}$ sq. in.
(e) £6 19s. $2\frac{1}{2}$ d.
(f) 6 tons 14 cwt. 1 qr. 18 lb. $0\frac{11}{15}$ oz.
(g) 4 lb. 8 oz. 7 dwt. $7\frac{3}{4}$ gr.
(h) £2 15s. $11\frac{3}{8}$ d.

THE METRIC SYSTEM

65. The metric system is the system of weights and measures used in all the countries of Europe except the United Kingdom and Russia. It is also used in the monetary system of these countries and in the United States of America ; and by scientists throughout the world. The name is derived from *metre* (from the Greek word *metron*, a measure), the unit from which all other units in this system are derived.

NOTE.—This system was legalized by Parliament a few years ago, but was not made compulsory. It is very desirable, however, that anyone likely to be engaged in drawing offices or workshops wherein much Continental work is undertaken, should be familiar with metric measurements, as the drawings supplied by Continental firms and those that have to be supplied to them are dimensioned in this system.

66. The system is a decimal one, that is, in any table of measures each unit is 10 times as great as the one below it; it therefore corresponds with the ordinary scale of numbers; hence, reduction is not required.

67. The **metre** is very nearly one ten-millionth part of the distance from the equator to the pole, and in 1794 a platinum bar was constructed measuring 1 metre and placed in the French Record Office. The length of the metre in inches is variously stated by different authorities; the value most generally accepted is 1 metre = 39.370432 inches. For the purposes of this arithmetic the length of the metre will be taken as 39.37 inches.

68. The metric system has three principal units, the only ones that will be considered here; they are: the **metre** (pronounced meeter), the unit of length; the **litre** (pronounced leeter), the unit of capacity; and the **gramme** (pronounced gram), the unit of weight. Each of the units has its multiples and subdivisions.

69. The names of the denominations higher than the leading unit are obtained by prefixing to the name of the unit the Greek names *dec'a* (or *dek'a*), *hec'to*, *kil'o*, and *myr'ia*. Thus, taking the metre as the unit:

Dec'a me'tre,	10 metres, deca	meaning	10;
Hec'to me'tre,	100 metres, hecto	meaning	100;
Kil'o me'tre,	1,000 metres, kilo	meaning	1,000;
Myr'ia me'tre,	10,000 metres, myria	meaning	10,000.

70. The names of the denominations lower than the leading unit are obtained by prefixing to the name of the unit the Latin names *dec'i*, *cent'i*, and *mil'li*. Thus:

Dec'i me'tre,	$\frac{1}{10}$ metre, deci	meaning	$\frac{1}{10}$;
Cent'i me'tre,	$\frac{1}{100}$ metre, centi	meaning	$\frac{1}{100}$;
Mil'li me'tre,	$\frac{1}{1000}$ metre, milli	meaning	$\frac{1}{1000}$.

The prefixes and their meanings, as given in this and the preceding article, should be carefully committed to memory.

MEASURES OF LENGTH

10 millimetres (<i>mm.</i>)	= 1 centimetre	<i>cm.</i> =	·3937 in.
10 centimetres	= 1 decimetre	<i>dm.</i> =	3·937 in.
10 decimetres	= 1 metre	<i>m.</i> =	3·281 ft.
10 metres	= 1 decametre	<i>Dm.</i> =	32·81 ft.
10 decametres	= 1 hectometre	<i>Hm.</i> =	328·09 ft.
10 hectometres	= 1 kilometre	<i>Km.</i> =	·62137 m.
10 kilometres	= 1 myriametre	<i>Mm.</i> =	6·2137 m.

It will be noticed that the abbreviations of the names of the units of the higher denominations begin with a *capital* letter, and those of the lower denominations with a *small* letter. A decimetre divided into centimetres and millimetres is shown in the figure below.



71. The decimetre, decametre, hectometre, and myriametre are rarely used. The metre is used for those measurements that would ordinarily be made in feet and yards; the centimetre and millimetre for those ordinarily made in inches and hundredths of an inch, and the kilometre for long distances, such as are ordinarily expressed in miles.

72. The *approximate* length of the metre is 40 inches, or, more closely, $39\frac{3}{8}$ inches (39·4 inches is a close approximation). When great exactness is not required, 40 inches is near enough. Hence, 1 decimetre equals $40 \times \frac{1}{10} = 4$ inches; 1 centimetre = ·4 inch; and 1 millimetre = ·04 inch. The length of 1 kilometre = $\frac{5}{8}$ mile, or 5 furlongs, approximately.

73. Since metres, centimetres, and millimetres have a decimal scale, 41·457 metres may be read as 41 and 457-thousandths metres, or as 41 metres 45 centimetres 7 millimetres. It might also be read 41 metres 45 and 7-tenths centimetres, or 41 metres 457 millimetres.

74. To express metric values decimally in terms of a given unit :

EXAMPLE.—Express 4 Km. 3 Hm. 1 m. 5 cm. and 9 mm. in metres.

SOLUTION.—Write the metres in units place, at the left of the decimal point; then, writing the other units in order, supply the missing denominations (if any) with ciphers, as 4301·059 metres. Ans.

Had it been required to express the above in kilometres, the result would have been 4·301059 Km.

75. Rule.—Write the number of given units ; then, the numbers of the higher denominations on the left, as integers, and those of the lower denominations on the right, as decimals, supplying any missing denomination with a cipher.

76. To reduce a metric number to higher or lower denominations, all that is necessary is to move the decimal point. Thus, 74.1026 Km. = 741.026 Hm. = 7,410.26 Dm. = 74,102.6 m. = 741,026 dm. = 7,410,260 cm. = 74,102,600 mm. Also, to reduce 97,452 cm. to kilometres, write 97,452 cm. Now, beginning with the decimal point (which, of course, follows the 2), count centi, deci, metre, deca, hecto, kilo, placing the decimal point after the last-named unit, and supplying any missing ones with ciphers. The result is 0.97452 Km.

EXAMPLES FOR PRACTICE

77. Solve the following :

- | | | |
|---------------------------------------|--------|-----------------|
| (a) Reduce 25.7 Km. to metres. | Ans. { | (a) 25,700 m. |
| (b) Reduce 43.4 m. to millimetres. | | (b) 43,400 mm. |
| (c) Reduce 4,823.6 m. to hectometres. | | (c) 48.236 Hm. |
| (d) Reduce 48,639 cm. to metres. | | (d) 486.39 m. |
| (e) Reduce 738.4 Dm. to centimetres. | | (e) 738,400 cm. |

Express the following in words :

- | | | |
|---------------|--------|--------------------------------------|
| (f) 14.5 m. | Ans. { | (f) 14 and 5-tenths metres |
| (g) 47.3 Dm. | | (g) 47 and 3-tenths decametres |
| (h) 568 Hm. | | (h) 568 hectometres |
| (i) 434.5 Km. | | (i) 434 and 5-tenths kilometres |
| (j) 27.4 mm. | | (j) 27 and 4-tenths millimetres |
| (k) 92.76 cm. | | (k) 92 and 76-hundredths centimetres |

SQUARE MEASURE

- | | | |
|---|---|---|
| 100 sq. millimetres }
(sq. mm. or mm ² .) | = | 1 sq. centimetre sq. cm. or cm ² . |
| 100 sq. centimetres . | = | 1 sq. decimetre sq. dm. or dm ² . |
| 100 sq. decimetres . | = | { 1 sq. metre or sq. m. or m ² .
1 centiare ca. |
| 100 sq. metres }
(100 centiares) | = | { 1 sq. decametre or . . . sq. Dm. or Dm ² .
1 are a. |
| 100 sq. decametres }
(100 ares) | = | { 1 sq. hectometre or . . . sq. Hm. or Hm ² .
1 hectare Ha. |
| 100 sq. hectometres }
(100 hectares) | = | 1 sq. kilometre sq. Km. or Km ² . |

COMMON EQUIVALENTS

1 sq. cm.	=	0.1550 sq. in.
1 sq. dm.	=	0.1076 sq. ft.
1 sq. m.	=	1.1960 sq. yd. = 10.7637 sq. ft.
1 are	=	3.954 sq. po., or approximately 4 sq. po.
1 hectare	=	2.471 ac., or approximately $2\frac{1}{2}$ ac.
1 sq. Km.	=	0.3861 sq. mi.

78. In square measure, the scale is 100 (10×10); hence, in reducing units of any denomination to a lower or higher denomination, the decimal point must be moved two places for each denomination. Thus, to reduce 42.09872 sq. m. to square centimetres, begin at the decimal point, point off two places to the right and say square decimetres, then two more places and say square centimetres, obtaining 420,987.2 sq. cm. If square millimetres were desired, it would be necessary to annex a cipher in reducing 42.09872 sq. m.; thus, 42.09872 sq. m. = 42,098,720 sq. mm. Reduction to higher denominations is performed in the same manner, moving the decimal point to the left. Thus, 42,098,720 mm². = 0.4209872 ares.

79. The square metre is used in measuring floors, ceilings, and other ordinary surfaces; the are and hectare in measuring land, and the square kilometre in measuring provinces and countries.

EXAMPLES FOR PRACTICE

80. Solve the following :

- Write 78.29 ares as centiares; also as hectares.
- Write 9 m². as square decimetres; also as square centimetres.
- In 3,246 ca., how many ares?
- Express 7,041.6 sq. dm. in ares.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \text{ 7,829 ca. or } .7829 \text{ Ha.} \\ (b) \text{ 900 dm}^2. \text{ or } 90,000 \text{ cm}^2. \\ (c) \text{ 32.46 a.} \\ (d) \text{ } .70416 \text{ a.} \end{array} \right.$$

MEASURES OF VOLUME OR CUBIC MEASURE

1,000 cubic millimetres } (<i>cu. mm.</i> or mm^3 .)	}	. . . = 1 cubic centimetre . . <i>cu. cm.</i> or cm^3 .
1,000 cubic centimetres . . .		
1,000 cubic decimetres . . .		
		. . . = 1 cubic decimetre . . <i>cu. dm.</i> or dm^3 .
		. . . = 1 cubic metre . . <i>cu. m.</i> or m^3 .

COMMON EQUIVALENTS

1 cu. cm. = .06102 cu. in.

1 cu. dm. = 61.024 cu. in.

1 cu. m. = 61,024 cu. in. = 35.3148 cu. ft. = 1.308 cu. yd.

In measuring wood, the cubic metre is called a *stere*.

81. Units higher than the cubic metre are not used, except in denoting the volume of planets.

In cubic measure, the scale is 1,000 ($10 \times 10 \times 10$); hence, to reduce units from one denomination to another, apply the method given in Art. 78, moving the decimal point three places each time, instead of two places, as in square measure.

MEASURES OF CAPACITY OR FLUID MEASURE

10 millilitres (<i>ml.</i>)	= 1 centilitre <i>cl.</i>
10 centilitres	= 1 decilitre <i>dl.</i>
10 decilitres	= 1 litre <i>l.</i>
10 litres	= 1 decalitre <i>Dl.</i>
10 decalitres	= 1 hectolitre <i>Hl.</i>
10 hectolitres	= 1 kilolitre <i>Kl.</i>
10 kilolitres	= 1 myrialitre <i>Ml.</i>

COMMON EQUIVALENTS

1 litre = 61.024 cu. in.

1 litre = 1.7598 (say, 1.76) pints.

1 hectolitre = 3.53148 cu. ft.

1 hectolitre = 21.9941 (say, 22) gallons.

1 hectolitre = 2.74926 (say, 2.75) bushels.

82. The litre is equal in volume to 1 cubic decimetre, i.e., to a cube whose edges measure 1 decimetre each side. The litre is the principal unit in measures of capacity, and is used for both *dry* and *liquid* measure; it is approximately equal to $1\frac{3}{4}$ pints.

83. One millilitre is equal in volume to 1 cubic centimetre, since $1 \text{ cu. cm.} = \frac{1}{1000} \text{ cu. dm.} = \frac{1}{1000} \text{ litre} = 1 \text{ millilitre.}$

The centilitre is a little more than $\frac{1}{14}$ gill; it is used for measuring small quantities of liquids, as medicines. The litre is used for the same purposes as the quart, and the hectolitre for the same purposes as the gallon and the bushel.

The units are reduced from one denomination to another in the same way as measures of length. (See Art. 76.)

EXAMPLES FOR PRACTICE

84. Solve the following :

- Express 8.53 l. as centilitres; as decilitres.
- Express 4.64 Kl. as litres; as hectolitres.
- How many decilitres in 8 litres? In 9.35 litres?
- How many litres in 6.358 cl.?
- In 8,500 litres how many kilolitres?

$$\text{Ans. } \begin{cases} (a) \text{ 853 cl. or 85.3 dl.} \\ (b) \text{ 4,640 l. or 46.4 Hl.} \\ (c) \text{ 80 dl. ; 93.5 dl.} \\ (d) \text{ .06358 l.} \\ (e) \text{ 8.5 Kl.} \end{cases}$$

MEASURES OF WEIGHT

10 milligrams (<i>mg.</i>)	=	1 centigram	<i>cg.</i>
10 centigrams	=	1 decigram	<i>dg.</i>
10 decigrams	=	1 gramme	<i>g.</i>
10 grammes	=	1 decagram	<i>Dg.</i>
10 decagrams	=	1 hectogram	<i>Hg.</i>
10 hectograms	=	1 kilogram, or kilo	<i>Kg.</i>
10 kilograms	=	1 myriagram	<i>Mg.</i>
10 myriagrams	=	1 quintal	<i>Ql.</i>
10 quintals	=	1 tonne, or ton	<i>T.</i>

COMMON EQUIVALENTS

1 gramme	=	$\begin{cases} 1 \text{ cu. cm., or} \\ 1 \text{ ml. of water.} \end{cases}$
1 kilogram	=	$\begin{cases} 1 \text{ cu. dm., or} \\ 1 \text{ litre of water.} \end{cases}$
1 metric ton	=	$\begin{cases} 1 \text{ cu. m., or} \\ 1 \text{ kilolitre of water.} \end{cases}$
1 gramme	=	15.4323 gr. troy.
1 gramme	=	0.03527 oz. avoirdupois.
1 kilogram	=	2.2046 lb. avoirdupois.
1 metric ton	=	.9842 of a ton of 2,240 lb.

NOTE.—In official spelling the ending *me* in the word *gramme* is omitted when a prefix is added, as *kilogram*.

85. The gramme is the principal unit of weight ; it is the weight of 1 cubic centimetre of distilled water at its temperature of maximum density, or 39.2° Fahrenheit. The gramme is used in weighing gold, silver, letters (for postage), and in mixing medicines.

The kilogram is generally called the **kilo**, and is used for the same purposes as the pound avoirdupois.

86. The units of weight most commonly used are the milligram (in chemical analysis), the gramme, the kilo, and the ton. It will be noticed that $1,000 \text{ mg.} = 1 \text{ g.}$, $1,000 \text{ g.} = 1 \text{ Kg.}$, and $1,000 \text{ Kg.} = 1 \text{ ton.}$ The milligram is approximately equal to $\frac{1}{63}$ of a grain ; the gramme, to $15\frac{1}{2}$ grains ; the kilo, to $2\frac{1}{3}$ pounds ; and the metric ton, to 1 English ton. Unless precise equivalents are desired, the values here given are accurate enough for all practical purposes. It should be firmly kept in mind that the weight of 1 litre of water is 1 kilogram, and that 1 litre of water will fill a space of 1 cubic decimetre.

OPERATIONS WITH METRIC UNITS

87. The rules for adding, subtracting, multiplying, and dividing metric numbers are the same as for the corresponding operations in decimals. Be sure, however, that all numbers are expressed in the same units.

EXAMPLE 1. —What is the sum of 45.68 Dm., 63.4 Hm., and 6,845 cm. ?

SOLUTION. —Reducing all the numbers to the same unit, say metres, and adding as in decimals, the sum is 6,865.25 m.

$$\begin{array}{r} 45.68 \\ 63.40 \\ 6.845 \\ \hline 686.525 \text{ m. Ans.} \end{array}$$

EXAMPLE 2. —From 5.462 kilos take 7 Hg. 4 g. 9 cg.

SOLUTION. Expressing both numbers in kilos, and subtracting, the result is 4.75791 Kg.

$$\begin{array}{r} 5.462 \\ -7.0409 \\ \hline 4.75791 \text{ Kg. Ans.} \end{array}$$

EXAMPLES FOR PRACTICE

88. Solve the following :

1. What is the difference between 8.5 Kg. and 976 grammes ?
Ans. 7.524 Kg.
2. How much silk is contained in $12\frac{1}{2}$ pieces, if each piece contains 48.75 m. ?
Ans. 609.375 m.
3. If 735 kilos of flour are equally distributed among 35 persons, how many kilos will each person receive ?
Ans. 21 Kg.
4. In example 3, how many pounds did each person receive, counting $2\frac{1}{5}$ pounds to a kilo ?
Ans. 46.2 lb.
5. Add 74 Hl., 147.2 l., 5,006.3 cl., and 6.5421 Kl., expressing the result in litres.
Ans. 14,139.363 l.
6. What is the weight in pounds of 197,862 cl. of water at 39.2° Fahrenheit ?
Ans. 4,362 lb.

ARITHMETIC

(PART 5)

INVOLUTION

1. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3×3 , or 9, is the *second power*, or **square**, of 3; $3 \times 3 \times 3$, or 27, is the *third power*, or **cube**, of 3; $3 \times 3 \times 3 \times 3$, or 81, is the **fourth power** of 3. Also, 3 is the **second root**, or **square root**, of 9; 3 is the **third root**, or **cube root**, of 27; 3 is the **fourth root** of 81.

2. For the sake of brevity,

3×3 is written 3^2 , and read **three squared**,
or *three to the second power*;

$3 \times 3 \times 3$ is written 3^3 , and read **three cubed**,
or *three to the third power*;

$3 \times 3 \times 3 \times 3$ is written 3^4 , and read **three fourth**,
or *three to the fourth power*;

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is known as the **index**. Thus, in 3^{12} , the number ¹² is the index, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

In an expression like 3^5 , the index ⁵ shows how often 3 is used as a factor. Hence, if the index of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

3. If the side of a square contains 5 inches, the area of the square contains 5×5 , or 5^2 , square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3 , cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

4. To find any power of a number :

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.—

$$35 \times 35 \times 35$$

$$\text{or} \quad \begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \\ 35 \\ \hline 6125 \\ 3675 \\ \hline \end{array}$$

$$\text{cube} = 42875 \quad \text{Ans.}$$

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.—

$$15 \times 15 \times 15 \times 15$$

$$\text{or} \quad \begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline \end{array}$$

$$\text{fourth power} = 50625 \quad \text{Ans.}$$

EXAMPLE 3.— $1.2^3 = \text{what?}$

SOLUTION.—

$$1.2 \times 1.2 \times 1.2$$

$$\text{or} \quad \begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 2.88 \\ 1.44 \\ \hline \end{array}$$

$$\text{cube} = 1.728 \quad \text{Ans.}$$

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

5. Rule.—I. To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the index.

II. To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the index.

EXAMPLES FOR PRACTICE

6. Raise the following to the powers indicated :

(a) 85^3 .

(b) $\left(\frac{12}{15}\right)^3$.

(c) $6 \cdot 5^3$.

(d) 14^4 .

(e) $\left(\frac{3}{4}\right)^3$.

(f) $\left(\frac{5}{8}\right)^3$.

(g) $\left(\frac{7}{2}\right)^3$.

(h) $1 \cdot 4^5$.

Ans. $\left\{ \begin{array}{l} (a) \ 7,225 \\ (b) \ \frac{144}{1500} \\ (c) \ 42 \cdot 25 \\ (d) \ 38,416 \\ (e) \ \frac{27}{64} \\ (f) \ \frac{125}{512} \\ (g) \ \frac{343}{8} \\ (h) \ 5 \cdot 37824 \end{array} \right.$

EVOLUTION

DEFINITIONS AND GENERAL REMARKS

7. **Evolution** is the reverse of involution. It is the process of finding the roots of numbers.

8. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2 (or 2^2) = 4.

9. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$ (or 3^3) = 27.

10. The **fourth root** of a number is that number which, when used four times as a factor, produces the number.

Thus, 9 is the fourth root of 6,561, since $9 \times 9 \times 9 \times 9$ (or 9^4) = 6,561.

11. The **fifth root** of a number is that number which, when used five times as a factor, produces the number.

Thus, 7 is the fifth root of 16,807, since $7 \times 7 \times 7 \times 7 \times 7$ (or 7^5) = 16,807.

12. The processes of finding squares and cubes and square roots and cube roots are very frequently employed in connection with the solution of problems pertaining to mensuration and engineering. The process of raising a number to some power, the index of the number being integral (*integral* is the adjective for integer, i.e., an integral number is one that does not contain a fraction or decimal), is very simple; but the reverse process, that of finding the roots, is very long and laborious, for which reason tables are generally employed. The tables so used are of two kinds: those giving the roots directly, and logarithms. While the roots of numbers can be found without the aid of a table, it is not customary to do this except in the case of square root, which is comparatively easy. At the same time, it is well to know some general method of finding the roots of numbers, as it may be necessary to find a root when a table is not at hand. Both a table and a general method are explained hereafter, and whichever is preferred may be used.

13. Some idea of the importance of the processes of involution and evolution may be obtained from the following:

In finding the area of a square or a circle, it is necessary to square the length of a certain line; conversely, in finding the side of a square or the diameter of a circle that will have a given area, it is necessary to find the square root. In finding the volume of a cube or a sphere, it is necessary to cube the length of a certain line; conversely, in finding the length of one of the edges of a cube or the diameter of a sphere that will have a given volume, it is necessary to find the cube root. There are many other cases where it is required to extract square root and cube root, but enough has been stated so far to show the importance of the processes.

14. Cube root is not required as often as square root, and fourth and fifth roots are required but seldom, the fifth root being required oftener than the fourth root. Roots higher

than the fifth are practically never required. As examples, it may be stated that the fourth root is required in finding the diameter of a revolving shaft that is to transmit a given power. The fifth root is required in finding the diameter of a pipe that will discharge a given amount of a fluid in a given time; in certain problems pertaining to mine ventilation.

15. At the end of this Section is given a very useful table, by means of which most of the ordinary problems in evolution (and also involution) may be readily solved. Before studying the explanations, however, those definitions and properties of numbers given in Part 3, and those which follow, should be carefully reviewed.

Each of the solutions given further on should be carefully studied, as it is intended to illustrate some feature not present in the other examples. *The actual work should be done on a separate sheet of paper, and each operation should be performed in the order given in the solution. It will be found advisable to work the Examples for Practice also. Anyone who follows these instructions will have no difficulty in using the table.*

16. The radical sign $\sqrt{}$, when placed before a number, indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. 17.

17. The index of the root is a *small figure placed over* and to the *left* of the radical sign, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100,

$\sqrt[3]{125}$ denotes the *cube root* of 125,

$\sqrt[4]{256}$ denotes the *fourth root* of 256,

$\sqrt[5]{243}$ denotes the *fifth root* of 243, and so on.

18. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

19. The smallest number that can be written with one figure is 1, and the largest is 9. Their corresponding squares are 1 and 81, respectively. The smallest number that can be written

with two figures is 10, and the largest is 99. Their corresponding squares are 100 and 9,801, respectively. Arrange the following numbers and their squares thus :

$$\begin{array}{ll} 1^2 = 1 & 9^2 = 81 \\ 10^2 = 100 & 99^2 = 9,801 \\ 100^2 = 10,000 & 999^2 = 998,001 \\ 1,000^2 = 1,000,000 & 9,999^2 = 99,980,001 \end{array}$$

It is seen that the square of a number containing one figure is written with one or two figures ; the square of a number containing two figures is written with three or four figures, etc. Or, the square of a number is always written with twice as many figures as the given number, or twice as many less one.

20. In order to find the square root of a number, the first step is to point off the number into periods, or groups, of two figures each, beginning at the right if the number is integral, and at the decimal point if the number is decimal. The number of periods will be equal to the number of figures in the root, if the number is a perfect square.

If the last period on the right of a decimal number contains but one figure, annex a cipher to complete the period.

Thus, the square root of 83,740,801 must contain four figures, since, pointing off the periods, it becomes 83'74'08'01, and has four periods ; consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain three figures, since there are (5'06'25) three periods. The extreme left-hand period may contain either one or two figures, according to the size of the number squared.

21. The square of any number wholly decimal always contains twice as many figures as the number squared. For example $\cdot 1^2 = \cdot 01$, $\cdot 13^2 = \cdot 0169$, $\cdot 751^2 = \cdot 564001$, etc.

The square of a number partly decimal contains twice as many decimal places as there are decimal places in the number. For example, $12\cdot 35^2 = 152\cdot 5225$.

22. It will also be noticed that the square of a decimal (or a proper fraction) is always less than the decimal. Hence, the square root of a number wholly decimal (or of a proper fraction) is greater than the number itself.

If it is required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to point off a decimal is to begin at the decimal point, and, going toward the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

If the decimal point of a number is moved one or more places to the right (or left), the decimal point in the square will be moved twice as many places to the right (or left), thus :

$$3.567^2 = 12'.72'34'.89$$

$$356.7^2 = 12'72'34'.89$$

$$.3567^2 = .12'72'34'.89$$

It will be observed that these squares differ only in the position of the decimal point, and when divided into periods, the corresponding group in each square contains the same figures.

Later it will be shown that numbers containing like periods have like figures in their roots.

23. There are comparatively few numbers that can be separated into exactly equal factors ; these numbers are called **perfect powers**, and the factors are called *rational factors*. Numbers that cannot be separated into exactly equal factors are called **imperfect powers**, and the factors are called *irrational factors*. In the numbers from 1 to 1,000, inclusive, there are only 48 perfect powers, not counting 1, and of these there are only 30 perfect squares and 9 perfect cubes. These perfect powers are as follows : Perfect squares, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961 ; perfect cubes, 8, 27, 64, 125, 216, 343, 512, 729, 1,000 ; perfect fourth powers, 16, 81, 256, 625 ; perfect fifth powers, 32, 243 ; perfect sixth powers, 64, 729 ; perfect seventh power, 128. Of these numbers, it will be noticed that two of the perfect cubes, 64 and 729, the four perfect fourth powers, and the two perfect sixth powers are duplicated among the squares and cubes ; hence, there are only 40 different numbers between 1 and 1,000 that are perfect powers.

24. The root of any number that cannot be divided into as many equal factors as there are units in the index of the

root contains an interminable decimal. For example, the number 20 lies between $16 (= 4^2)$ and $25 (= 5^2)$; hence, the square root of 20, or $\sqrt{20}$, is greater than 4 and less than 5, and is therefore equal to 4 plus an interminable decimal. In other words, no matter to how many figures the square root of 20 may be calculated, the root will never be found exactly.

25. Although the root of an imperfect power cannot be found exactly, as close an approximation may be obtained as is desired. In practice, five significant figures are all that are likely to be required, and four are generally sufficient. In the following examples, all roots will be calculated to five figures, unless the given number is a perfect power whose root contains five figures or less.

A table that gives five significant figures of the root will therefore be sufficiently correct for all practical purposes. Such a table will be found at the end of this Section, and its use will now be explained, beginning with square root and then continuing with cube, fourth, and fifth roots.

APPROXIMATE METHODS OF EXTRACTING ROOTS

SQUARE ROOT

26. The first step in finding the square root is to point off the number into periods of two figures each, as previously described. The second step is to move the decimal point until it falls between the first (left-hand) period containing a digit and the next period to the right; in other words, the second step is to make the first period the integral part of the number, if not already so. Call the result the **altered number**.

The third step is to search the table in the columns headed n^2 and find two consecutive numbers, one less and the other greater than the altered number. Opposite the smaller of the two numbers in the column headed n will be found the first three figures of the square root. All the numbers in the columns headed n are printed in heavy-face type, and it will be noticed that there are two such columns on each page.

The fourth step is to find two more figures of the root and the fifth step is to locate the decimal point in the root; these steps can be best illustrated by examples.

EXAMPLE 1.—What is the square root of 31,416?

SOLUTION.—Pointing off into periods, the result is $3'14'16$. In this case the given number is integral and the decimal point must be imagined to be at the right of the unit figure, as, $3'14'16.0$. Moving the decimal point so that the first period becomes the integral part of the number, the altered number is 3.1416. Searching the table in the columns headed n^2 , 3.1416 is found to lie between 3.1329 and 3.1684, on page 1. The number in the column headed n opposite 3.1329 is 1.77. The first three figures of the root are therefore 177. Find the difference between the two numbers between which the given number falls (call this the **first difference**), and the difference between the smaller number and the given number (call this the **second difference**); divide the second difference by the first difference, carrying the quotient to three decimal places and increasing the second figure by 1 if the third is 5 or a greater digit. The two figures of the quotient thus determined will be the fourth and fifth figures of the root. In the present example, dropping decimal points in the remainders, $3.1684 - 3.1329 = 355$, the first difference; $3.1416 - 3.1329 = 87$, the second difference; $87 \div 355 = .245 +$, or .25. Hence, the first five figures of the root are 17,725. The decimal point is located in all cases by reference to the original number, after pointing off into periods.

There will be as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and there will be as many ciphers following the decimal point in the root as there are complete cipher periods following the decimal point in the given number.

Applying this principle, there are three periods preceding the decimal point in the given number; hence, there are three figures preceding the decimal point in the root, and $\sqrt{31,416} = 177.25$. Ans.

The operations may be arranged thus:

$$\begin{array}{rcl}
 1.78^2 & = & 3.1684 \quad \text{altered number} = 3.1416 \\
 1.77^2 & = & 3.1329 \quad \quad \quad 1.77^2 = 3.1329 \\
 \text{first difference} & = & 355 \quad \quad \quad \text{second difference} = 87 \\
 355) 87.000 (.245, \text{ or } .25 & & \\
 \quad 710 & & \\
 \quad 1600 & & \\
 \quad 1420 & & \\
 \quad \quad 1800 & & \\
 \quad \quad 1775 & &
 \end{array}$$

First five significant figures of the root (the fifth figure being corrected) are 17725.

Locating decimal point, $\sqrt{31,416} = 177.25$. Ans.

NOTE.—Had the given number been 314.16, 3.1416, .031416, .00031416, etc., the significant figures of the root would have been the same as in the preceding case, since the altered number would have been 3.1416 in each instance; the decimal point would have been differently located, however. Thus, pointing off into periods, the given numbers are respectively 3'14'16, 3'14'16, .03'14'16, and .00'03'14'16, and the corresponding square roots are 17.725, 1.7725, .17725, and .017725.

EXAMPLE 2.—What is the square root of .0031416 ?

SOLUTION.—Pointing off into periods, the result is .00'31'41'60; moving the decimal point, the altered number is 0031.4160 or 31.4160. Referring to the table in the columns headed n^2 , 31.4160 is found to lie between 31.3600, opposite 5.60, and 31.4721, opposite 5.61, on page 5. The first three figures of the root are therefore 560. The first difference is $31.4721 - 31.3600 = 1121$; the second difference is $31.4160 - 31.3600 = 560$; $560 \div 1121 = .499 +$, or .50. Therefore, the first five figures of the root are 56050. Since there is one complete cipher period immediately following the decimal point in the given number, there is one cipher following the decimal point in the root, and $\sqrt{.0031416} = .056050$, or .05605. Ans.

EXAMPLE 3.—What is the square root of 7,500 ?

SOLUTION.—Pointing off into periods, the result is 75'00. Moving the decimal point, the altered number is 75.00 or 75. Referring to the table, in the columns headed n^2 , 75 is found to lie between 74.9956 and 75.1689, on page 8. The first difference is $75.1689 - 74.9956 = 1733$; the second difference is $75 - 74.9956 = 44$; $44 \div 1733 = .025 +$, or .03. The first three figures of the square root are 866, and the first five are 86603. Since there are two periods in the given number, there must be two figures in the integral part of the root; hence, $\sqrt{7,500} = 86.603$. Ans.

EXAMPLE 4.—What is the square root of 49,074,561,800 ?

SOLUTION.—Pointing off into periods, the result is 4'90'74'56'18'00. Moving the decimal point, the altered number is 4.90745618. Referring to the table in the columns headed n^2 , the altered number is found to lie between 4.8841 and 4.9284, on page 2. It is not necessary or advisable to retain more figures in the altered number than there are in the two numbers of the table between which it falls, in this case five figures; hence, throw off all figures after the fifth, increasing the fifth figure by 1 if the sixth is 5 or a greater digit. Doing so, the altered number becomes 4.9075. The first difference is $4.9284 - 4.8841 = 443$; the second difference is $4.9075 - 4.8841 = 234$; $234 \div 443 = .528 +$, or .53. The number opposite 4.8841 in the column headed n is 2.21; hence, the first five figures of the root are 22153. Since there are six periods on the left of the decimal point in the given number, there are six figures in the integral part of the root; and as only five figures were determined, write a cipher for the sixth figure, obtaining 221,530. Therefore, to five significant figures, $\sqrt{49,074,561,800} = 221,530$. Ans.

EXAMPLES FOR PRACTICE

27. Find the square root of :

(a) 5.	Ans. {	(a) 2.2361
(b) .005.		(b) .070711
(c) 149,263.		(c) 386.35
(d) 792.06.		(d) 28.144
(e) 88.527.		(e) 9.4089
(f) 1,000.		(f) 31.623

CUBE ROOT

28. An examination of the table will show that the columns headed n contain all the numbers between 1.00 and 9.99, inclusive, that is, all numbers that can be expressed by three figures, disregarding the decimal point. The columns headed n^2 contain the squares of all the numbers in the columns headed n . The columns headed n^3 , n^4 , and n^5 contain, respectively, the first six figures of the cubes and the fourth and fifth powers of the numbers in the columns headed n . The preceding explanation for square root will suffice for the cube, fourth, and fifth roots, the only difference being in the first step—that of pointing off into periods. For cube root each period (except the first) must contain *three* figures. The process is the same; that is, begin at the decimal point and point off to the left and to the right periods of three figures each. If the right-hand period is not complete, annex ciphers until it contains three figures. Then proceed exactly as before, using the columns headed n^3 , and locate the decimal point by means of the principle given in the explanation to example 1, Art. 26.

EXAMPLE 1.—What is the cube root of .0000062417 ?

SOLUTION.—Pointing off into periods of three figures each, the result is .000'006'241'700. Moving the decimal point until it immediately follows the first period that contains a digit, the altered number is 6.241700 or 6.24170, using but six figures, to correspond with the six figures of the table. Referring to the table, and looking in the columns headed n^3 , the altered number is found to lie between 6.22950 and 6.33163, on page 1. The first difference is $6.33163 - 6.22950 = 10213$; the second difference is $6.24170 - 6.22950 = 1220$; $1220 \div 10213 = .119 +$, or .12. The number opposite 6.22950 in the column headed n is 1.84; hence, the first five significant figures of the cube root are 18412. Since there is one

complete cipher period following the decimal point, there will be one cipher following the decimal point in the root; therefore, $\sqrt[3]{.0000062417} = .018412$. Ans.

EXAMPLE 2.—What is the cube root of 50,932,676?

SOLUTION.—Pointing off into periods of three figures each, the result is 50'932'676. Moving the decimal point, the altered number is 50.932676. Reducing to six figures and increasing the sixth figure by 1, since the seventh figure is 7, the altered number becomes 50.9327. Referring to the table in the columns headed n^3 , 50.9327 is found to lie between 50.6530 and 51.0648, on page 3, the first three figures of the root being 370. The first difference is $51.0648 - 50.6530 = 4118$; the second difference is $50.9327 - 50.6530 = 2797$; $2797 \div 4118 = .679 +$, or .68; hence, the first five figures of the root are 37068. Since the integral part of the given number contains three periods, there are three figures in the integral part of the root; therefore, $\sqrt[3]{50,932,676} = 370.68$. Ans.

EXAMPLE 3.—What is the cube root of .834?

SOLUTION.—There is but one period. Moving the decimal point, the altered number is 834, which falls between 833.238 and 835.897 in the columns headed n^3 on page 9 of the table. The first three figures of the root are 941. The first difference is $835.897 - 833.238 = 2659$; the second difference is $834 - 833.238 = 762$; $762 \div 2659 = .286 +$, or .29; hence, the first five figures of the root are 94129. Since the given number is wholly decimal, the root is wholly decimal; and since there is no complete cipher period between the decimal point and the first digit of the given number, there are no ciphers between the decimal point and the first digit of the root. Therefore, $\sqrt[3]{.834} = .94129$. Ans.

EXAMPLES FOR PRACTICE

29. Find the cube root of:

(a) 78,347.809639.

(b) 2.

(c) 4,180,769,192.462.

(d) .696.

(e) .375.

(f) 513,229.783302144.

Ans. $\left\{ \begin{array}{l} (a) 42.79 \\ (b) 1.2599 \\ (c) 1,611.0 \\ (d) .88621 \\ (e) .72112 \\ (f) 80.064 \end{array} \right.$

FOURTH AND FIFTH ROOTS

30. The processes for fourth and fifth roots are exactly the same as for square and cube roots, except that for fourth root the given number is pointed off into periods of *four* figures each

and the columns headed n^4 are used ; while for fifth root the given number is pointed off into periods of *five* figures each and the columns headed n^5 are used.

EXAMPLE 1.—What is the fourth root of 3,690·72 ?

SOLUTION.—Pointing off into periods of four figures each, the result is 3690·7200. In the present case it is not necessary to move the decimal point, since it already follows the first period containing a digit ; and since six figures only are required, throw off the two ciphers on the right and look in the table under the headings n^4 and find between what two numbers 3,690·72 falls. Referring to page 7, the given number falls between 3,682·56 and 3,701·51. The first difference is 3,701·51 — 3,682·56 = 1895 ; the second difference is 3,690·72 — 3,682·56 = 816 ; $816 \div 1895 = \cdot430 +$, or $\cdot43$. The number in the column headed n and opposite 3,682·56 is 7·79 ; hence, the first five figures of the root are 77943. Since there is but one integral period in the given number, there is but one integral place in the root ; therefore, $\sqrt[4]{3,690\cdot72} = 7\cdot7943$. Ans.

EXAMPLE 2.—What is the fifth root of ·7854 ?

SOLUTION.—Pointing off into periods of five figures each, the result is ·78540 ; moving the decimal point, the altered number is 78540. Referring to the table and looking in the columns headed n^5 , 78540 is found to lie between 78196·0 and 78607·6 on page 9. The first difference is 78607·6 — 78196·0 = 4116 ; the second difference is 78540·0 — 78196·0 = 3440 ; $3440 \div 4116 = \cdot835 +$, or $\cdot84 -$. In the column headed n and opposite 78196·0 is 9·52 ; hence the first five figures of the root are 95284. Since the given number is a decimal, and the first period contains digits, the root is a decimal, and there are no ciphers between the decimal point and the first digit ; therefore, $\sqrt[5]{\cdot7854} = \cdot95284$. Ans.

EXAMPLE 3.—What is the fifth root of 497·23 ?

SOLUTION.—As only six figures are required, annex a cipher, obtaining 497·230. Referring to the table in the columns headed n^5 , 497·230 is found to lie between 495·884 and 503·092 on page 3. The first difference is 503·092 — 495·884 = 7208 ; the second difference is 497·230 — 495·884 = 1346 ; $1346 \div 7208 = \cdot186 +$, or $\cdot19 -$. In the column headed n opposite 495·884 is 3·46 ; hence, the first five figures of the root are 34619. Since there is but one integral period, $\sqrt[5]{497\cdot23} = 3\cdot4619$. Ans.

GENERAL RULE FOR TABLE METHOD

31. The following is a general rule for using the table to find the square root, cube root, fourth root, or fifth root of any number.

Rule.—I. *Beginning at the decimal point, and going to the right and to the left, point off the given number whose root is to be found into periods having as many figures in each period as there are units in the index (see Art. 17) of the root.*

II. *If the decimal point does not immediately follow the right-hand figure of the first period containing a digit, move it from its position until it does follow the right-hand figure of the first period containing a digit. Call the result the altered number. If there are more than six figures in the altered number, drop all after the sixth figure, increasing the sixth figure by 1 if the seventh figure is 5 or a greater number. In the case of square root, retain only five figures, when the left-hand period contains but one significant figure.*

III. *Refer to the table of powers that is placed at the end of this Section, and looking in the columns having at the head "n" and an index of the same value as the index of the root, find between what two numbers in these columns the altered number falls. Subtract the smaller of these two numbers from the larger, and call the result the first difference. Subtract the smaller of the two numbers from the altered number, and call the result the second difference. Divide the second difference by the first difference, and find the quotient to three figures, which reduce to two figures, increasing the second figure by one if the third figure is 5 or a greater number. The two figures thus found are the fourth and fifth figures of the root. The first three figures will be found in the column headed "n," opposite the smaller of the two numbers in the table between which the given number falls.*

IV. *Locate the decimal point by means of the principle that there will be as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and if there are any complete cipher periods immediately following the decimal point, there will be as many ciphers following the decimal point in the root as there are complete cipher periods following the decimal point in the given number.*

32. The reason for the second rule, moving the decimal point, is to get the decimal point in the same relative position that it occupies in the corresponding numbers of the table.

33. On page 10 of the table are given the square, cube, and fourth and fifth powers of the first nine digits. By aid of this little table, the page on which the first three figures of the required root are to be found is instantly located. Thus, after moving the decimal point (if necessary) in the given number, find, in the column in which n has an exponent equal to the index of the root sought, between what two numbers the altered number falls; the number in the left-hand column opposite the smaller of these two numbers is the page of the table sought.

For example, on what pages will the first three figures of the (a) square, (b) cube, (c) fourth, and (d) fifth roots of .00432176 be found?

Pointing off and moving the decimal point, the altered numbers, reduced to six figures, become for each case, (a) 43·2176; (b) 4·32176; (c) 43·2176; (d) 432·176. Referring to page 10 of the table, in the column headed n^2 , 43·2176 falls between 36 and 49; hence, the first three figures of the square root will be found on page 6. 4·32176 falls between 1 and 8 in the column headed n^3 ; hence, the first three figures of the cube root will be found on page 1. 43·2176 falls between 16 and 81 in the column headed n^4 ; hence, the first three figures of the fourth root will be found on page 2. 432·176 falls between 243 and 1,024 in the column headed n^5 ; hence, the first three figures of the fifth root will be found on page 3.

ROOTS OF FRACTIONS

34. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.—
$$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8} \quad \text{Ans.}$$

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.— $\sqrt{\frac{5}{8}} = \sqrt{\cdot 625} = \cdot 79057$, since $\frac{5}{8} = \cdot 625$. Ans.

EXAMPLE 3.—What is the cube root of $\frac{27}{125}$?

SOLUTION.— $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$. Ans.

EXAMPLE 4.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = \cdot 25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\cdot 25} = \cdot 62996$. Ans.

35. Rule.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the root of the decimal.*

EXAMPLES FOR PRACTICE

36. Solve the following:

(a) $\sqrt{\frac{9}{16}} = ?$

(b) $\sqrt[3]{\frac{27}{1728}} = ?$

(c) $\sqrt[3]{\frac{27}{375}} = ?$

(d) $\sqrt[3]{\frac{560}{128}} = ?$

Ans. $\left\{ \begin{array}{l} (a) \frac{3}{4} \\ (b) \frac{1}{4} \\ (c) \cdot 41602 \\ (d) 1\cdot6355 \end{array} \right.$

EXACT METHODS OF EXTRACTING ROOTS

37. In the following articles will be described how more than five figures of the root can be found, and exact methods for extracting any root, the index being an integer, to any number of figures. *Everything from this point to the end of Art. 52 may be omitted if desired.*

38. If a root has been found to five significant figures and it is desired to obtain more figures, perhaps the easiest way is to proceed as follows: Raise the number indicated by the root to the power indicated by an index equal to the index of the root; if the result so obtained is less than the given number, add 1 to the right-hand figure of the root and raise the new number to the same power; but if the result so obtained is greater than the given number, subtract 1 from the right-hand figure of the root and raise the new number to the same power. The result of these operations is to obtain powers of two consecutive numbers having five significant figures each, one of the powers being a little greater and the other a little less

than the given number. Then, proceeding exactly as previously described, divide the second difference by the first difference and obtain four more figures of the root.

Consider example 1, Art. 26. The square root of 31,416 to five significant figures is $177\cdot25$. $177\cdot25^2 = 31,417\cdot5625$, which is a little greater than 31,416; hence, subtracting 1 from $177\cdot25$, $177\cdot24^2 = 31,414\cdot0176$, which is a little less than 31,416. The first difference is $31,417\cdot5625 - 31,414\cdot0176 = 3\cdot5449$. The second difference is $31,416 - 31,414\cdot0176 = 1\cdot9824$; $1\cdot9824 \div 3\cdot5449 = \cdot55922$, or $\cdot5592$ to four figures. Therefore, $\sqrt{31,416} = 177\cdot245592$ to nine significant figures.

Suppose it has been found that the cube root of 37,267 is $33\cdot402$ to five significant figures, and it is desired to obtain more figures. Proceed exactly as before. $33\cdot402^3 = 37,266\cdot397760808$, which is a little less than 37,267; hence, adding 1 to $33\cdot402$, $33\cdot403^3 = 37,269\cdot744941827$. The first difference is $37,269\cdot744941827 - 37,266\cdot397760808 = 3\cdot347181019$; the second difference is $37,267 - 37,266\cdot397760808 = \cdot602239192$; $\cdot602239192 \div 3\cdot347181019 = \cdot17992$ +, or $\cdot1799$ to four figures. Therefore, $\sqrt[3]{37,267} = 33\cdot4021799$ to nine significant figures.

39. As before stated, it is customary to use some kind of table instead of extracting the roots directly; still, the square root is so frequently required that it is well to learn how to extract the square root directly. There are several good methods, and none are much harder than long division. The method given here is the simplest and easiest to remember and apply of any. For cube root and higher roots, all exact methods are long and laborious. The method given in the pages that follow is a general one applicable to any root the index of which is an integer, and is very easy to remember.

SQUARE ROOT

40. The method is best explained by giving several examples with full explanations of each step. To make the work clearer and easier to follow, the figures in the root and the successive numbers resulting from their use are printed in light-face and *Italic* type alternately.

EXAMPLE 1.—Find the square root of 31,505,769.

SOLUTION.—

		<i>root</i>	
		3 1' 5 0' 5 7' 6 9	(5 6 1 3 Ans.
(a)	5	(b)	2 5
	<u>5</u>	(c)	<u>6 5 0</u>
(d)	1 0 0		<u>6 3 6</u>
	<u>6</u>	(e)	<u>1 4 5 7</u>
	1 0 6		<u>1 1 2 1</u>
	<u>6</u>		<u>3 3 6 6 9</u>
	1 1 2 0		<u>3 3 6 6 9</u>
	<u>1</u>		
	1 1 2 1		
	<u>1</u>		
	1 1 2 2 0		
	<u>3</u>		
	1 1 2 2 3		

EXPLANATION.—First point off into periods of two figures each. Now find the largest single number whose square is less than, or equal to, 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write 5 to the right, as in long division, and also to the left, as shown at (a). This is the first figure of the root. Now multiply the 5 at (a) by the 5 in the root, and write the result under the first period, as shown at (b). Subtract and obtain 6 as a remainder.

Add the root already found to the 5 at (a), getting 10, and annex a cipher to this 10, thus making it 100, as shown at (d), which call the **first trial divisor**. Bring down the next period, 50, and annex it to the remainder 6, as shown at (c), which call the **first dividend**. Divide the first dividend (c) by the first trial divisor (d) and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **first complete divisor**.

Multiply the first complete divisor, 106, by 6, the second figure in the root, and subtract the result from the first dividend (c); the remainder is 14. Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1120, which call the **second trial divisor**. Annex the next period to the remainder in the second column, making it 1457, as shown at (e), which call the **second dividend**. Dividing 1457 by 1120, 1 is obtained as the next figure of the root. Adding this last figure of the root to 1120, the result is 1121, the **second complete divisor**. Multiplying the second complete divisor by the third figure of the root and subtracting from the second dividend, 1457, the remainder is 336.

Now, adding the last figure of the root to 1121 and annexing a cipher as before, the result is 11220, the **third trial divisor**. Annexing the next and last period, 69, to the remainder in the second column, the result is 33669, the **third dividend**. Dividing 33669 by 11220, the

result is 3, the fourth figure of the root. Adding the fourth figure of the root to 11220, the result is 11223, the **third complete divisor**. Multiplying the third complete divisor by the fourth figure of the root, the result is 33669. Subtracting the product from the third dividend there is no remainder; hence, $\sqrt{31,505,769} = 5,613$.

EXAMPLE 2.—What is the square root of .000576 ?

SOLUTION.—

$$\begin{array}{r}
 \begin{array}{r}
 2 \\
 \underline{2} \\
 40 \\
 \underline{44} \\
 44
 \end{array}
 \qquad
 \begin{array}{r}
 \overset{\text{root}}{.00'05'76} \text{ (} \overset{\text{Ans.}}{.024} \text{)}
 \end{array}
 \end{array}$$

EXPLANATION.—Beginning at the decimal point and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

41. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

EXAMPLE 1.—What is the square root of 3 ? Find the result to five decimal places.

SOLUTION.—

$$\begin{array}{r}
 \begin{array}{r}
 1 \\
 \underline{1} \\
 20 \\
 \underline{7} \\
 27 \\
 \underline{7} \\
 340 \\
 \underline{3} \\
 343 \\
 \underline{3} \\
 3460 \\
 \underline{2} \\
 3462 \\
 \underline{2} \\
 346400 \\
 \underline{5} \\
 346405
 \end{array}
 \qquad
 \begin{array}{r}
 \overset{\text{root}}{3.00'00'00'00'00} \text{ (} \overset{\text{Ans.}}{1.73205+} \text{)} \\
 \underline{1} \\
 200 \\
 \underline{189} \\
 1100 \\
 \underline{1029} \\
 7100 \\
 \underline{6924} \\
 1760000 \\
 \underline{1732025} \\
 27975
 \end{array}
 \end{array}$$

42. If it is required to find the square root of a mixed number, begin at the decimal point and point off the periods both to the right and to the left. The manner of finding the root will then be exactly the same as in the previous cases.

EXAMPLE.—What is the square root of 258·2449 ?

SOLUTION.— $ \begin{array}{r} 1 \\ \underline{1} \\ 20 \\ \underline{6} \\ 26 \\ \underline{6} \\ 3200 \\ \underline{7} \\ 3207 \end{array} $	$ \begin{array}{r} 258\cdot2449 \text{ (} \overset{\text{root}}{16\cdot07} \text{) } \text{ Ans.} \\ \underline{1} \\ 158 \\ \underline{156} \\ 22449 \\ \underline{22449} \end{array} $
--	---

EXPLANATION.—In the above example, since 320 is greater than 224, a cipher is placed for the third figure of the root and another annexed to 320, making it 3200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258·2449 is 16·07.

43. PROOF.—To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.

44. Rule.—I. *Begin at units place and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there is any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period.

III. *Add the first figure of the root to the number in the first column on the left and annex a cipher to the result; this is the first trial divisor. Annex the second period to the remainder in the second column; this is the first dividend. Divide the dividend by the trial divisor for the second figure in the root and add this figure to the trial divisor to form the complete divisor. Multiply*

the complete divisor by the second figure in the root and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Add the second figure of the root to the complete divisor. Annex a cipher for a new trial divisor, and bring down the third period and annex it to the last remainder for the second dividend.

IV. Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods and continue the operation.

V. If at any time the trial divisor is larger than the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.

VI. If the root contains an interminable decimal and it is desired to terminate the operation at some point, say the fourth decimal place, carry the operation one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.

EXAMPLES FOR PRACTICE

45. Find the square root of :

(a) 186,624.	Ans. {	(a) 432
(b) 2,050,624.		(b) 1,432
(c) 29,855,296.		(c) 5,464
(d) .0116964.		(d) .10815
(e) 198.1369.		(e) 14.0761
(f) 994,009.		(f) 997
(g) 2.375 to four decimal places.		(g) 1.5411
(h) 1.625 to three decimal places.		(h) 1.275
(i) .3025.		(i) .55
(j) .571428.		(j) .75593
(k) .78125.		(k) .88388

CUBE ROOT

46. The process of extracting cube root is very similar to that just described for square root, the work being arranged in three columns instead of two. An example will best illustrate the method.

EXAMPLE.—What is the cube root of 375,741,853,696 ?

SOLUTION.—

(1)	(2)	(3)	root
		375'741'853'696	(7216 Ans.
7	49	343	
<u>7</u>	<u>98</u>	<u>32741</u>	
14	14700	30248	
<u>7</u>	<u>424</u>	2493853	
210	15124	1557361	
<u>2</u>	<u>428</u>	936492696	
212	1555200	936492696	
<u>2</u>	<u>2161</u>		
214	1557361		
<u>2</u>	<u>2162</u>		
2160	155952300		
<u>1</u>	<u>129816</u>		
2161	156082116		
<u>1</u>			
2162			
<u>1</u>			
21630			
<u>6</u>			
21636			

EXPLANATION.—Write the work in three columns as follows : On the right place the number whose cube root is to be extracted, and point it off into periods of *three* figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right, as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root 7, and write the product 49 at the head of column (2). Multiply the number in column (2) by the first figure of the root 7, and write the product 343 under the figures in the first period. Subtract, obtaining 32 for the remainder. Add the first figure of the root to the number in column (1), obtaining 14. Multiply the last number in column (1) by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the last number in column (1), and obtain 21. Annex *one* cipher to the number in column (1), and obtain 210. Also, annex *two* ciphers to the number in column (2), and obtain 14,700 for the first trial divisor. Bring down the next period, annexing it to the remainder in column (3), and obtain 32,741 for the first dividend. Dividing the first dividend by the first trial divisor, the result is $\frac{32741}{14700} = 2 +$; write

EXAMPLE 2.—What is the cube root of 78,347·809639 ?

SOLUTION.—			78'347·809'639 (^{root} 42·79 Ans.
4	16	64	
<u>4</u>	<u>32</u>	<u>14347</u>	
8	4800	<u>10088</u>	
<u>4</u>	<u>244</u>	<u>4259809</u>	
120	5044	<u>3766483</u>	
<u>2</u>	<u>248</u>	<u>493326639</u>	
122	529200	<u>493326639</u>	
<u>2</u>	<u>8869</u>		
124	538069		
<u>2</u>	<u>8918</u>		
1260	54698700		
<u>7</u>	<u>115371</u>		
1267	54814071		
<u>7</u>			
1274			
<u>7</u>			
12810			
<u>9</u>			
12819			

EXPLANATION.—Since the number is mixed, begin at the decimal point and point off periods of three figures each in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the second dividend by the second trial divisor for the third figure of the root, the quotient was 8+; but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore 7 was used instead of 8.

EXAMPLE 3.—What is the cube root of 5 to five decimal places?

SOLUTION.—		5.000'000'000'000'000'000 (^{root} 1.709975+ Ans.
1	1	1
1	2	4000
2	300	3913
1	259	87000000
30	559	78443829
7	308	8556171000
37	8670000	7889992299
7	45981	666178701000
44	8715981	614014317973
7	46062	52164383027
5100	876204300	
9	461511	
5109	876665811	
9	461592	
5118	87712740300	
9	3590839	
51270	87716331139	
9		
51279		
9		
51288		
9		
512970		
7		
512977		

Root correct to five decimal places is 1.70998—. Ans.

EXPLANATION.—In the preceding example annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. In practice it is not necessary to write these cipher periods after the given number; when one cipher is added to the number in column (1) and two to the number in column (2), three ciphers would be added to the number in column (3). Since the quotient obtained by dividing the fourth dividend by the fourth trial divisor is $666178701000 \div 87712740300 = 7.5+$, the root correct to five decimal places is 1.70998—.

EXAMPLE 4.—What is the cube root of .5 to four decimal places?

SOLUTION.—		^{root} ·500'000'000'000 (·7937+ Ans.
7	40	343
<u>7</u>	<u>98</u>	<u>157000</u>
14	14700	<u>150039</u>
<u>7</u>	<u>1971</u>	6961000
210	16671	<u>5638257</u>
<u>9</u>	<u>2052</u>	1322743000
219	1872300	<u>1321748953</u>
<u>9</u>	<u>7119</u>	994047
228	1879419	
<u>9</u>	<u>7128</u>	
2370	188654700	
<u>3</u>	<u>166579</u>	
2373	188821279	
<u>3</u>		
2376		
<u>3</u>		
23790		
<u>7</u>		
23797		

EXPLANATION.—In the above example annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The largest number whose cube is less than 500 is 7; write this as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples. Since the quotient obtained by dividing the third dividend by the third trial divisor is $1322743000 \div 188654700 = 7.0+$, the root is correct as found to the fourth decimal place.

column. Add the first figure of the root to the number in the first column. Annex one cipher to the last number in the first column, two ciphers to the last number in the second column, and annex the second period to the remainder in the third column. The last numbers in the second and third columns are, respectively, the first trial divisor and the first dividend.

III. Divide the first dividend by the first trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the first trial divisor to form the first complete divisor. Multiply the first complete divisor by the second figure of the root, and subtract the result from the first dividend in the third column. Add the second figure of the root to the number in the first column; multiply the sum by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the number in the first column. Annex one cipher to the number in the first column, and two ciphers to the last number in the second column to form the second trial divisor. Annex the third period to the remainder in the third column for the second dividend.

IV. If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.

V. If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fifth significant figure, carry the operation one place farther, and, if the sixth figure is 5 or greater, increase the fifth figure by 1 and omit the sign +.

EXAMPLES FOR PRACTICE

50. Find the cube root of :

(a) $\frac{27}{512}$.

(b) 2 to five decimal places.

(c) 4,180,769,192.462 to five decimal places.

(d) $\frac{87}{125}$.

(e) $\frac{3}{8}$.

(f) 513,229.783302144 to three decimal places.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \frac{3}{8} \\ (b) 1.25992 + \\ (c) 1,610.96233 \\ (d) .8862 + \\ (e) .7211 + \\ (f) 80.064 \end{array} \right.$$

FOURTH, FIFTH, AND OTHER ROOTS

51. If the application of the foregoing rules for square and cube root is thoroughly understood, one should be able to extend the process to other roots. For instance, consider the last example in Art. 47, $\sqrt[3]{.05} = ?$ The first step is to point off into periods of *three* figures each. Note: First, that the number *three* corresponds in value to the index figure ³; second, that the work is arranged in *three* columns, the number of columns corresponding in value to the index; third, that the number of operations (additions) in the first column for each figure of the root is *three* (which number also corresponds to the index), in the second column one less than in the first (or two), and in the third column one less than in the second (or one, subtraction); fourth, that one cipher is added to the number in the first column, two ciphers to the number in the second column, three ciphers (or a period of three figures) to the number in the third column, the number of operations in all cases corresponding to the number of the column, counting from the right, and the number of ciphers added corresponding to the number of the column, counting from the left. Bearing these facts in mind, it is unnecessary to remember a rule for each root, and by substituting for *three*, in the preceding sentence, *four*, *five*, etc., the fourth, fifth, etc., roots are readily found.

52. It is usually easier to extract the square root and then extract the square root of the result than to extract the fourth root direct. Thus,

$$\sqrt[4]{.05} = \sqrt{\sqrt{.05}} = \sqrt{.2236067977} = .47287.$$

If the fifth root of a number is required the exact method is very laborious and the table method is preferable.

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
I'00	I'0000	I'00000	I'00000	I'00000	I'50	2'2500	3'37500	5'06250	7'59375
I'01	I'0201	I'03030	I'04060	I'05101	I'51	2'2801	3'44295	5'19886	7'85027
I'02	I'0404	I'06121	I'08243	I'10408	I'52	2'3104	3'51181	5'33795	8'11368
I'03	I'0609	I'09273	I'12551	I'15927	I'53	2'3409	3'58158	5'47981	8'38411
I'04	I'0816	I'12486	I'16986	I'21665	I'54	2'3716	3'65226	5'62449	8'66171
I'05	I'1025	I'15763	I'21551	I'27628	I'55	2'4025	3'72388	5'77201	8'94661
I'06	I'1236	I'19102	I'26248	I'33823	I'56	2'4336	3'79642	5'92241	9'23896
I'07	I'1449	I'22504	I'31080	I'40255	I'57	2'4649	3'86989	6'07573	9'53890
I'08	I'1664	I'25971	I'36049	I'46933	I'58	2'4964	3'94431	6'23201	9'84658
I'09	I'1881	I'29503	I'41158	I'53862	I'59	2'5281	4'01968	6'39129	10'1622
I'10	I'2100	I'33100	I'46410	I'61051	I'60	2'5600	4'09600	6'55360	10'4858
I'11	I'2321	I'36763	I'51807	I'68506	I'61	2'5921	4'17328	6'71898	10'8176
I'12	I'2544	I'40493	I'57352	I'76234	I'62	2'6244	4'25153	6'88748	11'1577
I'13	I'2769	I'44290	I'63047	I'84244	I'63	2'6569	4'33075	7'05912	11'5064
I'14	I'2996	I'48154	I'68896	I'92541	I'64	2'6896	4'41094	7'23395	11'8637
I'15	I'3225	I'52088	I'74901	2'01136	I'65	2'7225	4'49213	7'41201	12'2298
I'16	I'3456	I'56090	I'81064	2'10034	I'66	2'7556	4'57430	7'59333	12'6049
I'17	I'3689	I'60161	I'87389	2'19245	I'67	2'7889	4'65746	7'77796	12'9892
I'18	I'3924	I'64303	I'93878	2'28776	I'68	2'8224	4'74163	7'96594	13'3828
I'19	I'4161	I'68516	2'00534	2'38635	I'69	2'8561	4'82681	8'15731	13'7858
I'20	I'4400	I'72800	2'07360	2'48832	I'70	2'8900	4'91300	8'35210	14'1986
I'21	I'4641	I'77156	2'14359	2'59374	I'71	2'9241	5'00021	8'55036	14'6211
I'22	I'4884	I'81585	2'21533	2'70271	I'72	2'9584	5'08845	8'75213	15'0537
I'23	I'5129	I'86087	2'28887	2'81531	I'73	2'9929	5'17772	8'95745	15'4964
I'24	I'5376	I'90662	2'36421	2'93163	I'74	3'0276	5'26802	9'16636	15'9495
I'25	I'5625	I'95313	2'44141	3'05176	I'75	3'0625	5'35938	9'37891	16'4131
I'26	I'5876	2'00038	2'52047	3'17580	I'76	3'0976	5'45178	9'59513	16'8874
I'27	I'6129	2'04838	2'60145	3'30384	I'77	3'1329	5'54523	9'81506	17'3727
I'28	I'6384	2'09715	2'68435	3'43597	I'78	3'1684	5'63975	10'0388	17'8690
I'29	I'6641	2'14669	2'76923	3'57231	I'79	3'2041	5'73534	10'2663	18'3766
I'30	I'6900	2'19700	2'85610	3'71293	I'80	3'2400	5'83200	10'4976	18'8957
I'31	I'7161	2'24809	2'94500	3'85795	I'81	3'2761	5'92974	10'7328	19'4264
I'32	I'7424	2'29997	3'03596	4'00746	I'82	3'3124	6'02857	10'9720	19'9690
I'33	I'7689	2'35264	3'12901	4'16158	I'83	3'3489	6'12849	11'2151	20'5237
I'34	I'7956	2'40610	3'22418	4'32040	I'84	3'3856	6'22950	11'4623	21'0906
I'35	I'8225	2'46038	3'32151	4'48403	I'85	3'4225	6'33163	11'7135	21'6700
I'36	I'8496	2'51546	3'42102	4'65259	I'86	3'4596	6'43486	11'9688	22'2620
I'37	I'8769	2'57135	3'52275	4'82617	I'87	3'4969	6'53920	12'2283	22'8669
I'38	I'9044	2'62807	3'62674	5'00490	I'88	3'5344	6'64467	12'4920	23'4849
I'39	I'9321	2'68562	3'73301	5'18888	I'89	3'5721	6'75127	12'7599	24'1162
I'40	I'9600	2'74400	3'84160	5'37824	I'90	3'6100	6'85900	13'0321	24'7610
I'41	I'9881	2'80322	3'95254	5'57308	I'91	3'6481	6'96787	13'3086	25'4195
I'42	2'0164	2'86329	4'06587	5'77353	I'92	3'6864	7'07789	13'5895	26'0919
I'43	2'0449	2'92421	4'18162	5'97971	I'93	3'7249	7'18906	13'8749	26'7785
I'44	2'0736	2'98598	4'29982	6'19174	I'94	3'7636	7'30138	14'1647	27'4795
I'45	2'1025	3'04863	4'42051	6'40973	I'95	3'8025	7'41488	14'4590	28'1951
I'46	2'1316	3'11214	4'54372	6'63383	I'96	3'8416	7'52954	14'7579	28'9255
I'47	2'1609	3'17652	4'66949	6'86415	I'97	3'8809	7'64537	15'0614	29'6709
I'48	2'1904	3'24179	4'79785	7'10082	I'98	3'9204	7'76239	15'3695	30'4317
I'49	2'2201	3'30795	4'92884	7'34398	I'99	3'9601	7'88060	15'6824	31'2080
I'50	2'2500	3'37500	5'06250	7'59375	2'00	4'0000	8'00000	16'0000	32'0000

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
2'00	4'0000	8'00000	16'0000	32'0000	2'50	6'2500	15'6250	39'0625	97'6563
2'01	4'0401	8'12060	16'3224	32'8080	2'51	6'3001	15'8133	39'6913	99'6251
2'02	4'0804	8'24241	16'6497	33'6323	2'52	6'3504	16'0030	40'3276	101'626
2'03	4'1209	8'36543	16'9818	34'4731	2'53	6'4009	16'1943	40'9715	103'658
2'04	4'1616	8'48966	17'3189	35'3306	2'54	6'4516	16'3871	41'6231	105'723
2'05	4'2025	8'61513	17'6610	36'2051	2'55	6'5025	16'5814	42'2825	107'820
2'06	4'2436	8'74182	18'0081	37'0968	2'56	6'5536	16'7772	42'9497	109'951
2'07	4'2849	8'86974	18'3604	38'0060	2'57	6'6049	16'9746	43'6247	112'115
2'08	4'3264	8'99891	18'7177	38'9329	2'58	6'6564	17'1735	44'3077	114'314
2'09	4'3681	9'12933	19'0803	39'8778	2'59	6'7081	17'3740	44'9986	116'546
2'10	4'4100	9'26100	19'4481	40'8410	2'60	6'7600	17'5760	45'6976	118'814
2'11	4'4521	9'39393	19'8212	41'8227	2'61	6'8121	17'7796	46'4047	121'116
2'12	4'4944	9'52813	20'1996	42'8232	2'62	6'8644	17'9847	47'1200	123'454
2'13	4'5369	9'66360	20'5835	43'8428	2'63	6'9169	18'1914	47'8435	125'828
2'14	4'5796	9'80034	20'9727	44'8817	2'64	6'9696	18'3997	48'5753	128'239
2'15	4'6225	9'93838	21'3675	45'9401	2'65	7'0225	18'6096	49'3155	130'686
2'16	4'6656	10'0777	21'7678	47'0185	2'66	7'0756	18'8211	50'0641	133'171
2'17	4'7089	10'2183	22'1737	48'1170	2'67	7'1289	19'0342	50'8212	135'693
2'18	4'7524	10'3602	22'5853	49'2360	2'68	7'1824	19'2488	51'5869	138'253
2'19	4'7961	10'5035	23'0026	50'3756	2'69	7'2361	19'4651	52'3611	140'851
2'20	4'8400	10'6480	23'4256	51'5363	2'70	7'2900	19'6830	53'1441	143'489
2'21	4'8841	10'7939	23'8544	52'7183	2'71	7'3441	19'9025	53'9358	146'166
2'22	4'9284	10'9410	24'2891	53'9219	2'72	7'3984	20'1236	54'7363	148'883
2'23	4'9729	11'0896	24'7297	55'1473	2'73	7'4529	20'3464	55'5457	151'640
2'24	5'0176	11'2394	25'1763	56'3949	2'74	7'5076	20'5708	56'3641	154'438
2'25	5'0625	11'3906	25'6289	57'6650	2'75	7'5625	20'7969	57'1914	157'276
2'26	5'1076	11'5432	26'0876	58'9579	2'76	7'6176	21'0246	58'0278	160'157
2'27	5'1529	11'6971	26'5524	60'2739	2'77	7'6729	21'2539	58'8734	163'079
2'28	5'1984	11'8524	27'0234	61'6133	2'78	7'7284	21'4850	59'7282	166'044
2'29	5'2441	12'0090	27'5006	62'9763	2'79	7'7841	21'7176	60'5922	169'052
2'30	5'2900	12'1670	27'9841	64'3634	2'80	7'8400	21'9520	61'4656	172'104
2'31	5'3361	12'3264	28'4740	65'7749	2'81	7'8961	22'1880	62'3484	175'199
2'32	5'3824	12'4872	28'9702	67'2109	2'82	7'9524	22'4258	63'2407	178'339
2'33	5'4289	12'6493	29'4730	68'6720	2'83	8'0089	22'6652	64'1425	181'523
2'34	5'4756	12'8129	29'9822	70'1583	2'84	8'0656	22'9063	65'0539	184'753
2'35	5'5225	12'9779	30'4980	71'6703	2'85	8'1225	23'1491	65'9750	188'029
2'36	5'5696	13'1443	31'0204	73'2082	2'86	8'1796	23'3937	66'9059	191'351
2'37	5'6169	13'3121	31'5496	74'7725	2'87	8'2369	23'6399	67'8465	194'720
2'38	5'6644	13'4813	32'0854	76'3633	2'88	8'2944	23'8879	68'7971	198'136
2'39	5'7121	13'6519	32'6281	77'9811	2'89	8'3521	24'1376	69'7576	201'599
2'40	5'7600	13'8240	33'1776	79'6262	2'90	8'4100	24'3890	70'7281	205'111
2'41	5'8081	13'9975	33'7340	81'2990	2'91	8'4681	24'6422	71'7087	208'672
2'42	5'8564	14'1725	34'2974	82'9998	2'92	8'5264	24'8971	72'6995	212'283
2'43	5'9049	14'3489	34'8678	84'7289	2'93	8'5849	25'1538	73'7005	215'942
2'44	5'9536	14'5268	35'4454	86'4867	2'94	8'6436	25'4122	74'7118	219'653
2'45	6'0025	14'7061	36'0300	88'2735	2'95	8'7025	25'6724	75'7335	223'414
2'46	6'0516	14'8869	36'6219	90'0898	2'96	8'7616	25'9343	76'7656	227'226
2'47	6'1009	15'0692	37'2210	91'9358	2'97	8'8209	26'1981	77'8083	231'091
2'48	6'1504	15'2530	37'8274	93'8120	2'98	8'8804	26'4636	78'8615	235'007
2'49	6'2001	15'4382	38'4412	95'7187	2'99	8'9401	26'7309	79'9254	238'977
2'50	6'2500	15'6250	39'0625	97'6563	3'00	9'0000	27'0000	81'0000	243'000

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
3'00	9'0000	27'0000	81'0000	243'000	3'50	12'2500	42'8750	150'063	525'219
3'01	9'0601	27'2709	82'0854	247'077	3'51	12'3201	43'2436	151'785	532'765
3'02	9'1204	27'5436	83'1817	251'209	3'52	12'3904	43'6142	153'522	540'397
3'03	9'1809	27'8181	84'2889	255'395	3'53	12'4609	43'9870	155'274	548'117
3'04	9'2416	28'0945	85'4072	259'638	3'54	12'5316	44'3619	157'041	555'925
3'05	9'3025	28'3726	86'5365	263'936	3'55	12'6025	44'7389	158'823	563'822
3'06	9'3536	28'6526	87'6770	268'292	3'56	12'6736	45'1180	160'620	571'808
3'07	9'4249	28'9344	88'8287	272'704	3'57	12'7449	45'4993	162'432	579'884
3'08	9'4864	29'2181	89'9918	277'175	3'58	12'8164	45'8827	164'260	588'051
3'09	9'5481	29'5036	91'1662	281'704	3'59	12'8881	46'2683	166'103	596'310
3'10	9'6100	29'7910	92'3521	286'292	3'60	12'9600	46'6560	167'962	604'662
3'11	9'6721	30'0802	93'5495	290'939	3'61	13'0321	47'0459	169'836	613'107
3'12	9'7344	30'3713	94'7585	295'647	3'62	13'1044	47'4379	171'725	621'646
3'13	9'7969	30'6643	95'9792	300'415	3'63	13'1769	47'8321	173'631	630'279
3'14	9'8596	30'9591	97'2117	305'245	3'64	13'2496	48'2285	175'552	639'009
3'15	9'9225	31'2559	98'4560	310'136	3'65	13'3225	48'6271	177'489	647'835
3'16	9'9856	31'5545	99'7122	315'091	3'66	13'3956	49'0279	179'442	656'758
3'17	10'0489	31'8550	100'980	320'108	3'67	13'4689	49'4309	181'411	665'779
3'18	10'1124	32'1574	102'261	325'189	3'68	13'5424	49'8360	183'397	674'899
3'19	10'1761	32'4618	103'553	330'334	3'69	13'6161	50'2434	185'398	684'119
3'20	10'2400	32'7680	104'858	335'544	3'70	13'6900	50'6530	187'416	693'440
3'21	10'3041	33'0762	106'174	340'820	3'71	13'7641	51'0648	189'450	702'861
3'22	10'3684	33'3862	107'504	346'162	3'72	13'8384	51'4788	191'501	712'385
3'23	10'4329	33'6983	108'845	351'571	3'73	13'9129	51'8951	193'569	722'012
3'24	10'4976	34'0122	110'200	357'047	3'74	13'9876	52'3136	195'653	731'742
3'25	10'5625	34'3281	111'566	362'591	3'75	14'0625	52'7344	197'754	741'577
3'26	10'6276	34'6460	112'946	368'204	3'76	14'1376	53'1574	199'872	751'518
3'27	10'6929	34'9658	114'338	373'886	3'77	14'2129	53'5826	202'007	761'565
3'28	10'7584	35'2876	115'743	379'638	3'78	14'2884	54'0102	204'158	771'719
3'29	10'8241	35'6129	117'161	385'460	3'79	14'3641	54'4399	206'327	781'981
3'30	10'8900	35'9370	118'592	391'354	3'80	14'4400	54'8720	208'514	792'352
3'31	10'9561	36'2647	120'036	397'320	3'81	14'5161	55'3063	210'717	802'832
3'32	11'0224	36'5944	121'493	403'358	3'82	14'5924	55'7430	212'938	813'424
3'33	11'0889	36'9260	122'964	409'469	3'83	14'6689	56'1819	215'177	824'126
3'34	11'1556	37'2597	124'447	415'654	3'84	14'7456	56'6231	217'433	834'942
3'35	11'2225	37'5954	125'945	421'914	3'85	14'8225	57'0666	219'707	845'870
3'36	11'2896	37'9331	127'455	428'249	3'86	14'8996	57'5125	221'998	856'913
3'37	11'3569	38'2728	128'979	434'660	3'87	14'9769	57'9606	224'308	868'070
3'38	11'4244	38'6145	130'517	441'147	3'88	15'0544	58'4111	226'635	879'344
3'39	11'4921	38'9582	132'068	447'712	3'89	15'1321	58'8639	228'980	890'734
3'40	11'5600	39'3040	133'634	454'354	3'90	15'2100	59'3190	231'344	902'243
3'41	11'6281	39'6518	135'213	461'075	3'91	15'2881	59'7765	233'726	913'869
3'42	11'6964	40'0017	136'806	467'876	3'92	15'3664	60'2363	236'126	925'615
3'43	11'7649	40'3536	138'413	474'756	3'93	15'4449	60'6985	238'545	937'482
3'44	11'8336	40'7076	140'034	481'717	3'94	15'5236	61'1630	240'982	949'470
3'45	11'9025	41'0636	141'670	488'760	3'95	15'6025	61'6299	243'438	961'580
3'46	11'9716	41'4217	143'319	495'884	3'96	15'6816	62'0991	245'913	973'814
3'47	12'0409	41'7819	144'983	503'092	3'97	15'7609	62'5708	248'406	986'172
3'48	12'1104	42'1442	146'662	510'383	3'98	15'8404	63'0448	250'918	998'655
3'49	12'1801	42'5085	148'355	517'758	3'99	15'9201	63'5212	253'450	1011'26
3'50	12'2500	42'8750	150'063	525'219	4'00	16'0000	64'0000	256'000	1024'00

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
4'00	16·0000	64·0000	256·000	1024·00	4'50	20·2500	91·1250	410·063	1845·28
4'01	16·0801	64·4812	258·570	1036·86	4'51	20·3401	91·7339	413·720	1865·88
4'02	16·1604	64·9648	261·159	1049·86	4'52	20·4304	92·3454	417·401	1886·65
4'03	16·2409	65·4508	263·767	1062·98	4'53	20·5209	92·9597	421·107	1907·62
4'04	16·3216	65·9393	266·395	1076·23	4'54	20·6116	93·5767	424·838	1928·76
4'05	16·4025	66·4301	269·042	1089·62	4'55	20·7025	94·1964	428·594	1950·10
4'06	16·4836	66·9234	271·709	1103·14	4'56	20·7936	94·8188	432·374	1971·62
4'07	16·5649	67·4191	274·396	1116·79	4'57	20·8849	95·4440	436·179	1993·34
4'08	16·6464	67·9173	277·103	1130·58	4'58	20·9764	96·0719	440·009	2015·24
4'09	16·7281	68·4179	279·829	1144·50	4'59	21·0681	96·7026	443·865	2037·34
4'10	16·8100	68·9210	282·576	1158·56	4'60	21·1600	97·3360	447·746	2059·63
4'11	16·8921	69·4265	285·343	1172·76	4'61	21·2521	97·9722	451·652	2082·11
4'12	16·9744	69·9345	288·130	1187·10	4'62	21·3444	98·6111	455·583	2104·80
4'13	17·0569	70·4450	290·938	1201·57	4'63	21·4369	99·2528	459·541	2127·67
4'14	17·1396	70·9579	293·766	1216·19	4'64	21·5296	99·8973	463·524	2150·75
4'15	17·2225	71·4734	296·615	1230·95	4'65	21·6225	100·545	467·533	2174·03
4'16	17·3056	71·9913	299·484	1245·85	4'66	21·7156	101·195	471·567	2197·50
4'17	17·3889	72·5117	302·374	1260·90	4'67	21·8089	101·848	475·628	2221·18
4'18	17·4724	73·0346	305·285	1276·09	4'68	21·9024	102·503	479·715	2245·07
4'19	17·5561	73·5601	308·217	1291·43	4'69	21·9961	103·162	483·828	2269·16
4'20	17·6400	74·0880	311·170	1306·91	4'70	22·0900	103·823	487·968	2293·45
4'21	17·7241	74·6185	314·144	1322·55	4'71	22·1841	104·487	492·134	2317·95
4'22	17·8084	75·1514	317·139	1338·33	4'72	22·2784	105·154	496·327	2342·66
4'23	17·8929	75·6870	320·156	1354·26	4'73	22·3729	105·824	500·547	2367·59
4'24	17·9776	76·2250	323·194	1370·34	4'74	22·4676	106·496	504·793	2392·72
4'25	18·0625	76·7656	326·254	1386·58	4'75	22·5625	107·172	509·066	2418·07
4'26	18·1476	77·3088	329·335	1402·97	4'76	22·6576	107·850	513·367	2443·63
4'27	18·2329	77·8545	332·439	1419·51	4'77	22·7529	108·531	517·694	2469·40
4'28	18·3184	78·4028	335·564	1436·21	4'78	22·8484	109·215	522·049	2495·40
4'29	18·4041	78·9536	338·711	1453·07	4'79	22·9441	109·902	526·322	2521·61
4'30	18·4900	79·5070	341·880	1470·08	4'80	23·0400	110·592	530·842	2548·04
4'31	18·5761	80·0630	345·071	1487·26	4'81	23·1361	111·285	535·279	2574·69
4'32	18·6624	80·6216	348·285	1504·59	4'82	23·2324	111·980	539·744	2601·57
4'33	18·7489	81·1827	351·521	1522·09	4'83	23·3289	112·679	544·238	2628·67
4'34	18·8356	81·7465	354·780	1539·74	4'84	23·4256	113·380	548·759	2655·99
4'35	18·9225	82·3129	358·061	1557·57	4'85	23·5225	114·084	553·308	2683·54
4'36	19·0096	82·8819	361·365	1575·55	4'86	23·6196	114·791	557·886	2711·32
4'37	19·0969	83·4535	364·692	1593·70	4'87	23·7169	115·501	562·491	2739·33
4'38	19·1844	84·0277	368·041	1612·02	4'88	23·8144	116·214	567·126	2767·57
4'39	19·2721	84·6045	371·414	1630·51	4'89	23·9121	116·930	571·789	2796·05
4'40	19·3600	85·1840	374·810	1649·16	4'90	24·0100	117·649	576·480	2824·75
4'41	19·4481	85·7661	378·229	1667·99	4'91	24·1081	118·371	581·200	2853·69
4'42	19·5364	86·3509	381·671	1686·99	4'92	24·2064	119·095	585·950	2882·87
4'43	19·6249	86·9383	385·137	1706·16	4'93	24·3049	119·823	590·728	2912·29
4'44	19·7136	87·5284	388·626	1725·50	4'94	24·4036	120·554	595·536	2941·95
4'45	19·8025	88·1211	392·139	1745·02	4'95	24·5025	121·287	600·373	2971·84
4'46	19·8916	88·7165	395·676	1764·71	4'96	24·6016	122·024	605·239	3001·98
4'47	19·9809	89·3146	399·236	1784·59	4'97	24·7009	122·763	610·134	3032·37
4'48	20·0704	89·9154	402·821	1804·64	4'98	24·8004	123·506	615·060	3063·00
4'49	20·1601	90·5188	406·430	1824·87	4'99	24·9001	124·251	620·015	3093·87
4'50	20·2500	91·1250	410·063	1845·28	5'00	25·0000	125·000	625·000	3125·00

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
5'00	25'0000	125'000	625'000	3125'00	5'50	30'2500	166'375	915'063	5032'84
5'01	25'1001	125'752	630'015	3156'38	5'51	30'3601	167'284	921'736	5078'76
5'02	25'2004	126'506	635'060	3188'00	5'52	30'4704	168'197	928'445	5125'02
5'03	25'3009	127'264	640'136	3219'88	5'53	30'5809	169'112	935'191	5171'61
5'04	25'4016	128'024	645'241	3252'02	5'54	30'6916	170'031	941'974	5218'54
5'05	25'5025	128'788	650'378	3284'41	5'55	30'8025	170'954	948'794	5265'81
5'06	25'6036	129'554	655'544	3317'05	5'56	30'9136	171'880	955'651	5313'42
5'07	25'7049	130'324	660'742	3349'96	5'57	31'0249	172'809	962'544	5361'37
5'08	25'8064	131'097	665'970	3383'13	5'58	31'1364	173'741	969'475	5409'67
5'09	25'9081	131'872	671'230	3416'56	5'59	31'2481	174'677	976'444	5458'32
5'10	26'0100	132'651	676'520	3450'25	5'60	31'3600	175'616	983'450	5507'32
5'11	26'1121	133'433	681'842	3484'21	5'61	31'4721	176'558	990'493	5556'67
5'12	26'2144	134'218	687'195	3518'44	5'62	31'5844	177'504	997'574	5606'37
5'13	26'3169	135'006	692'579	3552'93	5'63	31'6969	178'454	1004'69	5656'42
5'14	26'4196	135'797	697'995	3587'70	5'64	31'8096	179'406	1011'85	5706'84
5'15	26'5225	136'591	703'443	3622'73	5'65	31'9225	180'362	1019'05	5757'61
5'16	26'6256	137'388	708'923	3658'04	5'66	32'0356	181'321	1026'28	5808'74
5'17	26'7289	138'188	714'434	3693'62	5'67	32'1489	182'284	1033'55	5860'24
5'18	26'8324	138'992	719'978	3729'48	5'68	32'2624	183'250	1040'86	5912'10
5'19	26'9361	139'798	725'553	3765'62	5'69	32'3761	184'220	1048'21	5964'33
5'20	27'0400	140'608	731'162	3802'04	5'70	32'4900	185'193	1055'60	6016'92
5'21	27'1441	141'421	736'802	3838'74	5'71	32'6041	186'169	1063'03	6069'89
5'22	27'2484	142'237	742'475	3875'72	5'72	32'7184	187'149	1070'49	6123'22
5'23	27'3529	143'056	748'181	3912'99	5'73	32'8329	188'133	1078'00	6176'94
5'24	27'4576	143'878	753'920	3950'54	5'74	32'9476	189'119	1085'54	6231'02
5'25	27'5625	144'703	759'691	3988'38	5'75	33'0625	190'109	1093'13	6285'49
5'26	27'6676	145'532	765'496	4026'51	5'76	33'1776	191'103	1100'75	6340'34
5'27	27'7729	146'363	771'334	4064'93	5'77	33'2929	192'100	1108'42	6395'57
5'28	27'8784	147'198	777'205	4103'64	5'78	33'4084	193'101	1116'12	6451'18
5'29	27'9841	148'036	783'110	4142'65	5'79	33'5241	194'105	1123'87	6507'18
5'30	28'0900	148'877	789'048	4181'95	5'80	33'6400	195'112	1131'65	6563'57
5'31	28'1961	149'721	795'020	4221'56	5'81	33'7561	196'123	1139'47	6620'35
5'32	28'3024	150'569	801'026	4261'46	5'82	33'8724	197'137	1147'34	6677'52
5'33	28'4089	151'419	807'066	4301'66	5'83	33'9889	198'155	1155'25	6735'08
5'34	28'5156	152'273	813'139	4342'16	5'84	34'1056	199'177	1163'19	6793'04
5'35	28'6225	153'130	819'248	4382'97	5'85	34'2225	200'202	1171'18	6851'40
5'36	28'7296	153'991	825'390	4424'09	5'86	34'3396	201'230	1179'21	6910'16
5'37	28'8369	154'854	831'567	4465'51	5'87	34'4569	202'262	1187'28	6969'32
5'38	28'9444	155'721	837'778	4507'25	5'88	34'5744	203'297	1195'39	7028'89
5'39	29'0521	156'591	844'025	4549'29	5'89	34'6921	204'336	1203'54	7088'86
5'40	29'1600	157'464	850'306	4591'65	5'90	34'8100	205'379	1211'74	7149'24
5'41	29'2681	158'340	856'622	4634'32	5'91	34'9281	206'425	1219'97	7210'04
5'42	29'3764	159'220	862'973	4677'31	5'92	35'0464	207'475	1228'25	7271'24
5'43	29'4849	160'103	869'359	4720'62	5'93	35'1649	208'528	1236'57	7332'86
5'44	29'5936	160'989	875'781	4764'25	5'94	35'2836	209'585	1244'93	7394'90
5'45	29'7025	161'879	882'239	4808'20	5'95	35'4025	210'645	1253'34	7457'36
5'46	29'8116	162'771	888'731	4852'47	5'96	35'5216	211'709	1261'78	7520'23
5'47	29'9209	163'667	895'260	4897'07	5'97	35'6409	212'776	1270'27	7583'53
5'48	30'0304	164'567	901'825	4942'00	5'98	35'7604	213'847	1278'81	7647'26
5'49	30'1401	165'469	908'426	4987'26	5'99	35'8801	214'922	1287'38	7711'42
5'50	30'2500	166'375	915'063	5032'84	6'00	36'0000	216'000	1296'00	7776'00

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
6·00	36·0000	216·000	1296·00	7776·00	6·50	42·2500	274·625	1785·06	11602·9
6·01	36·1201	217·082	1304·66	7841·02	6·51	42·3801	275·894	1796·07	11692·4
6·02	36·2404	218·167	1313·37	7906·47	6·52	42·5104	277·168	1807·13	11782·5
6·03	36·3609	219·256	1322·12	7972·35	6·53	42·6409	278·445	1818·25	11873·1
6·04	36·4816	220·349	1330·91	8038·68	6·54	42·7716	279·726	1829·41	11964·3
6·05	36·6025	221·445	1339·74	8105·45	6·55	42·9025	281·011	1840·62	12056·1
6·06	36·7236	222·545	1348·62	8172·65	6·56	43·0336	282·300	1851·89	12148·4
6·07	36·8449	223·649	1357·55	8240·31	6·57	43·1649	283·593	1863·21	12241·3
6·08	36·9664	224·756	1366·51	8308·41	6·58	43·2964	284·890	1874·58	12334·7
6·09	37·0881	225·867	1375·53	8376·96	6·59	43·4281	286·191	1886·00	12428·7
6·10	37·2100	226·981	1384·58	8445·96	6·60	43·5600	287·496	1897·47	12523·3
6·11	37·3321	228·099	1393·69	8515·42	6·61	43·6921	288·805	1909·00	12618·5
6·12	37·4544	229·221	1402·83	8585·33	6·62	43·8244	290·118	1920·58	12714·2
6·13	37·5769	230·346	1412·02	8655·70	6·63	43·9569	291·434	1932·21	12810·5
6·14	37·6996	231·476	1421·26	8726·54	6·64	44·0896	292·755	1943·89	12907·4
6·15	37·8225	232·608	1430·54	8797·83	6·65	44·2225	294·080	1955·63	13004·9
6·16	37·9456	233·745	1439·87	8869·59	6·66	44·3556	295·408	1967·42	13103·0
6·17	38·0689	234·885	1449·24	8941·82	6·67	44·4889	296·741	1979·26	13201·7
6·18	38·1924	236·029	1458·66	9014·52	6·68	44·6224	298·078	1991·16	13300·9
6·19	38·3161	237·177	1468·12	9087·68	6·69	44·7561	299·418	2003·11	13400·8
6·20	38·4400	238·328	1477·63	9161·33	6·70	44·8900	300·763	2015·11	13501·3
6·21	38·5641	239·483	1487·19	9235·45	6·71	45·0241	302·112	2027·17	13602·3
6·22	38·6884	240·642	1496·79	9310·05	6·72	45·1584	303·464	2039·28	13704·0
6·23	38·8129	241·804	1506·44	9385·13	6·73	45·2929	304·821	2051·45	13806·2
6·24	38·9376	242·971	1516·14	9460·69	6·74	45·4276	306·182	2063·67	13909·1
6·25	39·0625	244·141	1525·88	9536·74	6·75	45·5625	307·547	2075·94	14012·6
6·26	39·1876	245·314	1535·67	9613·28	6·76	45·6976	308·916	2088·27	14116·7
6·27	39·3129	246·492	1545·50	9690·31	6·77	45·8329	310·289	2100·65	14221·4
6·28	39·4384	247·673	1555·39	9767·83	6·78	45·9684	311·666	2113·09	14326·8
6·29	39·5641	248·858	1565·32	9845·85	6·79	46·1041	313·047	2125·59	14432·7
6·30	39·6900	250·047	1575·30	9924·37	6·80	46·2400	314·432	2138·14	14539·3
6·31	39·8161	251·240	1585·32	10003·4	6·81	46·3761	315·821	2150·74	14646·6
6·32	39·9424	252·436	1595·40	10082·9	6·82	46·5124	317·215	2163·40	14754·4
6·33	40·0689	253·636	1605·52	10162·9	6·83	46·6489	318·612	2176·12	14862·9
6·34	40·1956	254·840	1615·69	10243·5	6·84	46·7856	320·014	2188·89	14972·0
6·35	40·3225	256·048	1625·90	10324·5	6·85	46·9225	321·419	2201·72	15081·8
6·36	40·4496	257·259	1636·17	10406·0	6·86	47·0596	322·829	2214·61	15192·2
6·37	40·5769	258·475	1646·48	10488·1	6·87	47·1969	324·243	2227·55	15303·3
6·38	40·7044	259·694	1656·85	10570·7	6·88	47·3344	325·661	2240·55	15415·0
6·39	40·8321	260·917	1667·26	10653·8	6·89	47·4721	327·083	2253·60	15527·3
6·40	40·9600	262·144	1677·72	10737·4	6·90	47·6100	328·509	2266·71	15640·3
6·41	41·0881	263·375	1688·23	10821·6	6·91	47·7481	329·939	2279·88	15754·0
6·42	41·2164	264·609	1698·79	10906·2	6·92	47·8864	331·374	2293·11	15868·3
6·43	41·3449	265·848	1709·40	10991·4	6·93	48·0249	332·813	2306·39	15983·3
6·44	41·4736	267·090	1720·06	11077·2	6·94	48·1636	334·255	2319·73	16098·9
6·45	41·6025	268·336	1730·77	11163·5	6·95	48·3025	335·702	2333·13	16215·3
6·46	41·7316	269·586	1741·53	11250·3	6·96	48·4416	337·154	2346·59	16332·3
6·47	41·8609	270·840	1752·33	11337·6	6·97	48·5809	338·609	2360·10	16449·9
6·48	41·9904	272·098	1763·19	11425·5	6·98	48·7204	340·068	2373·68	16568·3
6·49	42·1201	273·359	1774·10	11513·9	6·99	48·8601	341·532	2387·31	16687·3
6·50	42·2500	274·625	1785·06	11602·9	7·00	49·0000	343·000	2401·00	16807·0

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
7'00	49'0000	343'000	2401'00	16807'0	7'50	56'2500	421'875	3164'06	23730'5
7'01	49'1401	344'472	2414'75	16927'4	7'51	56'4001	423'565	3180'97	23889'1
7'02	49'2804	345'948	2428'56	17048'5	7'52	56'5504	425'259	3197'95	24048'6
7'03	49'4209	347'429	2442'43	17170'3	7'53	56'7009	426'958	3214'99	24208'9
7'04	49'5616	348'914	2456'35	17292'7	7'54	56'8516	428'661	3232'10	24370'1
7'05	49'7025	350'403	2470'34	17415'9	7'55	57'0025	430'369	3249'29	24532'1
7'06	49'8436	351'896	2484'38	17539'8	7'56	57'1536	432'081	3266'53	24695'0
7'07	49'9849	353'393	2498'40	17664'3	7'57	57'3049	433'798	3283'85	24858'8
7'08	50'1264	354'895	2512'66	17789'6	7'58	57'4564	435'520	3301'24	25023'4
7'09	50'2681	356'401	2526'88	17915'6	7'59	57'6081	437'245	3318'69	25188'9
7'10	50'4100	357'911	2541'17	18042'3	7'60	57'7600	438'976	3336'22	25355'3
7'11	50'5521	359'425	2555'51	18169'7	7'61	57'9121	440'711	3353'81	25522'5
7'12	50'6944	360'944	2569'92	18297'8	7'62	58'0644	442'451	3371'47	25690'6
7'13	50'8369	362'467	2584'39	18426'7	7'63	58'2169	444'195	3389'21	25859'7
7'14	50'9796	363'994	2598'92	18556'3	7'64	58'3696	445'944	3407'01	26029'6
7'15	51'1225	365'526	2613'51	18686'6	7'65	58'5225	447'697	3424'88	26200'4
7'16	51'2656	367'062	2628'16	18817'6	7'66	58'6756	449'455	3442'83	26372'0
7'17	51'4089	368'602	2642'88	18949'4	7'67	58'8289	451'218	3460'84	26544'6
7'18	51'5524	370'146	2657'65	19081'9	7'68	58'9824	452'985	3478'92	26718'1
7'19	51'6961	371'695	2672'49	19215'2	7'69	59'1361	454'757	3497'08	26892'5
7'20	51'8400	373'248	2687'39	19349'2	7'70	59'2900	456'533	3515'30	27067'8
7'21	51'9841	374'805	2702'35	19483'9	7'71	59'4441	458'314	3533'60	27244'1
7'22	52'1284	376'367	2717'37	19619'4	7'72	59'5984	460'100	3551'97	27421'2
7'23	52'2729	377'933	2732'46	19755'7	7'73	59'7529	461'890	3570'41	27599'3
7'24	52'4176	379'503	2747'60	19892'7	7'74	59'9076	463'685	3588'92	27778'2
7'25	52'5625	381'078	2762'82	20030'4	7'75	60'0625	465'484	3607'50	27958'2
7'26	52'7076	382'657	2778'09	20168'9	7'76	60'2176	467'289	3626'16	28139'0
7'27	52'8529	384'241	2793'43	20308'2	7'77	60'3729	469'097	3644'89	28320'8
7'28	52'9984	385'828	2808'83	20448'3	7'78	60'5284	470'911	3663'69	28503'5
7'29	53'1441	387'420	2824'30	20589'1	7'79	60'6841	472'729	3682'56	28687'1
7'30	53'2900	389'017	2839'82	20730'7	7'80	60'8400	474'552	3701'51	28871'7
7'31	53'4361	390'618	2855'42	20873'1	7'81	60'9961	476'380	3720'52	29057'3
7'32	53'5824	392'223	2871'07	21016'3	7'82	61'1524	478'212	3739'62	29243'8
7'33	53'7289	393'833	2886'80	21160'2	7'83	61'3089	480'049	3758'78	29431'3
7'34	53'8756	395'447	2902'58	21304'9	7'84	61'4656	481'890	3778'02	29619'7
7'35	54'0225	397'065	2918'43	21450'5	7'85	61'6225	483'737	3797'33	29809'1
7'36	54'1696	398'688	2934'35	21596'8	7'86	61'7796	485'588	3816'72	29999'4
7'37	54'3169	400'316	2950'33	21743'9	7'87	61'9369	487'443	3836'18	30190'7
7'38	54'4644	401'947	2966'37	21891'8	7'88	62'0944	489'304	3855'71	30383'0
7'39	54'6121	403'583	2982'48	22040'5	7'89	62'2521	491'169	3875'32	30576'3
7'40	54'7600	405'224	2998'66	22190'1	7'90	62'4100	493'039	3895'01	30770'6
7'41	54'9081	406'869	3014'90	22340'4	7'91	62'5681	494'914	3914'77	30965'8
7'42	55'0564	408'518	3031'21	22491'6	7'92	62'7264	496'793	3934'60	31162'0
7'43	55'2049	410'172	3047'58	22643'5	7'93	62'8849	498'677	3954'51	31359'3
7'44	55'3536	411'831	3064'02	22796'3	7'94	63'0436	500'566	3974'50	31557'5
7'45	55'5025	413'494	3080'53	22949'9	7'95	63'2025	502'460	3994'56	31756'7
7'46	55'6516	415'161	3097'10	23104'4	7'96	63'3616	504'358	4014'69	31957'0
7'47	55'8009	416'833	3113'74	23259'6	7'97	63'5209	506'262	4034'90	32158'2
7'48	55'9504	418'509	3130'45	23415'7	7'98	63'6804	508'170	4055'19	32360'4
7'49	56'1001	420'190	3147'22	23572'7	7'99	63'8401	510'082	4075'56	32563'7
7'50	56'2500	421'875	3164'06	23730'5	8'00	64'0000	512'000	4096'00	32768'0

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
8'00	64'0000	512'000	4096'00	32768'0	8'50	72'2500	614'125	5220'06	44370'5
8'01	64'1601	513'922	4116'52	32973'3	8'51	72'4201	616'295	5244'67	44632'1
8'02	64'3204	515'850	4137'11	33179'7	8'52	72'5904	618'470	5269'37	44895'0
8'03	64'4809	517'782	4157'79	33387'0	8'53	72'7609	620'650	5294'15	45159'1
8'04	64'6416	519'718	4178'54	33595'4	8'54	72'9316	622'836	5319'02	45424'4
8'05	64'8025	521'660	4199'36	33804'9	8'55	73'1025	625'026	5343'98	45691'0
8'06	64'9636	523'607	4220'27	34015'4	8'56	73'2736	627'222	5369'02	45958'8
8'07	65'1249	525'558	4241'25	34226'9	8'57	73'4449	629'423	5394'15	46227'9
8'08	65'2864	527'514	4262'31	34439'5	8'58	73'6164	631'629	5419'37	46498'2
8'09	65'4481	529'475	4283'45	34653'1	8'59	73'7881	633'840	5444'68	46769'8
8'10	65'6100	531'441	4304'67	34867'8	8'60	73'9600	636'056	5470'08	47042'7
8'11	65'7721	533'412	4325'97	35083'6	8'61	74'1321	638'277	5495'57	47316'8
8'12	65'9344	535'387	4347'35	35300'4	8'62	74'3044	640'504	5521'14	47592'3
8'13	66'0969	537'368	4368'80	35518'3	8'63	74'4769	642'736	5546'81	47869'0
8'14	66'2596	539'353	4390'33	35737'3	8'64	74'6496	644'973	5572'56	48146'9
8'15	66'4225	541'343	4411'95	35957'4	8'65	74'8225	647'215	5598'41	48426'2
8'16	66'5856	543'338	4433'64	36178'5	8'66	74'9956	649'462	5624'34	48706'8
8'17	66'7489	545'339	4455'42	36400'7	8'67	75'1689	651'714	5650'36	48988'7
8'18	66'9124	547'343	4477'27	36624'1	8'68	75'3424	653'972	5676'48	49271'8
8'19	67'0761	549'353	4499'20	36848'5	8'69	75'5161	656'235	5702'68	49556'3
8'20	67'2400	551'368	4521'22	37074'0	8'70	75'6900	658'503	5728'98	49842'1
8'21	67'4041	553'388	4543'31	37300'6	8'71	75'8641	660'776	5755'36	50129'2
8'22	67'5684	555'412	4565'49	37528'3	8'72	76'0384	663'055	5781'84	50417'6
8'23	67'7329	557'442	4587'75	37757'1	8'73	76'2129	665'339	5808'41	50707'4
8'24	67'8976	559'476	4610'08	37987'1	8'74	76'3876	667'628	5835'07	50998'5
8'25	68'0625	561'516	4632'50	38218'2	8'75	76'5625	669'922	5861'82	51290'9
8'26	68'2276	563'560	4655'01	38450'3	8'76	76'7376	672'221	5888'66	51584'7
8'27	68'3929	565'609	4677'59	38683'7	8'77	76'9129	674'526	5915'59	51879'8
8'28	68'5584	567'664	4700'25	38918'1	8'78	77'0884	676'836	5942'62	52176'2
8'29	68'7241	569'723	4723'00	39153'7	8'79	77'2641	679'151	5969'74	52474'0
8'30	68'8900	571'787	4745'83	39390'4	8'80	77'4400	681'472	5996'95	52773'2
8'31	69'0561	573'856	4768'74	39628'3	8'81	77'6161	683'798	6024'26	53073'7
8'32	69'2224	575'930	4791'74	39867'3	8'82	77'7924	686'129	6051'66	53375'6
8'33	69'3889	578'010	4814'82	40107'4	8'83	77'9689	688'465	6079'15	53678'9
8'34	69'5556	580'094	4837'98	40348'8	8'84	78'1456	690'807	6106'73	53983'5
8'35	69'7225	582'183	4861'23	40591'2	8'85	78'3225	693'154	6134'41	54289'6
8'36	69'8896	584'277	4884'56	40834'9	8'86	78'4996	695'506	6162'19	54597'0
8'37	70'0569	586'376	4907'97	41079'7	8'87	78'6769	697'864	6190'05	54905'8
8'38	70'2244	588'480	4931'47	41325'7	8'88	78'8544	700'227	6218'02	55216'0
8'39	70'3921	590'590	4955'05	41572'9	8'89	79'0321	702'595	6246'07	55527'6
8'40	70'5600	592'704	4978'71	41821'2	8'90	79'2100	704'969	6274'22	55840'6
8'41	70'7281	594'823	5002'46	42070'7	8'91	79'3881	707'348	6302'47	56155'0
8'42	70'8964	596'948	5026'30	42321'4	8'92	79'5664	709'732	6330'81	56470'8
8'43	71'0649	599'077	5050'22	42573'4	8'93	79'7449	712'122	6359'25	56788'1
8'44	71'2336	601'212	5074'23	42826'5	8'94	79'9236	714'517	6387'78	57106'8
8'45	71'4025	603'351	5098'32	43080'8	8'95	80'1025	716'917	6416'41	57426'9
8'46	71'5716	605'496	5122'49	43336'3	8'96	80'2816	719'323	6445'14	57748'4
8'47	71'7409	607'645	5146'76	43593'0	8'97	80'4609	721'734	6473'96	58071'4
8'48	71'9104	609'800	5171'11	43851'0	8'98	80'6404	724'151	6502'87	58395'8
8'49	72'0801	611'960	5195'54	44110'1	8'99	80'8201	726'573	6531'89	58721'7
8'50	72'2500	614'125	5220'06	44370'5	9'00	81'0000	729'000	6561'00	59049'0

n	n^2	n^3	n^4	n^5	n	n^2	n^3	n^4	n^5
9'00	81'0000	729'000	6561'00	59049'0	9'50	90'2500	857'375	8145'06	77378'1
9'01	81'1801	731'433	6590'21	59377'8	9'51	90'4401	860'085	8179'41	77786'2
9'02	81'3604	733'871	6619'51	59708'0	9'52	90'6304	862'801	8213'87	78196'0
9'03	81'5409	736'314	6648'92	60039'7	9'53	90'8209	865'523	8248'44	78607'5
9'04	81'7216	738'763	6678'42	60372'9	9'54	91'0116	868'251	8283'11	79020'9
9'05	81'9025	741'218	6708'02	60707'6	9'55	91'2025	870'984	8317'90	79435'9
9'06	82'0836	743'677	6737'72	61043'7	9'56	91'3936	873'723	8352'79	79852'7
9'07	82'2649	746'143	6767'51	61381'3	9'57	91'5849	876'467	8387'79	80271'2
9'08	82'4464	748'613	6797'41	61720'5	9'58	91'7764	879'218	8422'91	80691'5
9'09	82'6281	751'089	6827'40	62061'1	9'59	91'9681	881'974	8458'13	81113'5
9'10	82'8100	753'571	6857'50	62403'2	9'60	92'1600	884'736	8493'47	81537'3
9'11	82'9921	756'058	6887'69	62746'8	9'61	92'3521	887'504	8528'91	81962'8
9'12	83'1744	758'551	6917'98	63092'0	9'62	92'5444	890'277	8564'47	82390'2
9'13	83'3569	761'048	6948'37	63438'6	9'63	92'7369	893'056	8600'13	82819'3
9'14	83'5396	763'552	6978'86	63786'8	9'64	92'9296	895'841	8635'91	83250'2
9'15	83'7225	766'061	7009'46	64136'5	9'65	93'1225	898'632	8671'80	83682'9
9'16	83'9056	768'575	7040'15	64487'8	9'66	93'3156	901'429	8707'80	84117'4
9'17	84'0889	771'095	7070'94	64840'5	9'67	93'5089	904'231	8743'91	84553'7
9'18	84'2724	773'621	7101'84	65194'9	9'68	93'7024	907'039	8780'14	84991'8
9'19	84'4561	776'152	7132'83	65550'7	9'69	93'8961	909'853	8816'48	85431'7
9'20	84'6400	778'688	7163'93	65908'2	9'70	94'0900	912'673	8852'93	85873'4
9'21	84'8241	781'230	7195'13	66267'1	9'71	94'2841	915'499	8889'49	86317'0
9'22	85'0084	783'777	7226'43	66627'7	9'72	94'4784	918'330	8926'17	86762'4
9'23	85'1929	786'330	7257'83	66989'8	9'73	94'6729	921'167	8962'96	87209'6
9'24	85'3776	788'889	7289'33	67353'5	9'74	94'8676	924'010	8999'86	87658'7
9'25	85'5625	791'453	7320'94	67718'7	9'75	95'0625	926'859	9036'88	88109'6
9'26	85'7476	794'023	7352'65	68085'5	9'76	95'2576	929'714	9074'01	88562'3
9'27	85'9329	796'598	7384'46	68454'0	9'77	95'4529	932'575	9111'26	89017'0
9'28	86'1184	799'179	7416'38	68824'0	9'78	95'6484	935'441	9148'62	89473'5
9'29	86'3041	801'765	7448'40	69195'6	9'79	95'8441	938'314	9186'09	89931'8
9'30	86'4900	804'357	7480'52	69568'8	9'80	96'0400	941'192	9223'68	90392'1
9'31	86'6761	806'954	7512'75	69943'7	9'81	96'2361	944'076	9261'39	90854'2
9'32	86'8624	809'558	7545'08	70320'1	9'82	96'4324	946'966	9299'21	91318'2
9'33	87'0489	812'166	7577'51	70698'2	9'83	96'6289	949'862	9337'14	91784'1
9'34	87'2356	814'781	7610'05	71077'9	9'84	96'8256	952'764	9375'20	92251'9
9'35	87'4225	817'400	7642'69	71459'2	9'85	97'0225	955'672	9413'37	92721'7
9'36	87'6096	820'026	7675'44	71842'1	9'86	97'2196	958'585	9451'65	93193'3
9'37	87'7969	822'657	7708'30	72226'7	9'87	97'4169	961'505	9490'05	93666'8
9'38	87'9844	825'294	7741'25	72613'0	9'88	97'6144	964'430	9528'57	94142'3
9'39	88'1721	827'936	7774'32	73000'9	9'89	97'8121	967'362	9567'21	94619'7
9'40	88'3600	830'584	7807'49	73390'4	9'90	98'0100	970'299	9605'96	95099'0
9'41	88'5481	833'238	7840'77	73781'6	9'91	98'2081	973'242	9644'83	95580'3
9'42	88'7364	835'897	7874'15	74174'5	9'92	98'4064	976'191	9683'82	96063'5
9'43	88'9249	838'562	7907'64	74569'0	9'93	98'6049	979'147	9722'93	96548'7
9'44	89'1136	841'232	7941'23	74965'2	9'94	98'8036	982'108	9762'15	97035'8
9'45	89'3025	843'909	7974'94	75363'1	9'95	98'0025	985'075	9801'50	97524'9
9'46	89'4916	846'591	8008'75	75762'7	9'96	99'2016	988'048	9840'96	98015'9
9'47	89'6809	849'278	8042'66	76164'0	9'97	99'4009	991'027	9880'54	98509'0
9'48	89'8704	851'971	8076'69	76567'0	9'98	99'6004	994'012	9920'24	99004'0
9'49	90'0601	854'670	8110'82	76971'7	9'99	99'8001	997'003	9960'06	99501'0
9'50	90'2500	857'375	8145'06	77378'1	10'00	100'0000	1000'000	10000'0	100000

n	n^2	n^3	n^4	n^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049

ARITHMETIC

(PART 6)

RATIO

1. Suppose that it is desired to compare two numbers, say 20 and 4. If it is desired to know how many times larger 20 is than 4, divide 20 by 4 and 5 is obtained for the quotient; thus, $20 \div 4 = 5$. Hence, it may be said that 20 is 5 times as large as 4; i.e., 20 contains 5 times as many units as 4. Again, suppose it is required to know what part of 20 is 4. Then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or $\cdot 2$. Hence, 4 is $\frac{1}{5}$, or $\cdot 2$, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison, and may be defined as the relationship which any magnitude (whether numerical or other) bears to another magnitude of the same kind. Observe that the magnitudes between which a ratio is established must be of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4.

2. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$, $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$, $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom, if ever, used; the second form, which is a corruption of the first, $\div$ (sign of

division), is the one most often met with, while the third is rapidly growing in favour, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is the one that will be ordinarily adopted in this Section. There is still another way of expressing a ratio, though seldom, if ever, used in the case of a simple ratio like that given above. Instead of the colon, a vertical line is used; thus, $20 \mid 4$.

3. The terms of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together they are called a **couplet**; when considered separately, the first term is called the **antecedent**, and the second term the **consequent**. Thus, in the ratio $20 : 4$, 20 and 4 form a couplet; 20 is the antecedent and 4 the consequent.

When a ratio is expressed in the fractional form, the antecedent becomes the numerator and the consequent the denominator. Thus, the ratio of £35 to £7 is written $\frac{£35}{£7}$, £35 and £7 being in both cases the antecedent and consequent, respectively.

4. A ratio may be **direct** or **inverse**. The *direct ratio* of 20 to 4 is $20 : 4$, while the *inverse ratio* of 20 to 4 is $4 : 20$. The direct ratio of 4 to 20 is $4 : 20$, and the inverse ratio is $20 : 4$. When one ratio is the inversion of another, each is said to be the **reciprocal** of the other. Thus, $20 : 4$ and $4 : 20$ are reciprocal ratios. So, also, are $£17 : £1$ and $£1 : £17$. The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i.e., the reciprocal of a fraction is the fraction inverted. The inverse ratio of 20 to 4 may be expressed as $4 : 20$, or as $\frac{1}{20} : \frac{1}{4}$. The two ratios

are identical, that is, they have equal values ; for,

$$4 \div 20 = \frac{1}{5}, \text{ and } \frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$$

5. The term **vary** implies a ratio. When it is said that two numbers vary as some other two numbers, it means that the relation between the first two numbers is the same as the relation between the other two numbers.

6. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5 ; it is the quotient obtained by dividing the antecedent by the consequent.

7. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}$$

8. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio, and dividing the antecedent or multiplying the consequent divides the ratio.

9. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, simply transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent ; or, if the ratio is expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, transpose the terms, as 4 : 20, or as $\frac{4}{20}$; or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE

10. What are the values of the following :

(a) The ratio of 98 to 49 ?

(b) The ratio of £45 to £9 ?

(c) The ratio of $6\frac{1}{4}$ to $\frac{2}{4}$?

(d) The ratio of 3.5 to 4.5 ?

(e) The inverse ratio of 76 to 19 ?

(f) The inverse ratio of 49 to 98 ?

(g) The inverse ratio of 18 to 24 ?

(h) The inverse ratio of 9 to 15 ?

(i) The ratio of 10 to 3, multiplied by 3 ?

(j) The ratio of 35 to 49, multiplied by 7 ?

(k) The ratio of 18 to 64, divided by 9 ?

(l) The ratio of 14 to 28, divided by 5 ?

(m) 111 gal. : 37 gal. ?

Ans. $\left\{ \begin{array}{ll} (a) & 2 \\ (b) & 5 \\ (c) & 12\frac{1}{2} \\ (d) & .77\frac{7}{9} \\ (e) & \frac{1}{4} \\ (f) & 2 \\ (g) & 1\frac{1}{3} \\ (h) & 1\frac{2}{3} \\ (i) & 10 \\ (j) & 5 \\ (k) & \frac{1}{32} \\ (l) & \frac{1}{10} \\ (m) & 3 \end{array} \right.$

11. Instead of expressing the value of a ratio by a single number as above, it is customary to express it by means of another ratio, in which the consequent is 1. This is called expressing the ratio in terms of unity. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, it becomes $\frac{45}{30}$. Dividing

both terms by 30, the consequent, one obtains $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$.

This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

12. Since ratios are merely abstract numbers, they may be compared. For example, which is the greater, 7 : 5 or 18 : 12 ? Performing the divisions, $7 : 5 = 1.4$ and $18 : 12 = 1.5$; therefore, the latter is the greater. A convenient way of comparing ratios is to express them as fractions and reduce the fractions to a common denominator. For example, it is desired to find which is the greater, the ratio of 8 to 9 or the

ratio of 9 to 10. Ratio $8 : 9 = \frac{8}{9}$ and $9 : 10 = \frac{9}{10}$. Reducing to a common denominator, $\frac{8}{9} = \frac{80}{90}$ and $\frac{9}{10} = \frac{81}{90}$. Since 81 is greater than 80, the ratio of 9 to 10 is the greater.

EXAMPLES FOR PRACTICE

13. 1. Which is the greater

(a) $10 : 7$ or $16 : 11$?

(b) $2 : 3$ or $7 : 10$?

(c) $22 : 7$ or $355 : 113$?

Ans. $\begin{cases} (a) 16 : 11 \\ (b) 7 : 10 \\ (c) 22 : 7 \end{cases}$

2. If the antecedent is 5 hours and the consequent is 50 minutes, what is the ratio ?

Ans. 6

3. What is the ratio of $\frac{3}{8}$ to $\frac{11}{16}$?

Ans. $\frac{6}{11}$

4. If in the ratio $40 : 12$ the antecedent is multiplied by 3, what is the value of the new ratio ?

Ans. 10

PROPORTION

14. **Proportion** is an equality of ratios, the equality being indicated by the double colon ($::$) or by the sign of equality ($=$). Thus, to write in the form of a proportion the two equal ratios, $8 : 4$ and $6 : 3$, one of the three following forms may be employed :

$$8 : 4 :: 6 : 3 \quad (1)$$

$$8 : 4 = 6 : 3 \quad (2)$$

$$\frac{8}{4} = \frac{6}{3} \quad (3)$$

15. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably supersede the first form. In this Section the second form will be adopted, unless some statement can be made clearer by using the third form.

16. A proportion may be *read* in two ways. One is—*8 is to 4 as 6 is to 3*; the other way is—*the ratio of 8 to 4 equals the ratio of 6 to 3*. Either way may be used, according to choice.

17. Each ratio of a proportion is termed a **couplet**. In the above proportion, $8 : 4$ is a couplet; so, also, is $6 : 3$.

18. The numbers forming a proportion are called **terms**; and they are numbered consecutively from left to right, thus:

$$\begin{array}{cccc} \text{first} & \text{second} & \text{third} & \text{fourth} \\ 8 : 4 = 6 : 3 \end{array}$$

Hence, in any proportion the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

19. The first and fourth terms of a proportion are called the **extremes**, and the second and third terms, the **means**. Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

20. A **direct proportion** is one in which both couplets are direct ratios.

21. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, if it is stated that 8 is to 4 inversely as 3 is to 6, the proportion must be written $8 : 4 = 6 : 3$; i.e., the second ratio (couplet) must be inverted.

22. Proportion forms one of the most useful sections of arithmetic. In the arithmetics of our grandfathers it was called "The Rule of Three."

23. To test a proportion the following principle is applied:

In any proportion, the product of the extremes is equal to the product of the means.

The truth of this principle may be shown by an example. The proportion $9 : 3 = 51 : 17$ is evidently true, since the value of the ratio of each couplet is three. Expressed in the fractional form, the proportion is $\frac{9}{3} = \frac{51}{17}$. To reduce these fractions to a common denominator, multiply both terms of the first by 17

and both terms of the second by 3; thus, $\frac{9 \times 17}{3 \times 17} = \frac{51 \times 3}{17 \times 3}$.

The fractions are still equal, because their values have not been changed by multiplying the numerator and denominator by the same number; and since the denominators are equal, each being the product 3×17 , the numerators 9×17 and 51×3 must be equal. But the numerator 9×17 is the product of the extremes, and the numerator 51×3 is the product of the means. Hence, in every case, these products are equal if the four numbers form a proportion.

EXAMPLE.—Can the four numbers, 16, 125, 12, and 94 form a proportion?

SOLUTION.—Taking 16 and 94 for the extremes and 125 and 12 for the means, the product $16 \times 94 = 1,504$ and $125 \times 12 = 1,500$ are not equal, and the numbers cannot form a proportion. Ans.

24. The problem that most frequently occurs in proportion is to find one of the terms when the other three terms are given. Suppose it is required to find a number that will form a proportion with the numbers 6, 13, and 30. Representing the missing term by a letter, such as x , the proportion will read, $6 : 13 = 30 : x$. Placing the product of the extremes equal to the product of the means,

$$6 \times x = 13 \times 30$$

Since 6 times $x = 13 \times 30$, x must be $\frac{1}{6}$ of 13×30 ; or

$$x, \text{ the required number} = \frac{13 \times 30}{6} = 65$$

The unknown extreme is therefore equal to the product of the means divided by the known extreme.

25. Rule.—I. *To find an unknown extreme, divide the product of the means by the given extreme.*

II. *To find an unknown mean, divide the product of the extremes by the given mean.*

EXAMPLE 1.—What is the third term in the proportion $17 : 51 = x : 42$?

SOLUTION.—In this case, the mean x is unknown. The extremes are 17 and 42, and the other mean is 51. Hence, from rule II,

$$x = \text{the third term} = \frac{17 \times 42}{51} = 14. \text{ Ans.}$$

EXAMPLE 2.—What is the first term in the proportion $x : 4 = 21 : 6$?

SOLUTION.—Applying rule I,

$$x = \text{the first term} = 4 \times 21 \div 6 = 14. \quad \text{Ans.}$$

26. Proportion is useful in solving a class of problems containing three concrete quantities, of which two are of the same kind and have a certain ratio, and it is required to find a fourth quantity which shall bear the same ratio to the remaining quantity. The following is an example of a problem of this kind :

EXAMPLE.—If 4 horses can be bought for £130 how much must be paid for 11 horses at the same price ?

SOLUTION.—Here the 4 horses and the 11 horses are the two concrete quantities of the same kind having a definite ratio, and it is required to find a fourth quantity to which £130 shall bear the same relation that 4 horses bear to 11 horses.

In solving a problem of this kind the following question must always be asked : “ What is it I wish to find ? ” In the present case it is pounds sterling. The 4 horses are bought for a known amount of money, and the 11 horses, at the same price, cost an amount as yet unknown, but to be found. Since the horses are all bought at the same price, it is evident that the cost of 11 horses bears the same relation to £130, the cost of 4 horses, that 11 horses do to 4 horses. In other words, the ratio of the cost of 11 horses to the cost of 4 horses is equal to the ratio of 11 horses to 4 horses.

These two equal ratios may be put in the form of a proportion ; thus,
cost of 11 horses : £130 = 11 horses : 4 horses

The unknown term is an extreme, and is found by dividing the product of the means by the known extreme.

$$\text{Cost of 11 horses} = \frac{£130 \times 11}{4} = £357 \text{ 10s.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

27. 1. Find the value of x in each of the following :

(a) £16 : £64 :: x : 4.

(b) x : 85 :: 10 : 17.

(c) 24 : x :: 15 : 40.

(d) 18 : 94 :: 2 : x .

(e) £75 : £100 = x : 100.

(f) 15 dwt. : x = 21 : 10.

(g) x : 75 yd. = £15 : £5.

$$\text{Ans.} \left\{ \begin{array}{l} (a) \quad x = 1 \\ (b) \quad x = 50 \\ (c) \quad x = 64 \\ (d) \quad x = 10\frac{4}{9} \\ (e) \quad x = 75 \\ (f) \quad x = 7\frac{1}{7} \text{ dwt.} \\ (g) \quad x = 225 \text{ yd.} \end{array} \right.$$

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days ? Ans. 36 days

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what is the circumference of a circle 31 inches in diameter ?

Ans. $97\frac{3}{7}$ in.

4. A grocer bought 2 tons 3 cwt. 3 qr. of sugar for £61 5s.; what would he give for 5 cwt. 1 qr. 16 lb. ?

Ans. £7 11s.

INVERSE PROPORTION

28. In the example given in Art. 26, the proportion was direct. The *greater* the number of horses bought the *greater* must be the amount paid for them ; that is, the cost of the horses varies *directly* as the number of horses. In some cases, the nature of the problem requires an inverse proportion. Such a case may be best shown by an example.

EXAMPLE.—If 6 men can do a piece of work in 8 days, in what time can 18 men do it, working at the same rate ?

SOLUTION.—It is clear that by employing 18 men instead of 6 the work will be done in *less* time ; that is, the *greater* the number of men the *less* the time required to do the work. The time varies *inversely* as the number of men, and the statement of the problem requires an inverse proportion. The ratio of the number of days required by 18 men to the 8 days required by 6 men is equal to the *inverse* ratio of 18 men to 6 men. Denoting the unknown number of days by x and writing the proportion as a direct proportion, the result is

$$6 : 18 = 8 : x$$

But since the proportion is an inverse one, one of the couplets must be inverted. If the first couplet is inverted, the result is $18 : 6 = 8 : x$; if the second, $6 : 18 = x : 8$. In either case,

$$x = \frac{8 \times 6}{18} = 2\frac{2}{3} \text{ days. Ans.}$$

29. Sometimes the word *inverse* occurs in the statement of the example ; in such cases the proportion can be written at once, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, one should always ascertain whether an *increase* in one of the quantities forming the proportion involves a proportional *decrease* in the other quantity forming the proportion, and vice versa. If it does, the proportion is *inverse*. If, on the other hand, an increase in one of the quantities

forming the proportion involves an increase in the other quantity forming the proportion (or decrease involves a decrease), the proportion is direct.

30. EXAMPLE 1.—A carriage wheel, 16 feet in circumference, makes 200 turns in passing over a certain distance. How many turns will a wheel 10 feet in circumference make in going the same distance ?

SOLUTION.—The quantities considered are the circumferences of the wheels and the number of turns made. The circumference is the measurement *round* the wheel. It is obvious that the larger the circumference of a wheel, the fewer turns it will make in going a certain distance. Therefore, as the circumference of a wheel *decreases*, so will the number of turns it makes to cover a certain distance *increase*. It is clear, therefore, that the question is one of inverse proportion.

Let x = the number of turns of the smaller wheel. As a direct proportion, the problem would be stated as

$$16 : 10 = 200 : x$$

But as it has been shown to be a problem in inverse proportion, invert the second couplet, and the problem reads correctly, as 16 feet circumference is to 10 feet circumference, so is x turns to 200 turns, or

$$16 : 10 = x : 200,$$

from which
$$x = \frac{16 \times 200}{10} = 320 \text{ turns. Ans.}$$

EXAMPLE 2.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it ?

SOLUTION.—The required term is the number of days it will take B to do the work. Now, since A's rate of working is slower than B's, it follows that B would take a less number of days to do the work than A. Therefore, the *increase* in the rates of working forming the first couplet of the proportion involves a proportional *decrease* in the number of days occupied on the work, and the problem is an inverse proportion. Stated as a direct proportion, it would be expressed $5 : 7 = 42 : x$ (the number of days B will take to do the work). Inverting the second couplet it becomes

$$5 : 7 = x : 42,$$

from which
$$x = \frac{5 \times 42}{7} = 30 \text{ days. Ans.}$$

Had the example been stated thus : The *time* that A requires to do a piece of work is to the time that B requires as 5 : 7 ; A can do it in 42 days, in what time can B do it ?—it is evident that it would take B a longer time to do that work than it would A. Hence, the *increase* in the time taken for one piece of work involves a proportional *increase*

in the number of days taken for another piece of work ; therefore, the proportion is direct, and the problem would be stated

$$5 : 7 :: 42 : x ;$$

$$x = \frac{7 \times 42}{5} = 58.8 \text{ days. Ans.}$$

31. The unknown term x may occupy the place of any term in a proportion without destroying the proportion. Thus, consider the example : If 4 men earn £5 in 1 week, how much will 12 men earn in the same time ? The proportion may be formed in any of the four following ways :

$$x : £5 = 12 \text{ men} : 4 \text{ men} ;$$

$$£5 : x = 4 \text{ men} : 12 \text{ men} ;$$

$$12 \text{ men} : 4 \text{ men} = x : £5 ;$$

$$4 \text{ men} : 12 \text{ men} = £5 : x$$

From any one of the above proportions,

$$x = \frac{£5 \times 12}{4} = £15$$

EXAMPLES FOR PRACTICE

32. Solve the following :

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it ? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal. ? Ans. 32 hr.

3. If a man can make a journey in 15 days, travelling 10 hr. per day, how long will it take him if he travels 8 hr. per day ? Ans. $18\frac{3}{4}$ days

4. A merchant sells £4,500 worth of goods and gains £1,300. At the same rate, what would he gain if he sold £8,000 worth ?

Ans. £2,311 2s. $2\frac{2}{3}$ d.

5. If 14 men can dig a cellar in 12 days, how long will it take 8 men to dig it ? Ans. 21 days

POWERS AND ROOTS IN RATIO

33. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is $105 : 63$, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen ;

for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$ it may be done by simply dividing the indices (indices is the plural of index) by 3, obtaining $105 : 63$. This may be proved in the same way as in the case of cubing the ratio. Thus,

$$105^3 : 63^3 = \left(\frac{105}{63}\right)^3,$$

and
$$\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$$

34. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$

(this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), and hence the antecedent and consequent may both be multiplied or divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the indices, $24^2 = 576$ and $18^2 = 324$. Hence,

$$576 : 324 = 1\frac{7}{9}, \text{ or } 1\frac{7}{9} : 1$$

Also, $4^2 = 16$ and $3^2 = 9$

Hence, $16 : 9 = 1\frac{7}{9}, \text{ or } 1\frac{7}{9} : 1,$

the same result as before. Also,

$$24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$$

The case of equal roots of a ratio may be proved for roots in a similar manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$, and $\sqrt[3]{18^3} = 18$; and $24 : 18 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$, and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different indices, or if different roots of those numbers are indicated, the operations described in Art. 34 cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$ and, hence, cannot be reduced as described above.

EXAMPLES FOR PRACTICE

35. Solve the following :

1. What is the cube root of the ratio of 135 to 5 ? Ans. 3
2. What is the square root of the ratio of 9 to 576 ? Ans. $\frac{1}{8}$ or $\cdot 125$
3. Which is the greater, the square of the ratio of 5 to 9, or the cube root of the ratio of 125 to 4,096 ? Ans. The latter
4. What is the square of the inverse ratio of $\frac{1}{2}$ to 3 ? Ans. 36

POWERS AND ROOTS IN PROPORTION

36. If all the terms of a proportion are raised to the same power, or if the same root of all the terms is taken, the proportion still holds. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the fractional form, it becomes $\frac{8}{4} = \frac{6}{3}$. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since

$$\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8,$$

and
$$\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8 \text{ also}$$

Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since

$$\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2},$$

and
$$\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2} \text{ also}$$

37. It was stated in Art. 33 that a ratio may be raised to any power, or any root of it may be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power, or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power, or the same root of both terms must be taken.*

38. EXAMPLE 1.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

SOLUTION.—This is evidently a direct proportion. Hence,

$$6^3 : 8^3 = x : 180$$

Dividing both terms of the first couplet by 2 (see Art. 34),

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence,
$$x = \frac{27 \times 180}{64} = 75\frac{15}{16} \text{ lb. Ans.}$$

EXAMPLE 2.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{16}$ pounds?

SOLUTION.—Since the weights of any two spheres are to each other as the cubes of their diameters, it follows that

$$180 : 75\frac{15}{16} = 8^3 : x^3$$

x , the required term must be cubed, because the other term of the couplet is cubed (see Art. 36). But, $8^3 = 512$; hence,

$$180 : 75\frac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{16} \times 512}{180} = 216;$$

whence,
$$x = \sqrt[3]{216} = 6 \text{ in. Ans.}$$

39. Since taking the same root of all the terms of a proportion does not change its value (Art. 36), the above example might have been solved by extracting the cube root of all of the numbers, thus obtaining

$$\begin{aligned} \sqrt[3]{180} : \sqrt[3]{75\frac{15}{16}} &= 8 : x; \\ \text{whence, } x &= \frac{8 \times \sqrt[3]{75\frac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\frac{75\frac{15}{16}}{180}} = 8 \sqrt[3]{\frac{1,215}{2,880}} \\ &= 8 \sqrt[3]{\frac{27}{64}} = 8 \times \frac{3}{4} = 6 \text{ inches} \end{aligned}$$

This process, however, is longer and is not so direct, and the first method is to be preferred.

40. If two cylinders have *equal* volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}$$

Since neither 12 nor 15 is a perfect square, all of the terms are squared (Arts. 36 and 39), obtaining

$$81 : x^2 = 15 : 12 ;$$

whence,
$$x^2 = \frac{81 \times 12}{15} = 64.8,$$

and
$$x = \sqrt{64.8} = 8.05 \text{ — inches}$$

$$= \text{diameter of 15-inch cylinder}$$

EXAMPLES FOR PRACTICE

41. Solve the following examples :

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet ? Ans. $2\frac{1}{4}$ times as bright

2. In example 1, suppose that the object had been 40 feet from the gas jet ; how bright would it have been compared with its brightness at 30 feet from the gas jet ? Ans. $\frac{9}{16}$ as bright

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the objects they illuminate. If a man can just distinguish the time indicated by his watch 50 feet from a certain light, at what distance could he just distinguish the time by a light 3 times as powerful ? Ans. 86.6 ft., nearly

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot ? Ans. 68,034 cu. ft. per min., nearly

5. In example 4, suppose that 70,000 cubic feet per minute had been required ; what would be the pressure necessary for this quantity ?

Ans. 3.81 lb. per sq. ft., nearly

PROPERTIES OF A PROPORTION

42. If the same operations (addition and subtraction excepted) are performed upon *all* the terms of a proportion, the proportion is not thereby destroyed. In other words, if all the terms of a proportion are (1) multiplied or (2) divided by the same number ; or (3) if both couplets are inverted, the proportion still holds. These statements will be proved by a numerical example, in which the fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$.

Expressing it in the fractional form, it becomes $\frac{8}{4} = \frac{6}{3}$. The fact to be proved is that if any of the three operations, enumerated above, are performed upon all the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since each equals $\frac{1}{2}$.

43. If both terms of either couplet are multiplied or both are divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. 7. Hence, in any proportion, equal factors may be cancelled from the terms of a couplet, before applying the rule of Art. 25. Thus, in the proportion $45 : 9 = x : 7.1$, both terms of the first couplet may be divided by 9 (that is, 9 may be cancelled from both terms), obtaining $5 : 1 = x : 7.1$; whence, $x = 7.1 \times 5 \div 1 = 35.5$. Without cancelling, $x = \frac{7.1 \times 45}{9} = 35.5$.

CAUSE AND EFFECT

44. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called the *cause*, and the change or alteration, or thing accomplished, the *effect*.

45. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the ratio of the first cause to the second cause equals the ratio of the first effect to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? 3 men and 7 men are termed the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the *effects*. If 3 men is called the first cause, 1,400 pounds is the first effect; 7 men is the second cause and x pounds is the second effect. Hence,

$$\begin{array}{cccc} \text{1st cause} & \text{2nd cause} & \text{1st effect} & \text{2nd effect} \\ 3 & : & 7 & = 1,400 : x, \end{array}$$

whence
$$x = \frac{7 \times 1,400}{3} = 3,266\frac{2}{3} \text{ pounds}$$

46. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as will now be shown.

COMPOUND PROPORTION

47. All the cases of proportion so far considered have been cases of **simple proportion**; i.e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion**. In all examples in compound proportion, both causes or both effects, or all four, consist of more than one number each. This will be illustrated by an example.

EXAMPLE.—If 40 men earn £200 in 16 days how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn that something, 16 days is an element of the cause. For the same reason, 36 men working for 31 days is also a cause. The effects, that which is earned, are £200 and £ x . Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. £200 is the first effect and £ x is the second effect.

Hence,

$$\begin{array}{ccccccc} & 1st\ cause & & 2nd\ cause & & 1st\ effect & & 2nd\ effect \\ & 40 & : & 36 & = & 200 & : & x \\ & 16 & : & 31 & & & & \end{array}$$

If, instead of using the colon to express the ratio, the vertical line is used (see Art. 2), the above becomes

$$\begin{array}{cc|cc} 40 & & 36 & \\ 16 & & 31 & \end{array} = 200 \quad | \quad x$$

In the last expression, the product of all the numbers included between the vertical lines must equal the product of all the numbers outside them ; i.e., $36 \times 31 \times 200 = 40 \times 16 \times x$.

$$\begin{array}{c} 9 \qquad 5 \\ Or, \quad x = \frac{36 \times 31 \times 200}{40 \times 16} = £348\ 15s. \quad Ans. \\ 4 \end{array}$$

48. The above might have been solved by cancelling factors of the numbers in the original proportion. For, if any number within the lines has a factor common to any number outside the lines, that factor may be cancelled from both numbers.

$$\begin{array}{cc|cc} & 9 & & 5 \\ 40 & 36 & = & 200 \\ 16 & 31 & & x \\ 4 & & & \end{array}$$

Thus, 40 is contained in 200, 5 times. Cancel 40 and 200, and write 5 above 200. 4 is contained in 16, 4 times, and in 36, 9 times. Cancel 16 and 36, and write 4 below 16 and 9 above 36. Now, since there are no more numbers that can be cancelled,

$$x = \frac{9 \times 31 \times 5}{4} = £348.75 = £348\ 15s.,$$

the same result as was obtained in the last article.

49. *Rule.*—Write all the numbers forming the first cause in a vertical column, on the left of a vertical line ; on the other side of this line, write in a vertical column all the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right ; on the other side of this line, write for the fourth column the numbers

composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those outside the lines (cancelling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x .

50. EXAMPLE 1.—If 40 men can dig a ditch 720 ft. long, 5 ft. wide, and 4 ft. deep in a certain time, how long a ditch 6 ft. deep and 3 ft. wide can 24 men dig in the same time?

SOLUTION.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{c|c|c|c}
 & & 3 & \\
 & & 15 & \\
 & & 720 & x \\
 40 & 24 = 5 & 3 & \text{whence, } x = 24 \times 5 \times 4 = 480 \text{ ft. Ans.} \\
 & 4 & 6 &
 \end{array}$$

EXAMPLE 2.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what is the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, then

$$\begin{array}{c|c|c|c}
 & 4 & & \\
 8 & 12 & & \\
 5 & 8 = 7,200 & x, \text{ or } x = 4 \times 7,200 = 28,800 \text{ lb. Ans.} & \\
 3 & 5 & &
 \end{array}$$

EXAMPLE 3.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines, the effects. Hence,

$$\begin{array}{c|c|c|c}
 3 & 8 & 5 & 2 \\
 12 & 18 & 25 & 64 \\
 & & 4 & \\
 30 & x = 16 & 12, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days. Ans.} & \\
 8 & & 4 & \\
 10 & 8 & 32 & 40 \\
 & & & 5
 \end{array}$$

51. The rules of compound proportion, whether direct or inverse, apply equally to proportions involving powers and roots.

EXAMPLE 1.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570·8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

SOLUTION.—In this example, either the dimensions or the volumes may be considered the causes; say one takes the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|c|c|c|c|c} 10^2 & 16^2 & = & 1,570\cdot8 & x, \text{ or} & \frac{100}{20} \left| \frac{256}{24} = 1,570\cdot8 \right| x; \\ 20 & 24 & & & & \frac{5}{6} \end{array}$$

whence, $x = \frac{256 \times 6 \times 1,570\cdot8}{5 \times 100} = 4,825\cdot4976 \text{ cu. in. Ans.}$

EXAMPLE 2.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the centre of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect we have

$$\begin{array}{c|c|c} 15 & 15 & \\ 20^2 & 24^2 = 187 & x \\ 12 & 18 & \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|c} 15 & 15 & \\ & 2 & \\ 25 & 36 = 187 & x, \text{ or } x = \frac{12 \times 2 \times 187}{25} = 179\cdot52 \text{ lb. Ans.} \\ 400 & 576 & \\ 15 & 12 & \end{array}$$

EXAMPLES FOR PRACTICE

52. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rd. long in 24 days of 10 hr. each, how many roods can 16 men dig in 18 days of 9 hr. each? Ans. 36 rd.

2. If a bar of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a bar weigh that is 16 ft. long, 8 in. wide, and 4 in. thick? Ans. $1,828\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rd. long, 6 ft. wide, and 5 ft. high in 60 days of 10 hr. each, how many days will it take 32 men to build a wall 96 rd. long, 4 ft. wide, and 8 ft. high, working 8 hr. a day? Ans. 80 days

4. An employer pays each of his workmen 27s. a week when they work 9 hr. a day ; what should he pay them when they work only 8 hr. a day, if in that case 12 men can do as much in 1 hr. as 13 in the former case ? Ans. 26s.

5. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360

6. If 60 yd. of carpet $\frac{3}{4}$ of a yard wide will cover a room 27 ft. long and 15 ft. wide, how many yards of carpet $\frac{7}{8}$ of a yard wide will cover a room 35 ft. long and 18 ft. wide ? Ans. 80 yd.

7. If it costs £72 10s. to ship 9,000 cwt. of freight a distance of 850 miles, how much will it cost to ship 22,000 cwt. 675 miles at the same rate per mile ? Ans. £140 14s. 8 $\frac{8}{17}$ d.

8. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop, if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch ? Ans. 204.8 horsepower

9. Referring to example 1, Art. 51, what will be the volume of a cylinder 20 inches in diameter and 24 inches long ? Ans. 7,539.84 cu. in.

PERCENTAGE

53. It has been shown that a ratio can be expressed either as a vulgar or a decimal fraction. In a vulgar fraction the consequent may be any number from 1 upward ; in a decimal fraction, the consequent is either 10 or a power of 10, that is, 10 multiplied by itself once, twice, thrice, or more times. There is a third way of expressing a ratio, which is much used in practice, namely, by fixing the consequent at 100. Thus the ratio of 3 to 20 may be expressed as $\frac{3}{20}$ or $\frac{15}{100}$, the ratio that 15 bears to 100 ; i.e., 15 is the number of units out of 100 which is taken or considered.

54. The number of parts out of 100 is called a **percentage**. It is also seen to be the ratio that the given number of things or parts of things bears to 100 of them.

55. Thus, 8 parts out of a hundred is spoken of as **8 per cent.** (per cent., or per centum, meaning by, or out of, one hundred) and is usually written 8%. The sign % is read *per cent.*

56. The term per cent. is also spoken of as **rate per cent.** Thus, if 12 parts in the hundred is considered, it is spoken of as a *rate* of 12 *per cent.*, or a *percentage* of 12, when the number of things is 100. If the number of things is greater or less than 100, the percentage is obtained by multiplying that number by the rate per cent. For example, if the number is 435 and the rate per cent. is 12, the percentage is $435 \times \frac{12}{100} = 435 \times .12 = 52.2$.

It is noted in the above definition of percentage that the terms "rate per cent." and "percentage" are synonymous *only* when the number of things considered is 100. Though this distinction is correct, it is not in general use, it being customary to consider these terms as synonymous also when applied to any other number than 100. Thus in the preceding example 12 may be called "rate per cent." or "percentage" while 52.2 would be called "the amount of percentage."

57. If one quantity is a fraction of another quantity it is possible to determine what percentage the one quantity is of the other. Thus, if a person occupies a house at a rent of £200 a year, and his income is £1,500, he pays in rent $\frac{200}{1500}$ of his income, or 200 parts in every 1,500. Reducing this fraction to one with a denominator of 100 by dividing numerator and denominator by 15, it becomes $\frac{13\frac{1}{3}}{100}$, or $13\frac{1}{3}$ parts in every 100 parts. So that he pays in rent $13\frac{1}{3}\%$ of his income.

58. If the ratio of one quantity to another is expressed as a fraction, it is converted into a rate per cent. by multiplying the fraction by 100, since the fraction denotes so many parts out of *one* unit, and the rate per cent. denotes so many units out of 100 units. Thus, if one litre of water contains $\frac{1}{3}$ litre of oxygen, the percentage of oxygen in the water is $\frac{1}{3} \times 100 = 33\frac{1}{3}$, which is also, in this case, the rate per cent.

59. Bearing in mind that the rate per cent. is a ratio with a fixed consequent of 100, all calculations involving percentages may be stated as proportions. The following examples will make this clear :

EXAMPLE 1.—How much per cent. is 7 lb. of 1 cwt. ?

SOLUTION.—Let x = rate per cent.

$$x : 100 = 7 \text{ lb.} : 1 \text{ cwt.}$$

$$= 7 \text{ lb.} : 112 \text{ lb.}$$

$$x = \frac{100 \times 7}{112} = \frac{700}{112} = 6.25\%. \quad \text{Ans.}$$

EXPLANATION.—What is sought is to find an unknown member x which shall bear the same proportion to 100 as 7 lb. bears to 1 cwt., or 112 lb. Observe that both of the terms of a ratio must be reduced to the same denomination. The problem is therefore one in simple proportion.

EXAMPLE 2.—How much is 14% of £250 ?

SOLUTION.—In this question the rate per cent. is given. The unknown quantity is the amount which bears the same proportion to £250 as 14 bears to 100. Let x = the amount required, that is, the antecedent of the second couplet in the proportion. Thus,

$$14 : 100 = x : £250 ;$$

$$\text{hence,} \quad x = \frac{£250 \times 14}{100} = £35. \quad \text{Ans.}$$

EXAMPLE 3.—A man bought a certain number of barrels of apples and sold 76% of them. If he sold 228 barrels, how many barrels did he buy ?

SOLUTION.—Here 76 barrels were sold out of every 100 barrels, and 228 barrels out of a proportional unknown number. The proportion therefore is :

$$76 : 100 = 228 : x$$

3

$$\text{and} \quad x = \frac{228 \times 100}{76} = 300 \text{ barrels.} \quad \text{Ans.}$$

EXAMPLE 4.—In a factory where 2,100 men are employed the force is increased 8%. How many new men are employed, and how many men are at work after the increase ?

SOLUTION.—The increase is 8% of 2,100 ; that is,

$$100 : 8 = 2,100 : x \text{ (or } 8 : 100 = x : 2,100)$$

$$\text{and} \quad x = \text{increase} = \frac{8 \times 2,100}{100} = 168 \text{ men.} \quad \text{Ans.}$$

And total number of men or force is increased to $2,100 + 168 = 2,268$ men. Ans.

ALTERNATE SOLUTION.—As every 100 men are increased by 8 men, they are increased to 108 men. Therefore,

$$100 : 108 = 2,100 : x \text{ (increased force)}$$

$$\text{and} \quad x = \text{increased force} = \frac{2,100 \times 108}{100} = 2,268 \text{ men.} \quad \text{Ans.}$$

And the increase is $2,268 - 2,100 = 168$ men. Ans.

EXAMPLE 5.—A speculator invested £2,650 in a business enterprise and lost 16% of his investment. How much had he left?

SOLUTION.—Here there are alternate methods. Either find how much he lost and subtract it from the amount invested, or find how much per cent. he saved of his money, and work from that.

(a) Of every £100 invested he lost £16; therefore, of £2,650 he lost in the proportion $100 : 16 = 2,650 : x$ (amount lost).

$$x = \frac{2,650 \times 16}{100} = £424 \text{ lost}$$

Therefore, the amount he had left was £2,650 - £424 = £2,226. Ans.

(b) Of every £100 invested he regained £100 - £16 = £84. Therefore, $100 : 84 = 2,650 : x$ (amount regained).

$$x = \text{amount regained} = \frac{2,650 \times 84}{100} = £2,226. \text{ Ans.}$$

EXAMPLE 6.—If in 1908 the population of a village is 4,130, which is 18% more than the population in 1903, what was the population in 1903?

SOLUTION.—Of every 100 in 1903 there were 18 more, or 118 altogether, in 1908. Therefore, $118 : 100 = 4,130 : x$ (population in 1903).

$$x = \text{population in 1903} = \frac{4,130 \times 100}{118} = 3,500. \text{ Ans.}$$

EXAMPLE 7.—A speculator lost 34% of an investment and had £10,560 remaining. What was the original investment?

SOLUTION.—In this case, of every £100 originally invested he lost £34, and had remaining £100 - £34 = £66. The proportion therefore is:

$$£66 : £100 = £10,560 : x \text{ (the amount of the original investment)}$$

and x , the original investment = $\frac{10,560 \times 100}{66} = £16,000. \text{ Ans.}$

60. In finding how much a given rate per cent. is of a given number or quantity, instead of multiplying the given quantity by the percentage and dividing by 100 (see Art. 59, example 2), the result can frequently be more easily arrived at by regarding the percentage as a vulgar fraction. Thus, 25% of anything is $\frac{1}{4}$ of it. Divide the quantity by 4, therefore, to obtain the required result.

61. The following table gives the number of rates per cent. in their equivalent fractional ratios :

Rate Per Cent.	Equivalent Fraction	Rate Per Cent.	Equivalent Fraction
2	$\frac{1}{50}$	$\frac{1}{4}$	$\frac{1}{400}$
5	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{200}$
10	$\frac{1}{10}$	$6\frac{1}{4}$	$\frac{1}{16}$
25	$\frac{1}{4}$	$8\frac{1}{3}$	$\frac{1}{12}$
50	$\frac{1}{2}$	$12\frac{1}{2}$	$\frac{1}{8}$
75	$\frac{3}{4}$	$16\frac{2}{3}$	$\frac{1}{6}$
100	1	$33\frac{1}{3}$	$\frac{1}{3}$
125	$1\frac{1}{4}$	$37\frac{1}{2}$	$\frac{3}{8}$
150	$1\frac{1}{2}$	$62\frac{1}{2}$	$\frac{5}{8}$
175	$1\frac{3}{4}$	$87\frac{1}{2}$	$\frac{7}{8}$

62. The seven examples given in Art. 59 should be carefully studied, as they cover practically all the variations in percentage problems, stated in their simplest forms. In practice, problems may arise much more intricate than those given in the examples, and it is sometimes difficult to determine the construction of the proportion. One should therefore first ask oneself what is required to be found, then seek for the ratio whose terms are given, which will form one couplet of the proportion. The remaining term given and the unknown quantity will form the other couplet, but care must be taken that the couplets are set out in their proper ratios ; that is, that neither of them is wrongly inverted. Common sense and practice are the best teachers.

EXAMPLES FOR PRACTICE

63. 1. What is :

(a) 36% of 1,762 ?

(b) 19% of £89 ?

(c) 47% of 2,400 bushels ?

(d) 113% of £1,640 ?

Ans. $\left\{ \begin{array}{l} (a) \text{ 634.32} \\ (b) \text{ £16 18s. 2.4d.} \\ (c) \text{ 1,128 bush.} \\ (d) \text{ £1,853 4s.} \end{array} \right.$

2. What percentage of :

(a) 360 is 90 ?

(b) £900 is £360 ?

(c) 125 is 25 ?

(d) 150 is 750 ?

(e) 280 horses is 112 horses ?

(f) 400 is 200 ?

(g) 47 is 94 ?

(h) 500 days is 250 days ?

(i) 42 is 6% of what number ?

(j) 126 is $31\frac{1}{3}\%$ of what number ?

(k) 198 is 36% of what number ?

Ans. $\left\{ \begin{array}{l} (a) \ 25\% \\ (b) \ 40\% \\ (c) \ 20\% \\ (d) \ 500\% \\ (e) \ 40\% \\ (f) \ 50\% \\ (g) \ 200\% \\ (h) \ 50\% \\ (i) \ 700 \\ (j) \ 400 \\ (k) \ 550 \end{array} \right.$

3. A merchant's income is £1,800 a year and he saves £225. (a) What percentage of his income does he save ? (b) What percentage of it does he spend ?

Ans. $\left\{ \begin{array}{l} (a) \ 12\frac{1}{2}\% \\ (b) \ 87\frac{1}{2}\% \end{array} \right.$

4. A man has 32% of his money invested in stocks, 18% in grain, and the remainder, which is £7,620, in real estate. What is the total value of his property ?

Ans. £15,240

5. If wool loses 32% of its weight in washing, how many pounds of unwashed wool are required to produce 35,360 lb. of washed wool ?

Ans. 52,000 lb.

6. If in 1910 the population of a town is 85,000, which is 36% more than the population in 1900, what was the population in 1900 ?

Ans. 62,500

7. If gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder ?

Ans. $\left\{ \begin{array}{l} \text{Saltpetre, } 1,680 \text{ lb.} \\ \text{Sulphur, } 224 \text{ lb.} \\ \text{Charcoal, } 336 \text{ lb.} \end{array} \right.$

8. A man bequeathed to a charity 32% of his estate. To another charity he gave £23,100, which was 23% less than the amount given to the first charity. (a) What was the value of the estate ? (b) What percentage of the estate was given to the second charity ?

Ans. $\left\{ \begin{array}{l} (a) \ £93,750 \\ (b) \ 24.64\% \end{array} \right.$

9. A man owning a ship worth £30,000, sells $\frac{1}{4}$ of it to A, 20% of the remainder to B, and 35% of what then remains to C. How much each do A, B, and C pay for their shares ?

Ans. $\left\{ \begin{array}{l} A, £7,500 \\ B, £4,500 \\ C, £6,300 \end{array} \right.$

APPLICATION OF FORMULÆ

ELEMENTARY PRINCIPLES

USE OF LETTERS

1. **Definition.**—A formula, as used in mathematics and in technical books, may be defined as *a rule in which a combination of symbols is used instead of words*. Any formula may be expressed in words, and when so expressed becomes a rule. A formula is much more convenient than a rule, because it shows at a glance all the operations which are to be performed, and does not require to be read several times to be understood, as is the case with most rules. In technical books, formulæ are very extensively employed, and it is hardly possible to read such a book intelligently without an understanding of formulæ.

2. **Quantity.**—In mathematics the word **quantity** is applied to any single symbol or combination of symbols and numbers which it is desired to subject to the ordinary operations of addition, multiplication, etc. Such operations may be performed correctly even without knowing what the various symbols may represent. The term *quantity* has therefore a much wider application than the arithmetical term *number*.

3. **Letters as Symbols.**—In a formula, the symbols used to express quantities are usually the letters of the alphabet. It is customary to use the initial letter of the word expressing a quantity; thus, to express a length, the letter *l* would often be used. This method has the advantage of readily suggesting the name of the quantity, wherever it occurs in the formula. It is also the practice, in technical works, to use the same letter throughout to indicate a quantity, as confusion would result if

the same letter designated different quantities in different parts of a book. Sometimes Greek letters are used in special cases. For example, the Greek letter π (π) is nearly always used to indicate the value 3.1416, the ratio between the circumference and the diameter of a circle.

4. Capital and small letters are used in formulæ to designate, for instance, quantities of the same kind, but expressed in different units. Thus, it might be necessary to distinguish between lengths in feet and in inches; L might indicate the former, or larger, unit, and l the latter, or smaller, one. Again, P might represent a pressure in pounds, or ounces, per square foot, and p a pressure in pounds, or ounces, per square inch.

5. **Signs Used.**—The signs used in formulæ are the ordinary arithmetical ones, namely, $+$, $-$, $\times$, $\div$, $\sqrt{}$, $=$, and also the symbols of aggregation, which are explained farther on.

In some mathematical works, the process of multiplication is indicated by a dot instead of the ordinary sign; thus $l.b$ means the same as $l \times b$. This sign should not be employed between figures, since it would readily be mistaken for the decimal point. It may also be easily overlooked if not carefully made.

Division is seldom indicated in a formula by the sign $\div$, but the dividend is usually written over the divisor in the form of a fraction. Thus, 25 divided by 17 would be written $\frac{25}{17}$, not $25 \div 17$. Likewise, to indicate that a is to be divided by b , the expression is written $\frac{a}{b}$.

An important principle to be kept in mind in dealing with formulæ is that when no sign immediately precedes a quantity, a + sign is always understood. Thus, 9 means $+ 9$; and a means $+ a$.

6. **Omission of Multiplication Sign.**—It is customary to omit the sign of multiplication between quantities represented by letters in a formula, or between a number and a letter; hence, when two or more letters, or a number and one or more letters, stand together in a formula, *multiplication is always understood*.

Thus, alh , used in a formula, indicates $a \times l \times h$. When replacing letters by numerical values, the signs of multiplication should always be inserted, because, if the sign between two numbers were omitted, it would be impossible to distinguish the separate numbers when written closely together. For instance, if alh are replaced by 3, 4, and 5, respectively, 345 is a very different number from $3 \times 4 \times 5$.

It is also the practice to omit the multiplication sign from expressions like $a \times \sqrt{b+c}$, $3 \times (b+c)$, and $(b+c) \times a$, and to write them $a \sqrt{b+c}$, $3(b+c)$, and $(b+c)a$.

7. Primes and Subs.—A formula sometimes requires the use of the same initial letter to indicate two or more different quantities of the same nature. In such cases, to distinguish between two or more similar letters, recourse is had to what are termed *primes* and *subscripts*, or *subs*. Primes are written ', ", ''', etc., and subs are written ₁, ₂, ₃, ₄, etc. Thus a' , B'' , c''' , etc., are read *small a prime*; *large B second*; *small c fourth*, etc. Similarly, x_1 , k_2 , A_3 , P_a , etc., are read *small x sub-one*; *small k sub-two*; *large A sub-three*; *large P sub-a*, etc. Small figures or letters should not be used for primes, as they are likely to be mistaken for *exponents*, which are explained in Art. 12. The following formula, given in works on mechanics, for finding the centre of gravity of three bodies or weights, is an example of the use of subscripts:

$$x = \frac{W_1x_1 + W_2x_2 + W_3x_3}{W_1 + W_2 + W_3}$$

Here, W_1 , W_2 , and W_3 are the weights of the three bodies, and x , x_1 , x_2 , and x_3 , the distances from a given line to the centre of gravity of the three weights and to that of each weight, respectively.

8. Symbols of Aggregation.—The symbols of aggregation are four in number, namely, the *vinculum*, $\overline{\quad}$; the *parentheses*, $(\quad)$; the *square brackets*, $[\quad]$; and the *braces*, $\{\quad\}$. The last three are known generally as *brackets*, except when necessary to distinguish between them. This term, where used hereafter, will, unless otherwise stated, refer to the parentheses, which are most commonly used,

Brackets are used when it is desired to indicate that all the quantities within them are to be subjected to the operation indicated by the sign immediately outside the brackets. Thus, to indicate that the sum of 5 and 8 is to be multiplied by 7, any of the four symbols of aggregation may be used, but ordinarily the operation would be expressed by using the parentheses as $5 + 8 \times 7$. The vinculum is rarely used, except in connection with the radical sign, as $\sqrt{5 + 8}$.

If more than one pair of symbols of aggregation are necessary, the square brackets and parentheses are used, the former outside, as in $[20 - 5] \div 3 \times 9$. If more than two symbols are needed, the braces are used outside the others, as in $\{[(20 - 5) \div 3] \times 9 - 21\} \div 8$.

9. Use and Omission of Brackets.—As stated in *Arithmetic*, when several quantities are connected by the signs of addition, subtraction, multiplication, and division, the operations indicated by the signs of multiplication and division must be performed before those indicated by the other signs. Thus, $2 + 3 \times 4 = 14$; and $5 - 9 \div 3 = 2$. The use of brackets, however, modifies this principle. To indicate that $2 + 3$ is to be multiplied by 4, the brackets should be used; thus $(2 + 3) \times 4 = 20$. Similarly, $35 - 3 \times 7 = 14$, but $(35 - 3) \times 7 = 224$. Hence, when brackets are used, the operation indicated within them is to be performed first.

In the expression $(2 + 3) \times 4 = 20$, it will be evident that the same result will follow if 2 and 3 are each multiplied by 4 and the results added. Thus, $2 \times 4 + 3 \times 4 = 20$. In the same way, if letters are used, $(a + b) \times c = ac + bc$. Hence, to multiply a quantity in brackets by another quantity is the same as to multiply each term of the quantity in the brackets by the other quantity, and to omit the brackets. Here the two quantities enclosed by brackets are considered as *one* quantity.

The same principles hold true of division. Thus, $15 - 9 \div 3 = 12$; whereas $(15 - 9) \div 3 = 2$. Also, $(15 - 9) \div 3 = 15 \div 3 - 9 \div 3$. Again, $(a + b) \div c = a \div c + b \div c = \frac{a}{c} + \frac{b}{c}$.

10. If several numbers or symbols are connected by plus signs, two or more of these quantities may be enclosed in brackets, leaving the + sign of the first quantity outside the brackets, without affecting the operations. Thus, $35 + 10 + 5 = 50$, and $35 + (10 + 5) = 50$. If, however, the quantity in brackets is placed after a - sign, the case is different. For, using the figures in the example just given, if 10 is subtracted from 35 and 5 from the remainder, the result is 20; that is, $35 - 10 - 5 = 20$. If the sum of 10 and 5 is subtracted from 35, the result is also 20; that is, $35 - (10 + 5) = 20$. Hence, $35 - (10 + 5) = 35 - 10 - 5$. Again, if 5 is subtracted from 10 and the remainder from 35, the result is 30; that is, $35 - (10 - 5) = 30$. Also, if 10 is subtracted from 35 and 5 added to the remainder, the sum is 30; that is, $35 - 10 + 5 = 30$. Hence, $35 - (10 - 5) = 35 - 10 + 5$. Therefore, *if quantities are to be enclosed in brackets, and the first quantity is preceded by a - sign, all the signs within the brackets must be changed from - to + or from + to -*.

It is frequently very convenient to enclose several quantities in brackets and to place outside of them a factor common to each quantity, as thereby operations are often much shortened. For example, in the expression $24 \times 22 + 24 \times 35$ it is required to multiply 22 and 35 each by 24 and to add the products. Only one addition and one multiplication are necessary, if 22 and 35 are first added, and the sum multiplied by 24; thus, $24(22 + 35)$. Likewise, $a b d + a b c + a b f$ may be written $a b(d + c + f)$. Again, $a - b c - c d$ becomes $a - c(b + d)$, if brackets are used.

11. If brackets contain several numbers or symbols connected by + or - signs and the brackets are preceded by a plus sign, then the brackets may be removed without affecting the quantity within the brackets in any way. Thus, $4 + (12 + 6)$ and $4 + (12 - 6)$ are the same as $4 + 12 + 6$ and $4 + 12 - 6$.

If, however, an expression contains brackets preceded by a minus sign, such brackets cannot be removed without affecting the value of the expression. For instance, in the example $50 - (20 + 10)$ the brackets indicate that 10 is to be added

to 20 and the sum subtracted from 50, the result being $50 - 30 = 20$. If the brackets are removed before performing the operation, indicated by the brackets, the result is $50 - 20 + 10 = 40$. If the sign inside the brackets had been changed before removing the brackets, the result would have been the same as that found in the first instance. Thus, $50 - (20 + 10) = 50 - 20 - 10 = 20$. The same rule applies if the sign inside the brackets is a minus sign. Performing the operations indicated in the example $50 - (20 - 10)$ the result is 40. Omitting the brackets, thus, $50 - 20 - 10$, the remainder is 20. But if the sign inside the brackets is changed, before removing them, as $50 - 20 + 10$, the remainder is 40. Therefore, *if brackets which are preceded by a minus sign are omitted, all the signs within the brackets must be changed from + to - or from - to +*. When brackets are omitted, the sign immediately before the brackets becomes the sign of the first quantity that was inside them.

The same principles apply when letters are used instead of figures. Thus $a - b(c + d)$ becomes $a - bc - bd$ when the brackets are omitted.

The principles relating to brackets have been fully explained, to indicate not only their use, but also to show the importance of setting down carefully the symbols when given in a formula, in order to avoid incorrect results.

12. Powers and Roots.—An **exponent** or **index** is a small figure placed at the right of a quantity to show that the quantity is to be multiplied by itself a number of times indicated by the exponent; or, in other words, that the quantity is to be raised to some **power**. Thus, 8^2 means that 8 is to be multiplied by itself; that is, $8^2 = 8 \times 8$. Also, $a^3 = a \times a \times a$.

The **radical sign**, $\sqrt{}$, indicates that some root of the quantity, before which it is placed, is to be taken. A small figure, or index, is placed within the sign to indicate which root is to be taken, except for square root, when the index is always omitted. Thus, $\sqrt{25}$ indicates the square root of 25; $\sqrt[3]{ab}$, the cube root of ab ; etc.

Another method of indicating roots is by the use of **fractional exponents**. For example, the square root of 37 may be indicated as $37^{\frac{1}{2}}$; the cube root of a as $a^{\frac{1}{3}}$; etc. These are read,

thirty-seven to the one-half power; a to the one-third power; etc. Sometimes the quantity is a power, and a root of it is to be taken. Thus, $24^{\frac{2}{3}}$, read *twenty-four to the two-thirds power*, means that 24 is to be squared and the cube root of the result taken; $(a-b)^{\frac{3}{5}}$ means to extract *the fifth root of $(a-b)$ cubed*. The numerator of the exponent always indicates the power, and the denominator is the index of the root. An expression like k_n^2 or c_2^3 is read *k sub-three squared; c sub-two cubed*. It will be noted that the subscript and exponent have no line between and are separated more than the figures in a fractional exponent.

13. Complex Fractions.—A formula sometimes contains a **complex fraction**, which is an expression similar to $\frac{160}{\frac{660}{25}}$, the heavy

line indicating that 160 is to be divided by $\frac{660}{25}$. If both lines were light, it would be impossible to know whether the expression meant 160 divided by $\frac{660}{25}$ or $\frac{160}{660}$ divided by 25. In an

expression like $\frac{160}{7 + \frac{660}{25}}$, the heavy line is not necessary, as it is

impossible to mistake the intended operation. But, since $7 + \frac{660}{25} = \frac{175 + 660}{25}$, if the latter expression is substituted, the heavy line becomes necessary, to make the resulting expression

clear. Thus, $\frac{160}{7 + \frac{660}{25}} = \frac{160}{\frac{175 + 660}{25}} = \frac{160}{\frac{835}{25}}$.

14. It was shown in *Arithmetic* that to divide a quantity by a fraction, the fraction is inverted, and the quantity multiplied by the inverted fraction. Thus, $2 \div \frac{1}{4} = 2 \times \frac{4}{1} = 8$;

$\frac{160}{\frac{835}{25}} = 160 \times \frac{25}{835} = \frac{4,000}{835}$; also, $\frac{ab}{\frac{c^2}{d}} = ab \div \frac{c^2}{d} = ab \times \frac{d}{c^2} = \frac{abd}{c^2}$.

Therefore, it follows that a complex fraction may be reduced to a simple fraction by multiplying the part above the heavy line by the denominator below the light line, and placing the product over the numerator of the fraction that was below the heavy line, which thus becomes the new denominator. If the quantity below the heavy line is a mixed number, it must be reduced to an improper fraction before the complex fraction

is simplified. Thus, $\frac{160}{7 + \frac{660}{25}}$ must be reduced to the form $\frac{160}{\frac{835}{25}}$

before it is reduced to $\frac{160 \times 25}{835}$.

15. Reciprocals.—The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 2 is $1 \div 2 = \frac{1}{2} = .5$; of a is $1 \div a = \frac{1}{a}$. Likewise, the reciprocal of $\frac{1}{2}$ is $1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 2$; of $\frac{1}{a} = 1 \div \frac{1}{a} = 1 \times \frac{a}{1} = a$. Hence, to divide any quantity by another is the same as multiplying the first quantity by the reciprocal of the second.

The principle of reciprocals is often very convenient in calculations, as by it the process of division may be replaced by the easier and shorter operation of multiplication. A formula can usually be simplified, if it is a fraction with a numerical quantity as the denominator, by considering the number as a reciprocal and reducing it to a whole number or a decimal. Thus, in the fraction $\frac{n A}{360}$, the denominator 360 may be considered as

a reciprocal, and the fraction may be written, $\frac{1}{360} n A$. The quantity $\frac{1}{360}$ reduced to a decimal fraction is .0028, and the whole fraction will therefore be .0028 $n A$.

APPLICATION OF PRINCIPLES

16. **Substitution.**—To apply a formula to any special case, all that is necessary is to substitute the known quantities for the letters which represent them, and to perform the indicated operations according to the principles explained in the preceding articles. Three points it is necessary to keep continually in mind are : first, to be sure that the quantities are expressed in the same units as the formula requires ; second, to substitute the proper quantity for the letter representing it ; and third, to perform the *exact* operations indicated. If a formula is at all complicated, a careful study of it will often show the easiest way to perform the operations. Sometimes brackets should be removed first, or a fraction reduced to a decimal, or a complex fraction reduced to a simple one, etc., before some other operation is done.

The foregoing principles and explanations will now be applied in the following examples. It should be distinctly understood that the examples are merely exercises in solving formulæ, and are *not* intended to be full explanations of how and when the formulæ should be used. It is beyond the scope of this Paper to consider any but the simpler and more common forms of formulæ. For the solution of more complex ones, a wider knowledge of algebra is required than can be included within the limits of this Section.

17. **Horsepower Formula.**—The following is a well-known rule for finding the horsepower of a steam engine : *Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000 ; the result will be the horsepower.*

This is a very simple rule, and very little will be saved by expressing it as a formula, so far as clearness is concerned ; but the formula, however, will occupy a great deal less space.

An examination of the rule will show that four quantities, namely, the mean effective pressure, the length of stroke, the area of the piston, and the number of strokes, are multiplied

together, and the result is divided by 33,000. Hence, the rule might be expressed as follows :

$$\text{Horsepower} = \frac{\text{mean effective pressure}}{(\text{in pounds per square inch})} \times \frac{\text{stroke}}{(\text{in feet})} \\ \times \frac{\text{area of piston}}{(\text{in square inches})} \times \frac{\text{number of strokes}}{(\text{per minute})} \div 33,000$$

This expression can be shortened by representing each quantity by a letter ; thus, if horsepower is represented by H , the mean effective pressure in pounds per square inch by P , the length of stroke in feet by L , the area of the piston in square inches by A , the number of strokes per minute by N , then, by substituting these letters for the quantities that they represent, the above expression reduces to the much simpler *formula* :

$$H = \frac{P \times L \times A \times N}{33,000}$$

As has been explained in Art. 6, it is usual to omit the multiplication sign between letters. Hence, the formula can be further simplified to

$$H = \frac{P L A N}{33,000}$$

EXAMPLE.—What is the horsepower of a steam engine whose piston has an area of 201 square inches and a stroke of 2 feet, the average steam pressure being 44 pounds per square inch and the revolutions 75 per minute ?

SOLUTION.—There are two strokes in each revolution, hence $N = 75 \times 2 = 150$; $P = 44$, $L = 2$, and $A = 201$. Substituting these values in the formula, and shortening the work by cancellation,

$$H = \frac{P L A N}{33,000} = \frac{44 \times 2 \times 201 \times 150}{33,000} = \frac{402}{5} = 80.4 \text{ H.P. Ans.}$$

18. **Temperature of Mixture Formula.**—The following formula for the final temperature of three liquid bodies when mixed together involves the use of subscripts, explained in Art. 7 :

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3}$$

The main thing to be remembered in such formulæ is that, when the same letters occur several times, all letters having the same primes or subs represent the same quantities, while those which differ in any respect represent different quantities. Here,

w_1 , w_2 , and w_3 represent the weights of the three different bodies; s_1 , s_2 , and s_3 , certain values known as their specific heats; and t_1 , t_2 , and t_3 , their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that the letters having the *same* subs refer to the *same* body. Thus, w_1 , s_1 , and t_1 all refer to one of the three bodies; w_2 , s_2 , t_2 , to another; etc.

In substituting the numerical values, the signs of multiplication should, of course, be written in their proper places, and all the multiplications are performed before the additions are made.

EXAMPLE.—If three liquid bodies are mixed and the values of w_1 , s_1 , and t_1 are, respectively, 2 pounds, $\cdot 0951$, and 80° ; of w_2 , s_2 , and t_2 , 7·8 pounds, 1, and 80° ; and of w_3 , s_3 , and t_3 , $3\frac{1}{4}$ pounds, $\cdot 1138$, and 780° , what is the final temperature of the mixture?

SOLUTION.—Substituting the values for their respective letters in the formula, the final temperature t is:

$$\begin{aligned} t &= \frac{2 \times \cdot 0951 \times 80 + 7\cdot 8 \times 1 \times 80 + 3\frac{1}{4} \times \cdot 1138 \times 780}{2 \times \cdot 0951 + 7\cdot 8 \times 1 + 3\frac{1}{4} \times \cdot 1138} \\ &= \frac{15\cdot 216 + 624 + 288\cdot 483}{\cdot 1902 + 7\cdot 8 + \cdot 36985} = \frac{927\cdot 699}{8\cdot 36005} = 110\cdot 97^\circ \end{aligned}$$

19. Area of Circular Segment Formula.—The area of any segment of a circle which is less than, or equal to, a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h)$$

in which A = area of segment;

π = $3\cdot 1416$, the ratio of the circumference to diameter of circle;

r = radius of circle;

E = angle obtained by drawing lines from the centre of circle to the extremities of arc of segment;

c = chord of segment;

h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is $83\cdot 46^\circ$, radius is 7·5 inches, and height of segment is 1·91 inches?

SOLUTION.—Here, $r^2 = 7\cdot 5 \times 7\cdot 5 = 56\cdot 25$; $E = 83\cdot 46^\circ$; $\frac{c}{2} = \frac{10}{2}$; and $h = 1\cdot 91$.

Substituting these values in the formula,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(\gamma - h) = \frac{3 \cdot 1416 \times 56 \cdot 25 \times 83 \cdot 46}{360} - \frac{10}{2}(7 \cdot 5 - 1 \cdot 91) \\ = \frac{14,748 \cdot 634}{360} - \frac{10}{2} \times 5 \cdot 59 = 40 \cdot 968 - 27 \cdot 95 = 13 \cdot 018 \text{ sq. in., nearly.} \\ \text{Ans.}$$

NOTE.—A trial should be made to see the effect of disregarding the brackets when preceded by a — sign and the incorrect answer which would result.

20. Area of Triangle Formula.—The area of any triangle may be found by means of the following formula, in which A is the area, and a , b , and c represent the lengths of the sides :

$$A = \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2}$$

This formula is general, that is, it applies to any triangle. Furthermore, the same result will be obtained no matter which side of a triangle is termed a , b , or c . As this formula is somewhat more complex than the previous examples, it requires careful study to follow the various steps.

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long ?

SOLUTION.—Let a represent the side which is 21 ft. long ; b the side 50 ft. long ; and c the side 46 ft. long. Then, $a^2 = 21^2 = 441$; $b^2 = 50^2 = 2,500$; $c^2 = 46^2 = 2,116$; $\frac{b}{2} = 25$; and $2b = 100$.

Substituting these values in the formula,

$$A = \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2} = \frac{50}{2} \sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50} \right)^2} \\ = \frac{50}{2} \sqrt{441 - \left(\frac{441 + 2,500 - 2,116}{100} \right)^2}$$

Performing the addition, subtraction, and division indicated inside the brackets, the formula becomes

$$A = 25 \sqrt{441 - \left(\frac{825}{100} \right)^2} = 25 \times 19 \cdot 312 = 482 \cdot 8 \text{ sq. ft., nearly.} \\ \text{Ans.}$$

21. Column Formula.—The Rankine-Gordon formula, for determining the least load in pounds which will cause a long column to fail, is

$$P = \frac{SA}{1 + q \frac{l^2}{G^2}}$$

in which P = load (pressure) in pounds ;

S = ultimate strength, in pounds per square inch,
of the material composing the column ;

A = area of cross-section of column in square inches ;

q = a factor or multiplier whose value depends upon
the shape of the ends of the column and on
the material composing the column ;

l^2 = square of length of column in inches ;

G^2 = square of least radius of gyration of cross-
section of column.

The values of S , q , and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load which will break a hollow steel flat-ended column having an outside diameter of 14 inches, an inside diameter of 11 inches, and a length of 20 feet ?

FIRST SOLUTION.—To solve this formula, it is necessary to solve two other formulæ which are involved. First, A , the area of cross-section, is found from the formula, $A = .7854 (d_1^2 - d_2^2)$, d_1 and d_2 being the outside and inside diameters, respectively, of the hollow column. Substituting the values given, $A = .7854 (14^2 - 11^2) = 58.905$ sq. in. Second, G^2 for the given column is found from the formula $G^2 = \frac{d_1^2 + d_2^2}{16}$.

Substituting the values, $G^2 = \frac{14^2 + 11^2}{16} = \frac{196 + 121}{16} = \frac{317}{16} = 19.8125$.

The solution of the main formula may now be proceeded with.

For steel, let $S = 70,000$, and $q = .00004$, for flat-ended columns. Since the length of the column is given in feet, and the formula requires it in inches, $l = 20 \times 12 = 240$ in., and $l^2 = 240^2 = 57,600$. Substituting the values, and remembering that to multiply a fraction by a number is the same as to multiply the numerator by the number,

$$\begin{aligned} P &= \frac{SA}{1 + q \frac{l^2}{G^2}} = \frac{70,000 \times 58.905}{1 + .00004 \times \frac{240^2}{19.8125}} = \frac{70,000 \times 58.905}{1 + \frac{.00004 \times 57,600}{19.8125}} \\ &= \frac{4,123,350}{1 + \frac{2.304}{19.8125}} = \frac{4,123,350}{1 + .1163} = \frac{4,123,350}{1.1163} = 3,693,765 \text{ lb. Ans.} \end{aligned}$$

SECOND SOLUTION.—The following solution involves the principles of complex fractions, explained in Art. 14. The denominator $1 + q \frac{l^2}{G^2}$,

reduced to an improper fraction, becomes $\frac{G^2 + q l^2}{G^2}$.

$$\begin{aligned}\text{Then, } \frac{S A}{1 + q \frac{l^3}{G^2}} &= \frac{S A}{\frac{G^2 + q l^2}{G^2}} = \frac{S A G^2}{G^2 + q l^2} = \frac{70,000 \times 58.905 \times 19.8125}{19.8125 + .00004 \times 57,600} \\ &= \frac{70,000 \times 58.905 \times 19.8125}{19.8125 + 2.304} = 3,693,797 \text{ lb.} \quad \text{Ans.}\end{aligned}$$

The slight difference in the two answers is due to decimals.

EXAMPLES FOR PRACTICE

22. Solve the following formulæ, using the values $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

$$1. \quad x = \frac{d + c^2}{d^2 - 40}. \quad \text{Ans. } x = \frac{7}{30}$$

$$2. \quad y_1 = \frac{\frac{2}{3}(A + e)}{c e}. \quad \text{Ans. } y_1 = 1\frac{1}{3}$$

$$3. \quad a = \sqrt{\frac{d^2}{2c}} + \sqrt{A B^2}. \quad \text{Ans. } a = 29$$

$$4. \quad P = \frac{A e}{\sqrt{16 B c}} + \frac{5}{16}. \quad \text{Ans. } P = 2$$

$$5. \quad x_2 = (c + 2e) \left(\sqrt[3]{B} - \frac{1}{c} \right) + \frac{e^2 - c^2}{e^2 + c^2}. \quad \text{Ans. } x_2 = 12\frac{5}{13}$$

$$6. \quad R_1 = \sqrt{\frac{B c d}{A \left(2 + \frac{d^3}{e^2} \right)}}. \quad \text{Ans. } R_1 = .396$$

7. When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following?

$$E = \sqrt[3]{\frac{B C D}{A \left(2 + \frac{D^2}{C^2} \right)}}. \quad \text{Ans. } E = 1.823$$

8. Solve for K , using the values for A , B , C , and D given in example 7.

$$K = \frac{A - \frac{3}{4}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}. \quad \text{Ans. } K = 3.008$$

9. If $c_1 = .97$, $g = 32.2$, and $h = 125$, what is the value of v in the equation $v = c_1 \sqrt{2 g h}$?

$$\text{Ans. } v = 87.03$$

EQUATIONS

TRANSFORMATION AND TRANSPOSITION

23. An **equation** is an expression of equality between two quantities. Thus, $2 \times 4 = 8$ is an equation; $a + b = c$ is also an equation. The former is a *numerical* equation, or one involving only numbers; the latter is a *literal* equation, or one involving letters. A literal equation becomes a numerical one when numbers are substituted for the letters. Each of the separate quantities, connected by $+$ and $-$ signs, in an equation is called a **term**.

It will be seen that a formula is an equation, in which the unknown quantity usually stands by itself at the left of the equality sign. Sometimes, however, it is desired to find one of the quantities in a formula other than that at the left of the equality sign. To effect this, some special operations are necessary, which are called *transformation* and *transposition*, and of which some of the more simple steps will now be explained.

TRANSFORMATION

24. If the same quantity is added to both sides of an equation, that is, to the quantity on the left, and to that on the right, of the equality sign, the equality will still remain. For example, in the equation $2 + 4 = 6$, if 2 is added to each side, thus, $2 + 4 + 2 = 6 + 2$, the equality still exists. It may be shown in the same way, that if the same quantity is subtracted from both sides of an equation, or if each side is multiplied by the same quantity, or divided by the same quantity, the equation still holds true. Thus, $2 + 4 - 3 = 6 - 3$; $(2 + 4) \times 3 = 6 \times 3$; and $\frac{2 + 4}{3} = \frac{6}{3}$. Again, both sides of an equation may be squared, or the square root of both sides extracted,

without affecting the equality. Thus, $(2 + 4)^2 = 6^2$; also, $\sqrt{4 \times 9} = \sqrt{36}$.

Consider the formula or equation, $H = \frac{P L A N}{33,000}$, and assume that it is desired to rearrange the formula so that N will be alone on one side of the equality sign. From what has been said, it is evident that if both sides are multiplied by 33,000, the equality will not be altered; thus,

$$H \times 33,000 = \frac{P L A N}{33,000} \times 33,000$$

Since multiplying a fraction by a quantity is the same as multiplying the numerator by the quantity, $\frac{P L A N}{33,000} \times 33,000$ is the same as $\frac{33,000 P L A N}{33,000}$. (It is customary to place

figures first, when they occur in combination with letters.)

Since as many numbers divided by itself is equal to 1, the 33,000 in numerator and denominator cancel, and the formula then becomes $33,000 H = P L A N$. This operation is called **clearing of fractions**.

The next step is to get N by itself on one side of the equality sign; it does not matter which side, although the left is the usual one. It should be clear that if the equation $33,000 H = P L A N$ is divided by $P L A$, the result will be $\frac{33,000 H}{P L A} = \frac{P L A N}{P L A}$. But the $P L A$ in the denominator cancels $P L A$ in the numerator, and the result is $\frac{33,000 H}{P L A} = N$, which is what

was sought. This entire operation constitutes a **transformation**, because the original formula has been *transformed* so as to make what is practically another formula.

Principles.—The preceding explanations should make clear the following:

I. *Any quantity may be removed from an equation by dividing both sides of the equation by it.* The quantity will cancel out on one side, and will occur as a denominator of the other side. The

quantity must be divided out of each term on both sides when there are more than one, or must be expressed as a denominator of any term which does not contain the quantity.

II. *To clear an equation of a fraction, multiply all the terms in the equation by the denominator.* The numerator of the fraction stands unchanged. If there is more than one fraction, however, it will first be necessary to find the least common denominator, which will then be used in clearing the equation of fractions.

EXAMPLE 1.—Transform the formula given in Art. 17, so as to obtain a value for A . Then substitute the following values and solve for A : $H = 67$; $P = 44$; $N = 150$; and $L = 2$.

SOLUTION.—By a transformation which should be clear from the preceding explanation, $A = \frac{33,000 H}{PLN}$. Substituting,

$$A = \frac{33,000 \times 67}{44 \times 2 \times 150} = 167.5 \text{ sq. in. Ans.}$$

EXAMPLE 2.—Solve the formula $W = \frac{2 P T}{R - r}$ for T .

SOLUTION.—In this example the quantity $R - r$ must be first removed from the denominator by the operation for clearing of fractions. If both sides of the equation are multiplied by $R - r$, it becomes

$$W(R - r) = \frac{2 P T (R - r)}{R - r}$$

When a quantity consisting of more than one term is used as a multiplier it should be placed in brackets as shown. On the right side of the equation, evidently the $R - r$ in numerator and denominator cancel, and the equation becomes

$$W(R - r) = 2 P T$$

Now, $2 P$ can be removed from the right side by dividing both sides of the equation by it; thus,

$$\frac{W(R - r)}{2 P} = \frac{2 P T}{2 P}$$

But, as the $2 P$ in numerator and denominator of the fraction on the right cancel, the equation becomes

$$\frac{W(R - r)}{2 P} = T, \text{ which is what is wanted. Ans.}$$

EXAMPLES FOR PRACTICE

25. The following examples involve the principles explained in Art. 24:

1. Solve the formula $F = .00034 WRN^2$ for R .

$$\text{Ans. } R = \frac{F}{.00034 WN^2}$$

2. Solve the formula in example 1 for N . $\text{Ans. } N = \sqrt{\frac{F}{.00034 WR}}$

3. Solve $l = \frac{rn}{57.3}$ for n . $\text{Ans. } n = \frac{57.3 l}{r}$

4. Solve $Q = G s(t_2 - t_1)$ for s . $\text{Ans. } s = \frac{Q}{G(t_2 - t_1)}$

5. Solve $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ for T_2 . $\text{Ans. } T_2 = \frac{T_1 V_2}{V_1}$

NOTE.—Multiply both sides by V_2 , cancelling V_2 from numerator and denominator on left hand; multiply the result by T_2 , and cancel; and finally divide both sides by V_1 .

TRANSPOSITION

26. In cases where two or more quantities on one side of an equation are connected by $+$ or $-$ signs, or both, and it is desired to arrange the equation so that one of these quantities will be alone on one side of the equality sign, a somewhat different operation to that explained in Art. 24 is necessary. It is called **transposition**, because it is necessary to *transpose* one or more of the quantities from one side of the equality sign to the other. To show this, consider the following formula for obtaining the length of the *hypotenuse* of a right triangle when the two *sides* are given:

$$h = \sqrt{a^2 + b^2}$$

in which

h = hypotenuse;

a = one side;

b = other side.

Suppose h and b are known, and it is desired to find a . It has been explained that the radical sign without an index indicates square root. Since the square root of a given number when multiplied by itself equals the given number, it follows that by multiplying $\sqrt{a^2 + b^2}$ by itself, the result is $a^2 + b^2$ *without the*

radical sign. But, as has been shown, h must also be multiplied by itself to maintain the equality. Hence, $h^2 = a^2 + b^2$. Now, if the b^2 on the right of the equality sign can be removed, the a^2 will stand by itself. Since the same quantity may be subtracted from each side of an equation without altering the equality, suppose b^2 to be subtracted, thus :

$$h^2 - b^2 = a^2 + b^2 - b^2$$

If any quantity is added to and subtracted from a second quantity, the operation leaves the second quantity unchanged. For instance, if 5 is added to 7, and 5 subtracted from the sum, the final result is still 7. Then the b^2 and $-b^2$ cancel, leaving

$$h^2 - b^2 = a^2$$

By inspecting this result, it will be seen that the b^2 has been *transposed* from the right to the left of the equation, with its sign changed (for the sign immediately preceding a quantity belongs to the quantity). Hence :

Principle.—*Any quantity, either a single term or several terms in brackets, may be transposed from one side of an equation to the other if its sign (or the sign before the brackets) is changed from + to -, or from - to +.*

The square root of the square of a number is the number itself. Thus, $\sqrt{6^2} = \sqrt{36} = 6$. Hence, by extracting the square root of both sides of the equation, $h^2 - b^2 = a^2$, the result is,

$$\sqrt{h^2 - b^2} = a$$

EXAMPLE 1.—Solve $Q = G s(t_2 - t_1)$ for t_1 .

SOLUTION.—Removing the brackets first, $Q = G s t_2 - G s t_1$. Next, transpose $G s t_1$ to the left hand, thus, $Q + G s t_1 = G s t_2$, remembering that the sign before $G s t_1$ belongs to it, and must be changed when $G s t_1$ is transposed. When no sign is given before a quantity, a + sign is always understood. Transpose Q to the right hand, changing the + sign (understood) to -, and making the equation read $G s t_1 = G s t_2 - Q$. Now, dividing both sides by $G s$, and cancelling $G s$ from the resulting left-hand fraction, the result is $t_1 = \frac{G s t_2 - Q}{G s}$, which is in the required form. Ans.

EXAMPLE 2.—Find the value of P in the formula $d = D \sqrt{\frac{P}{n S}}$.

SOLUTION.—The desired quantity P is under the radical sign; hence, the sign must be removed. This is done by squaring both sides of the

equation; thus, $d^2 = D^2 \left(\frac{P}{nS} \right)$. (The brackets have been used to indicate what the radical sign covered.) The quantity D^2 before the brackets is multiplied by $\frac{P}{nS}$; hence, $d^2 = \frac{D^2 P}{nS}$. Clearing of fractions, by the principles of Art. 24, $d^2 nS = D^2 P$; and dividing both sides by D^2 , the desired result is found, $\frac{d^2 nS}{D^2} = P$.

NOTE.—This example shows that when an expression contains a radical and a quantity outside (as D) multiplying the radical, the quantity outside must be squared (or in other cases, raised to the proper power) in getting rid of the radical sign.

27. The explanation in Art. 26 serves to show another important point, which is, that the operations indicated on either side of the equality sign must be performed *before* the square (or other) root is extracted, and that the radical sign must extend over all the quantities on either side when there are more than one. Thus, in extracting the square root of $h^2 - b^2$, the subtraction must be made *before* the root is extracted. Let $h^2 = 25$, and $b^2 = 16$; then $\sqrt{h^2 - b^2} = \sqrt{25 - 16} = \sqrt{9} = 3$; but $\sqrt{h^2} - \sqrt{b^2} = \sqrt{25} - \sqrt{16} = 5 - 4 = 1$. Hence, it is not correct to extract separately the square (or other) root of each quantity under one radical sign. The same caution also applies to raising both sides of an equation to a power. Thus, $(h + b)$ squared is $(h + b)^2$, not $h^2 + b^2$.

COMBINED OPERATIONS

28. It is sometimes necessary to combine the operations of transformation, transposition, and substitution in the solution of a formula. The steps may best be shown by examples.

EXAMPLE 1.—Solve the formula $K = \frac{2LS}{S-v}$ for S .

SOLUTION.—The first operation is to clear of fractions. By Art. 24, the equation becomes $K(S - v) = 2LS$. It will be noticed that the value S occurs on both sides of the equation, which must be transformed so as to bring the S on one side only. Removing the brackets,

$$KS - Kv = 2LS$$

Next, transposing $2LS$ to the left side, *changing its sign*, which is + (understood), the equation becomes, since nothing will remain on the right side,

$$KS - 2LS - Kv = 0$$

But $-Kv$ should be transposed to the right side, with its sign changed; thus,

$$KS - 2LS = +Kv$$

Placing the left side in brackets with S outside, and omitting the sign $+$ before Kv , as it is customary to do when the first term on either side would be preceded by the sign $+$,

$$S(K - 2L) = Kv$$

Dividing both sides by $K - 2L$ so that S will stand alone, the equation becomes

$$S = \frac{Kv}{K - 2L}. \quad \text{Ans.}$$

NOTE.—This example shows that when a term, like S , is contained in more than one quantity in an equation, all the quantities containing the term may be enclosed in brackets with the term outside, and the quantities in the brackets may be treated as if they were but a single quantity. Due attention must be paid to the signs.

EXAMPLE 2.—Substitute the value of e obtained from the formula

$$e^2 = \left(\frac{c}{2}\right)^2 + h^2 \text{ in the formula } l = \frac{8e - c}{3}.$$

SOLUTION.—In the formula $e^2 = \left(\frac{c}{2}\right)^2 + h^2$, $\left(\frac{c}{2}\right)^2 = \frac{c^2}{4}$; and h^2 may be written $\frac{4h^2}{4}$. Then, $e^2 = \frac{c^2}{4} + \frac{4h^2}{4} = \frac{c^2 + 4h^2}{4} = \frac{1}{4}(c^2 + 4h^2)$; and, by Arts. 26 and 27, $e = \sqrt{\frac{1}{4}(c^2 + 4h^2)}$. This operation may be carried one step farther, although not necessary. The square root of $\frac{1}{4}$ is $\frac{1}{2}$, and $\frac{1}{2}$ may be placed outside the radical sign; hence, $e = \frac{1}{2}\sqrt{c^2 + 4h^2}$.

Now, substituting this value in the formula, $l = \frac{8e - c}{3}$, the latter becomes, using the brackets merely to denote the value of e ,

$$l = \frac{8\left(\frac{1}{2}\sqrt{c^2 + 4h^2}\right) - c}{3} = \frac{4\sqrt{c^2 + 4h^2} - c}{3}. \quad \text{Ans.}$$

NOTE.—It will be seen that the radical does not extend over the final c , but only over the expression representing e .

EXAMPLES FOR PRACTICE

29. The following examples are solved according to the principles explained in the preceding articles:

1. Obtain a formula for C from the formula $r = \frac{C^2 + 4H^2}{8H}$.

Ans. $C = \sqrt{8Hr - 4H^2}$

2. Solve $p = wh + p_1$ for h .

Ans. $h = \frac{p - p_1}{w}$

3. Obtain the value of t_2 from the formula $t_1 = \frac{9}{5}t_2 + 32$.

$$\text{Ans. } t_2 = \frac{5}{9}(t_1 - 32)$$

NOTE.—Transpose 32 to left side, multiply by 5 and divide by 9 (or multiply by $\frac{5}{9}$).

4. Find the value of d in $W = \frac{b c d^2}{l}$.

$$\text{Ans. } d = \sqrt{\frac{W l}{b c}}$$

5. Solve the formula $r = V\sqrt{\frac{2y}{g}}$ for y .

$$\text{Ans. } y = \frac{g}{2} \frac{r^2}{V^2}$$

NOTE.—Square both sides of equation to remove radical sign.

GEOMETRY AND MENSURATION

(PART 1)

PRINCIPLES OF GEOMETRY

PRELIMINARY DEFINITIONS

1. **Geometry** is that branch of mathematics which treats of the properties of lines, angles, surfaces, and volumes.

2. **Mensuration** is that part of geometry which treats of the measurement of lines, surfaces, and solids.

LINES, ANGLES, AND ARCS

3. A point indicates position only ; it has no length, breadth, or thickness.

4. A line has only one dimension, length.

5. A straight line, Fig. 1, is a line that does not change its direction, and is the shortest distance between two points. A straight line is also frequently called a **right line**. Unless otherwise specified, the word *line* is understood to mean a *straight line*.



FIG. 1

6. A curved line, Fig. 2, changes its direction at every point.



FIG. 2

7. A broken line, Fig. 3, is one made up wholly of straight lines lying in different directions.



FIG. 3

8. A **surface** has the shape of a body but is without thickness. A surface has but two dimensions, length and breadth.

9. A **flat surface**, **plane surface**, or simply a **plane**, is a surface such that a straight line between any two of its points lies wholly in the surface. If a straightedge is laid on a plane surface in any direction, every point of the straightedge will touch the surface.

10. A **figure** is any combination of points and lines. A figure that lies entirely in one plane is a **plane figure**. In referring to a figure, a point is designated by a letter placed conveniently near it; thus, in Fig. 1, the left end of the line is referred to as the *point A*. The entire line is referred to as the *line AB*, the letters *A* and *B* designating two points, usually the ends of the line. If a line is broken or curved, as many points in the line are named as are considered necessary to designate it clearly and accurately.

ANGLES AND PERPENDICULARS

11. An **angle**, Fig. 4, is the opening between two straight lines that meet in a point. The two straight lines are the **sides**, and the point where the lines meet is the **vertex** of the angle. Thus, in Fig. 4, the straight lines *OA* and *OB* form an angle at the point *O*; the lines *OA* and *OB* are the sides of this angle, and the point *O* is its vertex. The plural of vertex is **vertices**.

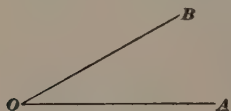


FIG. 4

12. An angle is usually referred to by naming a letter on each of its sides and a third letter at the vertex, the letter at the vertex being placed between the two others. Thus the angle in Fig. 4 is called angle *AOB* or angle *BOA*. An angle may also be designated by a letter placed between its sides near the vertex. Thus, the two angles *XCZ* and *YCZ*, Fig. 5, may be referred to as the angles *A* and *B*, respectively. An angle that stands alone, that is, one whose vertex is not the vertex of any other angle, may be designated by naming the letter at its vertex. For example, the angle in Fig. 4 may be called

the angle O . On the other hand, it would be incorrect to refer to the angle C , Fig. 5, inasmuch as there are two angles with their vertices at C , and it would be impossible to know whether ZCY or XCY was meant.

13. Two angles, as A and B , Fig. 5, having the same vertex and a common side CY , are called **adjacent angles**.

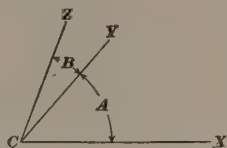


FIG. 5

14. An angle may be said to be formed, or **generated**, by a line turning about a pivot, the vertex. Thus, the angle AOB , Fig. 4, may be conceived as generated by a line turning about O from the position OA to the position OB . The size of the angle does not depend on the length of the sides, which are supposed to be of indefinite length, but it depends on the opening between those sides; or, what is the same thing, it depends on the amount of turning necessary to bring one side to the position of the other.



FIG. 6

15. If a straight line, as AB , Fig. 6, meets another straight line, as CD , so as to make with it two equal adjacent angles, each of these angles is a **right angle**, and the first line is said to be **perpendicular** to the second. The point B where the first line meets the second is called the **foot** of the perpendicular. It is evident that all right angles are equal.

The edges of the tongue of a carpenter's square are perpendicular to those of the body, or long blade, and the angles thus formed are right angles. Thus, in Fig. 7, AB is perpendicular to BC , and the angle at B is a right angle; DE is perpendicular to EF , and the angle at E is a right angle.

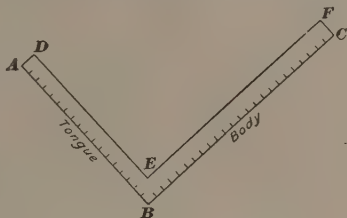


FIG. 7

MEASUREMENT OF ARCS AND ANGLES



FIG. 8

16. A circle, Fig. 8, is a plane figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **centre**.

17. The **diameter** of a circle is a straight line passing through the centre and terminated at both ends by the circumference, as AB , Fig. 9.



FIG. 9

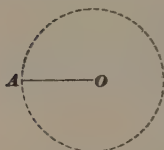


FIG. 10

18. The **radius** of a circle, OA , Fig. 10, is a straight line drawn from the centre to the circumference. It is equal in length to one-half of the diameter. The plural of the word radius is **radii**. All radii of any one circle are equal in length.

19. An **arc** of a circle is any part of its circumference, as AEB , Fig. 11.

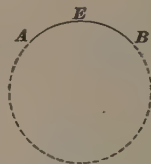


FIG. 11

20. It has been found convenient to divide the whole circumference of a circle into 360 equal parts; each of these equal parts is called a **degree of arc**. A degree of arc is divided into 60 equal parts called **minutes of arc**; and a minute of arc is

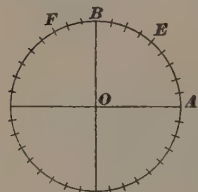


FIG. 12

divided into 60 equal parts called **seconds of arc**. Degrees, minutes, and seconds of arc are used as units for measuring circular arcs.

21. In Fig. 12, the circumference is divided into 36 equal parts; hence, each length of arc between two consecutive division lines equals $360 \div 36 = 10$ degrees. Arc AEB is one-quarter of a circumference. A quarter of a circumference is called a **quadrant**; its length is $360 \div 4 = 90$ degrees. The arc EBF

extends over seven divisions; hence, its length is $7 \times 10 = 70$ degrees.

22. Angles are measured by finding the number of degrees in the arc of the circle lying between the lines forming the sides of the angle, the centre of the circle being at the vertex of the angle. For example, in Fig. 13, the sides of the angle CAB cut off an arc of 40 degrees from the circumference whose centre is at A ; hence, angle CAB is 40 degrees in magnitude. Since the sides of a right angle whose vertex is at the centre of a circle include between them an arc of 90 degrees, each right angle is equal to 90 degrees.

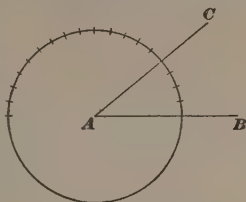


FIG. 13

In practice, angles are commonly measured by means of a *protractor* or a similar instrument, explanations of which are given in works on drawing.

23. Degrees, minutes, and seconds, both of angle and of arc, are indicated by the marks ($^{\circ}$), ($'$), ($''$); thus, $32^{\circ} 15' 10''$ is read *thirty-two degrees fifteen minutes and ten seconds*.

24. An **oblique angle** is any angle that is not a right angle.

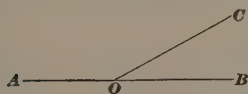


FIG. 14

An **acute angle** is an oblique angle that is less than a right angle. An **obtuse angle** is an oblique angle that is greater than a right angle. In Fig. 14, BOC and AOC are oblique

angles, BOC being an acute angle and AOC an obtuse angle.

In carpentry, any angle not a right angle is known as a **bevel angle**.

25. Two angles are said to be **complementary** when their sum is equal to one right angle, or 90° . Each of the two complementary angles is called the **complement** of the other. Thus, in Fig. 15, in which AB is perpendicular to BD , the angles M and N are complementary, their sum being equal to the right angle ABD .

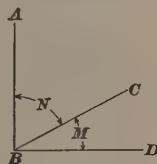


FIG. 15

EXAMPLE.—If angle CBD , Fig. 15, is 31° , what is the complementary angle ABC ?

SOLUTION.—Since ABD is a right angle, the angle ABC is
 $90^\circ - \text{angle } CBD = 90^\circ - 31^\circ = 59^\circ$. Ans.

26. Two angles are said to be **supplementary** when their sum is equal to two right angles, or 180° . Each of two supplementary angles is called the **supplement** of the other. In Fig. 16, AOD and DOB are supplementary angles, their sum being evidently equal to the sum of the two right angles POB and POA .

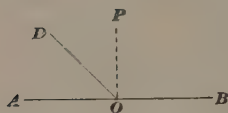


FIG. 16

EXAMPLE.—In Fig. 16, angle AOD is 45° ; what is the supplementary angle DOB ?

SOLUTION.—Since angle $AOD + \text{angle } DOB = 180^\circ$,
 angle $DOB = 180^\circ - \text{angle } AOD = 180^\circ - 45^\circ = 135^\circ$. Ans.

27. The sum of all the angles formed on the same side of a straight line about the same point in the line is equal to two right angles, or 180° .

EXAMPLE.—In Fig. 17, assuming that $R = 30^\circ$, $Q = 25^\circ$, $P = 40^\circ$, and $N = 28^\circ$, find M .



FIG. 17

SOLUTION.—Since $R + Q + P + N + M = 180^\circ$,
 $M = 180^\circ - (R + Q + P + N) = 180^\circ - (30^\circ + 25^\circ + 40^\circ + 28^\circ)$
 $= 180^\circ - 123^\circ = 57^\circ$. Ans.

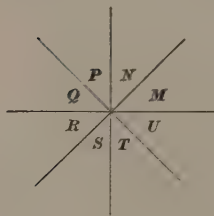


FIG. 18

28. The sum of all the angles formed in the same plane about one point is equal to four right angles. Thus, in Fig. 18,

$$M + N + P + Q + R + S + T + U = \text{four right angles} = 360^\circ.$$

EXAMPLE.—If the angles N , M , U , etc., Fig. 18, are equal to one another, what is the magnitude of each angle?

SOLUTION.—There are eight angles and their sum is 360° . Hence, each angle is $360^\circ \div 8 = 45^\circ$. Ans.

29. When two lines, as AB and CD , Fig. 19, cut or cross each other, they are said to **intersect**. The point where they cross is called their **point of intersection**, or simply their **intersection**.

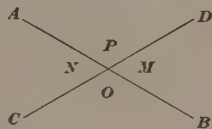


FIG. 19

30. Two intersecting straight lines determine four angles, as M , P , N , and O , Fig. 19, having a common vertex. Any one of these angles and the angle on the opposite side of both lines, as the angles M and N , Fig. 19, are called **vertical** or **opposite angles** with respect to each other. Vertical angles may also be defined as those having a common vertex and in which the sides of the one are the prolongations of the sides of the other. Since M and N are each the supplement of P , they are equal to each other. Any angle is equal to its vertical angle.

PARALLELS

31. Parallel lines, AB and CD , Fig. 20, are straight lines that lie in the **same plane** and never meet, however far they are produced. Any two parallel lines have the same direction and are everywhere equally distant from each other.

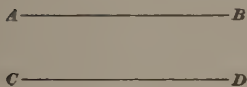


FIG. 20

32. A horizontal line, Fig. 21, is a line parallel to the horizon, or to the surface of still water. On a drawing or in a diagram in a book, lines parallel to the lower edge of the paper are usually considered to be horizontal.

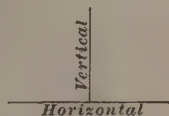


FIG. 21

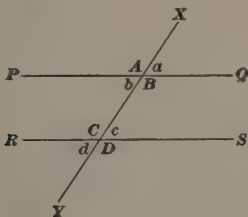


FIG. 22

33. A **vertical** line, Fig. 21, is a line perpendicular to a horizontal line, or it has the direction of a plumb-line.

34. When two parallel lines, as PQ and RS , Fig. 22, are cut by a third line, as XY , the angles denoted by the capital letters are equal to one another; that

is, $A = B = C = D$. The angles designated by the small letters are also equal to one another: $a = b = c = d$.

35. If the corresponding sides of two angles are parallel and extend in the same or in opposite directions from the respective

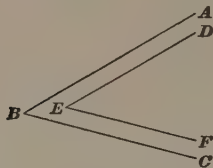


FIG. 23

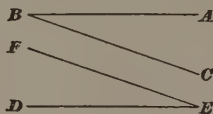


FIG. 24

vertices, the angles are equal. Thus, if the side AB , Fig. 23 or Fig. 24, is parallel to the side DE , and if the side BC is parallel to the side EF , the angle E is equal to the angle B .

36. If one of the sides of one angle, however, extends in the same direction as the corresponding side of the other angle, and the other two corresponding sides extend in opposite directions, the angles are supplementary. Thus, in Fig. 25, GH is parallel to and lies in the same direction as DE , and HI is parallel to but lies in the opposite direction to EF ; then, as GHI is the supplement of $G HK$, which, by Art. 35, is equal to DEF , GHI is the supplement of DEF .

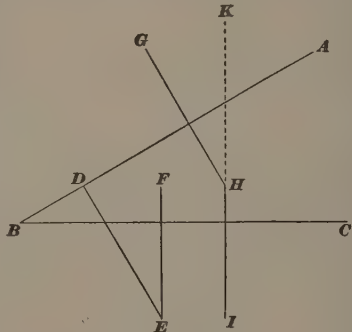


FIG. 25

37. If two sides of an angle are perpendicular to two sides of another angle, the two angles are equal or supplementary. Thus, if DE and GH , Fig. 25, are perpendicular to BA , and if EF and HK are perpendicular to BC , then will angle $E = \text{angle } B = \text{angle } GHK$; also, GHI is the supplement of ABC .

EXAMPLES FOR PRACTICE

38. 1. In Fig. 26, find the number of degrees in the angle EOA . Ans. 156°
 2. In Fig. 26, find the number of degrees in the arc AD . Ans. 113°

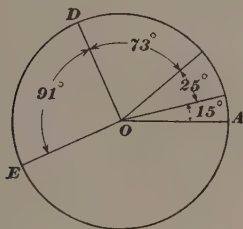


FIG. 26

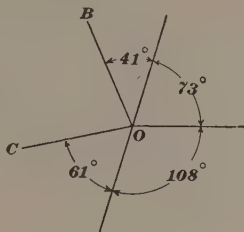


FIG. 27

3. In Fig. 27, find the number of degrees in the angle BOC . Ans. 77°
 4. In a wheel with fourteen spokes spaced equal distances apart, what is the angle included between the centre lines of any two adjacent spokes? Ans. $25\frac{5}{7}^\circ$
 5. If one straight line meets another straight line so as to form an angle equal to 137° , what does its adjacent angle equal? Ans. 43°
 6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°

PLANE FIGURES

39. A **plane figure** is any part of a plane surface enclosed or bounded by straight or curved lines.

40. When a plane figure is bounded by *straight lines*, it is called a **polygon**. The bounding lines are called the *sides*, and the length of the broken line that bounds it, or the whole distance round it, is called the **perimeter** of the polygon.

The most common kinds of plane figure bounded by curved lines are the *circle* and *ellipse*, which are explained later. The perimeter of these figures is the length of the bounding curved line.

41. The angles formed by the sides of a polygon are called the *angles*. Thus, $ABCDE$, Fig. 28, is a polygon.

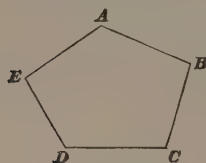


FIG. 28

AB , BC , etc., are the sides; EAB , ABC , etc., are the angles; and the length of the broken line $ABCDEA$ is the perimeter.

42. Polygons are classified according to the number of their sides: One of three sides is called a **triangle**; one of four sides, a **quadrilateral**; one of five sides, a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**; one of eight sides, an **octagon**; one of ten sides, a **decagon**; one of twelve sides, a **dodecagon**; etc.

43. **Equilateral polygons** are those in which the sides are all equal. Thus, in Fig. 29, $AB = BC = CD = DA$; hence, $ABCD$ is an equilateral

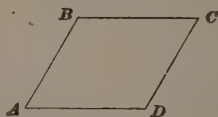


FIG. 29

44. An **equiangular polygon** is a polygon whose angles are all equal. Thus, in Fig. 30, angle $A =$ angle $B =$ angle $D =$ angle C ; hence, $ABDC$ is an equiangular polygon.



FIG. 30

45. A **regular polygon** is a polygon in which all the sides and all the angles are equal. Thus, in Fig. 31, $AB = BD = DC = CA$; and angle $A =$ angle $B =$ angle $D =$ angle C ; hence, $ABDC$ is a regular polygon. A regular polygon having four sides is

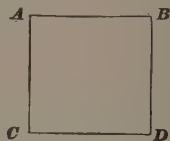


FIG. 31

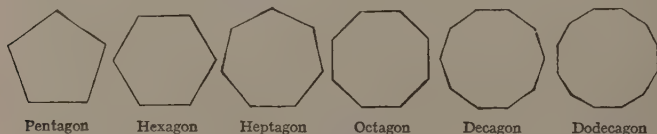


FIG. 32

called a **square**. Some other regular polygons are shown in Fig. 32.

THE TRIANGLE

46. Triangles are named according to their sides as *isosceles*, *equilateral*, and *scalene* triangles, and according to their angles as *right* and *oblique* triangles.



FIG. 33

47. An **isosceles triangle**, Fig. 33, is one having two of its sides equal.

48. An **equilateral triangle**, Fig. 34, is one that has its three sides equal.



FIG. 34

49. A **scalene triangle**, Fig. 35, is one not having any two of its sides equal



FIG. 35

50. A **right-angled triangle**, Fig. 36, is any triangle having one right angle. When the

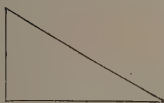


FIG. 36

sides of a right-angled triangle are referred to, the short sides are usually meant. The side opposite the right angle is called the **hypotenuse**. For brevity, a right-angled triangle is termed a **right triangle**.

51. An **oblique-angled**, or **oblique**, triangle, Fig. 37, is one that has no right angle.



FIG. 37

52. The **base** of any triangle is the side on which the triangle is supposed to stand. Any side of a triangle may be taken as the base.

The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base extended. Thus, in Figs. 38 and 39, if

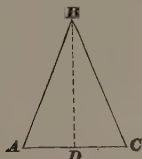


FIG. 38

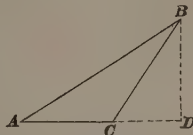


FIG. 39

the side AC is the base of the triangle, the line BD is the altitude.

In a right triangle, if one of the short sides is taken as the base, the other short side will be the altitude of the triangle.

53. In an isosceles triangle, the angles opposite the equal sides are equal. Thus, in Fig. 40, $AB = BC$; hence, angle $C =$ angle A .



FIG. 40

In any isosceles triangle, if a perpendicular is drawn from the vertex opposite the unequal side to that side, it bisects (cuts in halves) the side. Thus, AC , Fig. 40, is the unequal side in the isosceles triangle ABC ; hence, the perpendicular BD bisects AC , or $AD = DC$.

If two angles of any triangle are equal, it follows that two of its sides are equal, and that the triangle is isosceles.

54. In any triangle, the sum of the three angles is equal to two right angles, or 180° . Thus, in Fig. 41, the sum of the angles at A , B , and $C =$ two right angles; that is, $A + B + C = 180^\circ$. Hence, if any two angles of a triangle are given, the third may be found by subtracting the sum of the two from 180° .

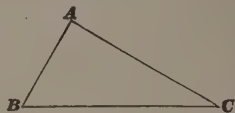


FIG. 41

EXAMPLE.—If, in Fig. 41, $A = 90^\circ$ and $C = 35^\circ$, how many degrees are there in B ?

SOLUTION.—Since the sum of A , B , and C is 180° , B is equal to the difference between 180° and the sum of A and C . Therefore,

$$B = 180^\circ - (90^\circ + 35^\circ) = 180^\circ - 125^\circ = 55^\circ. \text{ Ans.}$$

55. In any right triangle there can be but one right angle, and since the sum of all the angles equals two right angles, it is evident that the sum of the two acute angles must be equal to a right angle. Therefore, if in any right triangle one acute angle is known, the other can be found by subtracting the known angle from 90° .

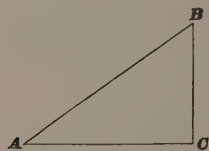


FIG. 42

Thus, the right triangle ABC , Fig. 42, is right angled at C . Then, the angle $A +$ angle $B =$ one right angle, or 90° . If $A = 40^\circ 30'$, $B = 90^\circ - 40^\circ 30' = 49^\circ 30'$.

56. Rise, Run, and Pitch.—In roof framing and other branches of carpentry, angles are sometimes expressed, not directly by the number of degrees contained in them, but indirectly by the *rise* and *run* or by the *pitch*. In Fig. 43, consider that AB and BC represent two rafters placed in position. Then the rise is the vertical distance BD , or the elevation of B above A or C . The run of the rafter AB is the distance AD , or the horizontal distance from A , the lower end of AB , to a plumb-line dropped from B . Similarly, the run of the rafter CB is the distance CD . The pitch is the proportion that the rise bears to the span, which is the distance AC , and is expressed by $\frac{BD}{AC}$.

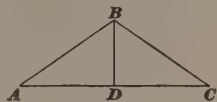


FIG. 43

If BD is 8 feet and AC 24 feet, then the pitch of AB will be

$$\frac{8}{24}, \text{ or } \frac{1}{3}.$$

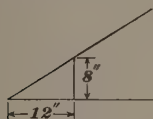


FIG. 44

57. The slope of rafters or other inclined pieces is often expressed as the rise for a run of 1 foot. Thus, a rise of 8 inches to the foot means that the difference in elevation, for each foot measured horizontally, is 8 inches. See Fig. 44.

58. Fig. 45 further illustrates the use of the terms rise, run, and pitch. The length of the rafter is measured on a line passing

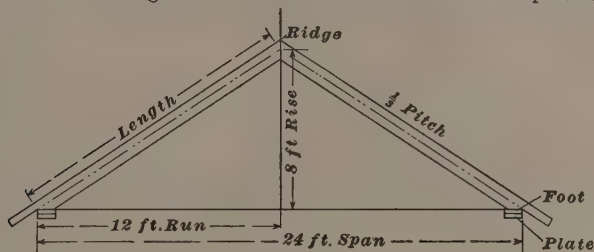


FIG. 45

through the outside edge of the plate parallel to the edge of the rafter; and the run is measured from the same point in the plate to a point directly below the ridge.

EXAMPLES FOR PRACTICE

59. 1. Find the number of degrees in each of the angles B and C of the triangle ABC , Fig. 46.

$$\text{Ans. } \begin{cases} B = 36^\circ \\ C = 108^\circ \end{cases}$$

2. How many degrees are there in each of the acute angles of an isosceles right triangle?

$$\text{Ans. } 45^\circ$$

3. One of the acute angles of a right triangle contains 30° ; how many degrees are there in the other acute angle?

$$\text{Ans. } 60^\circ$$



FIG. 46

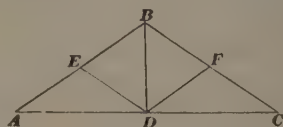


FIG. 47

4. Fig. 47 represents a roof truss. Angle $A = 35^\circ$, and $AE = ED$; find the number of degrees in angles AED , EDA , and ABD .

$$\text{Ans. } \begin{cases} AED = 110^\circ \\ EDA = 35^\circ \\ ABD = 55^\circ \end{cases}$$

5. The rise of a rafter is 8 feet 6 inches and the run is 10 feet; what is the rise per foot?

$$\text{Ans. } 10.2 \text{ in., or } 10\frac{1}{4} \text{ in., nearly}$$

6. The roof of a building 26 feet wide is $\frac{1}{3}$ pitch; what is: (a) the height of the ridge above the foot of the rafters? (b) the rise per foot?

$$\text{Ans. } \begin{cases} (a) \text{ 8 ft. 8 in.} \\ (b) \text{ 8 in.} \end{cases}$$

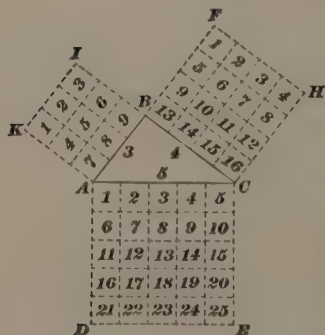


FIG. 48

PRINCIPLE OF THE RIGHT TRIANGLE

60. In any right triangle, the square described on the hypotenuse is equal to the sum of the squares described on the other two sides. If ABC , Fig. 48, is a right triangle, right angled at B , then the square described on the hypotenuse AC is equal to the sum of the squares described on the sides AB and BC ; consequently,

if the lengths of the sides AB and BC are known, the length of the hypotenuse can be found by adding the squares of the lengths of the sides AB and BC , and then extracting the square root of the sum.

Rule.—*To find the hypotenuse of a right triangle, add together the squares of the sides and extract the square root of the sum.*

Let a = length of one short side ;
 b = length of other short side ;
 c = length of hypotenuse.

Then, $c = \sqrt{a^2 + b^2}$

EXAMPLE 1.—In Fig. 49, BC is a brace joining the two pieces AB and AC , which are perpendicular to each other. Find the length of BC , B being 24 inches from A , and C 36 inches from A .

SOLUTION.— ABC is a right triangle, of which AB and AC are the sides and BC the hypotenuse. The length of BC is found by adding the squares of the lengths of AB and AC and extracting the square root of the sum. Thus, the square of AB is

$$24^2 = 24 \times 24 = 576$$

The square of AC is $36^2 = 36 \times 36 = 1,296$

The sum of these squares is $576 + 1,296 = 1,872$;
the square root of $1,872 = 43.27$. The length of BC is therefore 43.27 in. Ans.

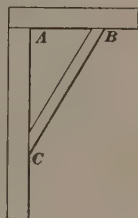


FIG. 49

EXAMPLE 2.—The width of a house is 20 feet, and the roof is half pitch, that is, the height of the gable is equal to one-half the width; what is the length of a rafter ?

SOLUTION.—Let one-half the width be called $a = 10$ ft. ; the height of the gable b is one-half of 20, or 10 ft. ; and the hypotenuse c is to be found. Substituting these values in the formula,

$$c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14 \text{ ft. Ans.}$$

61. When one side and the hypotenuse of a right triangle are known, the other side may be found by applying the following rule :

Rule.—*To find one side of a right triangle when the other side and the hypotenuse are known, subtract the square of the given side from the square of the hypotenuse and extract the square root of the difference.*

Otherwise stated, using letters having the same meaning as those given in the formula of Art. 60,

$$a = \sqrt{c^2 - b^2}, \text{ or } b = \sqrt{c^2 - a^2}$$

EXAMPLE 1.—It is desired to ascertain the height of the ceiling in a room. One end of a 10-foot rod is set in the angle of the ceiling and wall, and the other end is 6 feet from the base of the wall; what is the height of the ceiling?

SOLUTION.—In this example, the 10-ft. rod is the hypotenuse c and the short side b is 6 ft. Hence, applying the formula,

$$a = \sqrt{c^2 - b^2} = \sqrt{100 - 36} = 8 \text{ ft. Ans.}$$

EXAMPLE 2.—The equal sides of an isosceles triangle are 10 inches long, and the base is 12 inches long; find the length of the perpendicular from the vertex to the base.

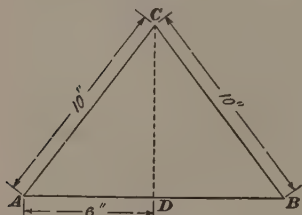


FIG. 50

SOLUTION.—Let ABC , Fig. 50, be the isosceles triangle. Then, AD is one-half of the base; therefore, $AD = 6$ in. In the right triangle ACD , the side CD is equal to the square root of the remainder obtained by subtracting the square of

the side AD from the square of the hypotenuse AC . Hence,

$$CD = \sqrt{10^2 - 6^2} = \sqrt{64} = 8 \text{ in. Ans.}$$

62. The hypotenuse of a right triangle whose sides are of equal length is equal to the square root of twice the square of one side. ABC , Fig. 51, is a right triangle, the side AB being equal to the side BC ; then,

$$AC = \sqrt{2 \times AB^2} = 1.4142 AB$$

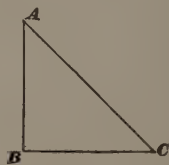


FIG. 51

Rule.—The hypotenuse of a right triangle having two equal sides is equal to 1.4142 times the length of a side.

NOTE.—When great accuracy is not required, $1\frac{1}{2}$ is sometimes used instead of 1.4142, as given in the foregoing rule. For example, when each side is 12 inches the hypotenuse is practically $1\frac{1}{2} \times 12 = 17$ inches.

EXAMPLE.—What is the length of a slope of a roof having half pitch, the width of the house being 30 feet?

SOLUTION.—Since the roof is half pitch, the height of the roof DG , Fig. 52, is equal to one-half of the width of the house: $DG = \frac{1}{2}$ of $EF = EG$.

The slope ED is therefore the hypotenuse of a right triangle whose sides are equal.

$$EG = \frac{1}{2} \text{ of } 30 = 15 \text{ ft.}$$

$$ED = 1.4142 \times 15 = 21.21 \text{ ft.}$$

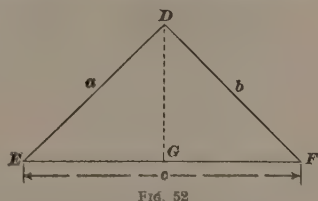
$$\cdot 21 \text{ ft.} = \cdot 21 \times 12 = 2.52 \text{ in.}$$

$$= 2\frac{1}{2} \text{ in., nearly}$$

Therefore, the length of the slope is 21 ft. $2\frac{1}{2}$ in. Ans.

The diagonal of a square is the hypotenuse of a right triangle

having equal sides; hence, the diagonal of a square is equal to 1.4142 times the side of the square. The side of a square can be obtained when the diagonal is known by dividing the diagonal by 1.4142, or by multiplying the diagonal by .7071.



63. A right angle can be set out by the following method, when the three sides of any right triangle are known. Suppose that AB , Fig. 53, is to be one line of the foundation of a building

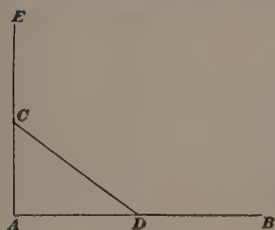


FIG. 53

and it is desired to set out from A another line that will make a right angle with AB . It is known that a triangle whose sides are, respectively, 6, 8, and 10 feet is a right triangle, since $6^2 + 8^2 = 10^2$. On AB cut off AD equal to 8 feet. Fasten one end of a tape, or line, at A and draw the other end toward E , so that AE will be as nearly perpendicular to AB as can be estimated by the eye. Make AC on AE 6 feet. Measure the distance CD ; if this is more or less than 10 feet, swing the line AE to the right or left, as may be necessary, until DC is exactly 10 feet, when the angle at A will be a right angle.

Any distances that will form three sides of a right triangle may be used in the preceding method; the numbers 3, 4, and 5, or some multiple, as 6, 8, and 10, or 9, 12, and 15, are most convenient.

64. The principle of the right triangle is of very great value in practical work, and it is necessary to become thoroughly familiar with it in all its variations.

The following is an example showing a double application of the right-triangle principle :

EXAMPLE.—Fig. 54 represents a skylight, which forms the four sides of a pyramid. The length of the base AD is 9 feet and the width CD is 7 feet 6 inches. The point O is 2 feet above the plane of $ABCD$. What is the length of the hip rafter OD ?

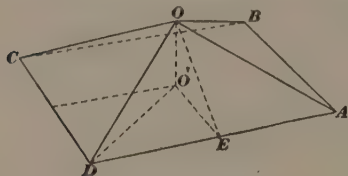


FIG. 54

SOLUTION.—The point O' is directly under O and 2 ft. below it, so that O' is on the same plane as $ABCD$. DO is simply the hypotenuse of a right triangle,

whose base is DO' and whose altitude is OO' . But DO' is also the hypotenuse of a right triangle, whose sides are EO' and ED . EO' is one-half of 7 ft. 6 in., that is, 3.75 ft.; and ED is one-half of 9 ft., that is, 4.5 ft.

Then, $DO' = \sqrt{3.75^2 + 4.5^2} = 5.86$ ft.

In the triangle $DO'O$, $DO' = 5.86$ ft., and $OO' = 2$ ft.; whence,

$$DO = \sqrt{5.86^2 + 2^2} = 6.19 \text{ ft.} = 6 \text{ ft. } 2\frac{1}{4} \text{ in., nearly}$$

One extraction of square root may be dispensed with, thus :

$$\begin{aligned} DO &= \sqrt{(DO')^2 + (OO')^2} = \sqrt{[(EO')^2 + (ED)^2] + (OO')^2} \\ &= \sqrt{3.75^2 + 4.5^2 + 2^2} = \sqrt{38.31} = 6.19 \text{ ft., as before. Ans.} \end{aligned}$$

Again, if it is possible to find the length of a line along the skylight at right angles to an edge, as OE to AD , then the hip OD is the hypotenuse of a right triangle of which OE and ED are sides; and

$$OE = \sqrt{(O'E)^2 + (OO')^2}$$

The result will be the same either way.

NOTE.—In solving the examples in *Geometry and Mensuration*, Parts 1 and 2, it is sufficient to state the results to two decimal places (except in constants, as .7854, .5236, etc.), and if the third decimal figure is greater than 5, the second decimal figure is to be increased by 1; thus, 13.537 should be written 13.54, not 13.53. To convert feet and inches to feet and decimals of a foot, feet and decimals of a foot to feet and inches, or to change a fraction of an inch to a decimal, or vice versa, Tables 1 and 11, Art. 95, may be used.

SIMILAR POLYGONS AND TRIANGLES

65. Similar polygons are those whose corresponding angles are equal and whose corresponding sides are proportional.

In the quadrilaterals $ABCD$ and $A''B''C''D''$, Fig. 55, the corresponding angles are equal; that is, angle A equals angle A'' ; angle B equals angle B'' ; angle C equals angle C'' ; and angle D equals angle D'' . Moreover, the corresponding sides of the figures are proportional; that is, $A''B''$ bears the

same ratio to AB as does $B''C''$ to BC , $C''D''$ to CD , and $D''A''$ to DA . The quadrilaterals $ABCD$ and $A''B''C''D''$ are therefore similar.

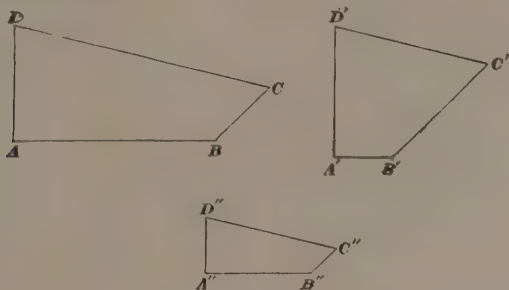


FIG. 55

The quadrilaterals $ABCD$ and $A'B'C'D'$, Fig. 55, have their corresponding angles equal, but the figures are not similar, since the corresponding sides are not proportional. For example, $A'D'$ does not bear the same ratio to AD as does $A'B'$ to AB . Therefore, quadrilaterals $ABCD$ and $A'B'C'D'$ are not similar.

Regular polygons of the same number of sides are also similar polygons.

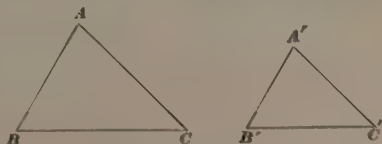


FIG. 56

66. Two triangles are similar when the angles of one are equal to the angles of the other. In the triangles ABC and $A'B'C'$, Fig. 56, angle A equals angle A' , angle B equals angle B' , and angle C equals angle C' ; the triangles are therefore similar, and their corresponding sides are proportional.

The following are some of the proportions that may be formed from these similar triangles :

$$AB : A'B' = AC : A'C'$$

$$AB : A'B' = BC : B'C'$$

$$AC : A'C' = BC : B'C'$$

$$AB : BC = A'B' : B'C'$$

EXAMPLE.—In Fig. 56, $AB = 6$ feet, $AC = 7$ feet, $BC = 8$ feet, and $B'C' = 6$ feet; find $A'B'$ and $A'C'$.

SOLUTION.—The ratio between the lengths of the corresponding sides BC and $B'C'$ is $8 : 6$, or $\frac{8}{6}$. The ratio between other corresponding sides

will be the same; that is, $\frac{AB}{A'B'} = \frac{8}{6}$, and $\frac{AC}{A'C'} = \frac{8}{6}$.

Since $AB : A'B' = 8 : 6$, or $6 : A'B' = 8 : 6$

$$A'B' = \frac{6 \times 6}{8} = 4\frac{1}{2} \text{ ft.} = 4 \text{ ft. } 6 \text{ in.} \quad \text{Ans.}$$

Again, $AC : A'C' = BC : B'C'$, or $7 : A'C' = 8 : 6$

$$\text{Then, } A'C' = \frac{7 \times 6}{8} = 5\frac{1}{4} \text{ ft.} = 5 \text{ ft. } 3 \text{ in.} \quad \text{Ans.}$$

67. Triangles having their corresponding sides parallel or in the same straight line are similar.

EXAMPLE.—It is required to find the distance AB , Fig. 57, across a stream.

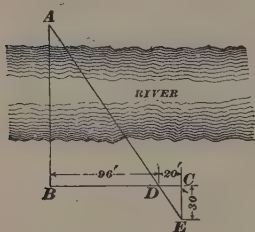


FIG. 57

SOLUTION.—The line BC making any angle with AB is measured, and CE is made parallel with AB , and of any convenient length. The point D is marked where AE intersects BC , and BD and DC are measured. Then, since the triangles ABD and CDE have their corresponding sides parallel, they are similar, and $AB : CE = BD : DC$; or

$$AB = \frac{30 \times 96}{20} = 144 \text{ ft.} \quad \text{Ans.}$$

68. If a straight line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. Thus, let the line DE be drawn parallel to the side BC in the triangle ABC , Fig. 58. Then, $AD : AB = AE : AC$, or $AD : DB = AE : EC$. It is to be noticed that the triangles ADE and ABC are similar, and their sides are proportional. Thus $AB : AD = BC : DE$, and $AC : AE = BC : DE$.

If a straight line, as DF , is drawn from D , Fig. 58, parallel to AC , the triangles ADE , ABC , and DBF are all similar, and a number of other proportions may be formed.

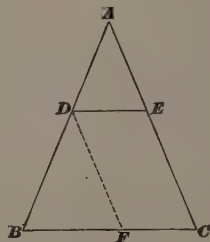


FIG. 58

EXAMPLE 1.—In Fig. 59, DE is parallel to BC , and AD is one-third of AB ; find the length of AE .

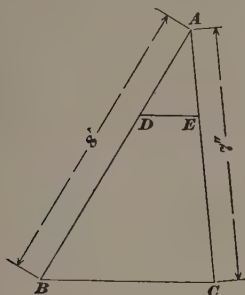


FIG. 59

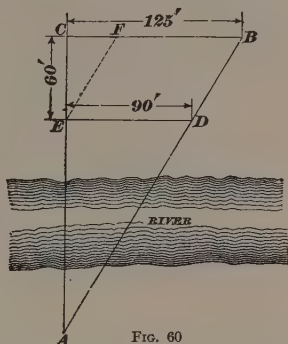


FIG. 60

SOLUTION.—Since DE is parallel to BC , $AD : AB = AE : AC$. But, $AD : AB = 1 : 3$; then, $AE : AC = 1 : 3$. Substituting for AC its value, $AE : 7 = 1 : 3$; hence

$$AE = \frac{7 \times 1}{3} = 2\frac{1}{3} \text{ in. Ans.}$$

EXAMPLE 2.—Referring to Fig. 60, it is desired to find the length of the line CA extending across a river. E is on the line AC , D is on the line BA , and DE is parallel to CB ; the lengths of CE , ED , and CB are as shown in the figure.

SOLUTION.—Draw EF parallel to AB , then $CF = 125 - 90 = 35$ ft. The triangles CAB and CEF being similar, $CA : CE = CB : CF$, or $CA : 60 = 125 : 35$; whence

$$CA = \frac{60 \times 125}{35} = 214.3 \text{ ft., nearly. Ans.}$$

EXAMPLE 3.—On a drawing, it is required to divide a line 8 inches long into twelve equal parts.

SOLUTION.—Let AB , Fig. 61, be the 8-in. line. Through A draw any line AC 12 in. long, and mark the inch points on it. Connect C and B , and through each inch mark on AC , draw a parallel to CB , as DE , etc., cutting AB at E , etc., which will be the points required; for by similar triangles, $AE : AD = AB : AC$; or $AE : 1 \text{ in.} = 8 \text{ in.} : 12 \text{ in.}$; hence,

$$AE = \frac{1 \times 8}{12} = \frac{2}{3} \text{ in. Ans.}$$

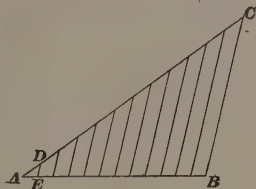


FIG. 61

NOTE.—This principle is very useful when an exact measurement, as, for example $\frac{2}{3}$ inch, cannot be obtained by a scale or rule. The equal spaces on AC need not be 1 inch in length; they may be of any length that will be most convenient.

EXAMPLES FOR PRACTICE

69. 1. A ladder 65 feet long reaches to the top of a house when its foot is 25 feet from the house; how high is the house, supposing the ground to be level? Ans. 60 ft.

2. In a triangle ABC , side $AB = 32$ feet, $BC = 34$ feet, and $AC = 48$ feet; if side AB of a similar triangle is 72 feet long, what are the lengths of the two other sides? Ans. $\begin{cases} AC = 108 \text{ ft.} \\ BC = 76.5 \text{ ft.} \end{cases}$

3. The base of a right triangle is 24 inches, and its altitude 72 inches; at what distance from the top is the triangle 16 inches wide? Ans. 48 in.

4. What length of rafter is required for each side of a span roof, the width of which is 24 feet, and whose height is three-fourths of the span? Ans. 21.63 ft.

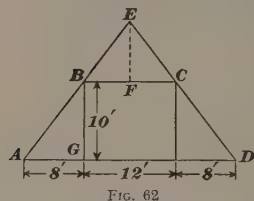


FIG. 62

distances EF and EA ?

5. $ABCD$, Fig. 62, is a section of a roof. It is desired to extend the sides AB and CD to the ridge E ; what are the distances EF and EA ? Ans. $\begin{cases} EF = 7 \text{ ft. } 6 \text{ in.} \\ EA = 22.41 \text{ ft., or } 22 \text{ ft. } 5 \text{ in., nearly} \end{cases}$

6. The distance EF , Fig. 63, is $16\frac{2}{3}$ feet. If the distance between the pieces parallel to AB , measured from centre to centre, is 20 inches, and the length of AB is 5 feet 6 inches, what is the length of the piece CD ?

Ans. 3 ft. $3\frac{3}{5}$ in.



FIG. 63

7. In Fig. 64, $CBAEC$ represents a gable at right angles to the main roof, the line AB ,

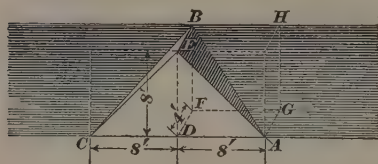


FIG. 64

where they meet, being called a *valley*. What is the length of the valley rafter AB , if CA is 16 feet, ED , the rise of the rafter, 8 feet, and $EB = DF$, the distance of B behind ACE , 4 feet? The

principle is the same as explained for hip rafters. Ans. 12 ft.

8. Each of the two sides of a right triangle is $12\frac{1}{2}$ inches long; what is the length of the hypotenuse? Ans. 17.7 in.

9. The diagonal of a square is 56.57 inches long; what is the length of a side? Ans. 40 in.

GEOMETRICAL PRINCIPLES OF THE CIRCLE

70. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc. Thus, in Fig. 65, AB is the chord of the arc AEB .

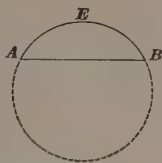


FIG. 65

71. A **segment** of a circle is the space included between an arc and its chord. Thus, in Fig. 65, the space between the arc AEB and the chord AB is a segment.

72. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc. Thus, in Fig. 66, the space included between the arc AB and the radii OA and OB is a sector of the circle.

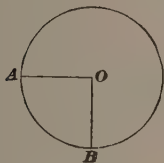


FIG. 66

73. Two circles are equal when the radius or diameter of one is respectively equal to the radius or diameter of the other. Two arcs are equal when the radius and chord of one are equal to the radius and chord of the other.

74. An **inscribed angle** is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 67, ABC is an inscribed angle, and it is measured by one-half the arc ADC .

EXAMPLE.—If in Fig. 67 the arc $ADC = 144^\circ$, what is the measure of the inscribed angle ABC ?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or

$$\frac{1}{2} \times 144^\circ = 72^\circ. \text{ Ans.}$$

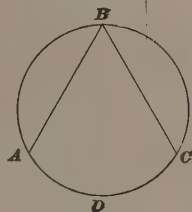


FIG. 67

75. All angles inscribed in the same segment are equal. Thus, in Fig. 68, the inscribed angles ABE , ACE , and ADE are all equal, for each is measured by one-half of the arc $A E$.

For example, if arc AE is 86° , each of the angles ABE , ACE , and ADE is equal to one-half of 86° , or 43° . It is thus seen that angles ABE , ACE , and ADE are each

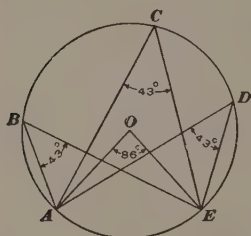


FIG. 68

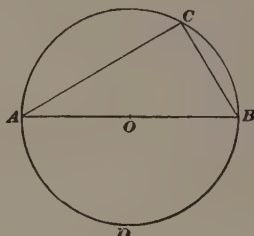


FIG. 69

equal to one-half of the angle AOE , which is formed by drawing radii from the centre O of the circle to A and E .

Any angle inscribed in a semicircle is a right angle. Thus, the angle ACB , Fig. 69, is a right angle, for it is measured by one-half of the semi-circumference ADB , and one-half of a semi-circumference is a quadrant.

76. If, in any circle, a radius is drawn perpendicular to any chord, it will bisect (cut in halves) the chord. Thus, if the radius OC , Fig. 70, is perpendicular to the chord AB , $AD = DB$.

A radius that bisects a chord bisects also the angle included between radii drawn to the ends of the chord. Thus, in Fig. 70, the radius OC bisects the angle AOB .

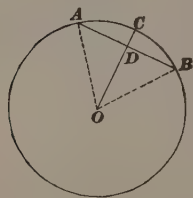


FIG. 70

If a straight line is drawn perpendicular to any chord at its middle point, it must pass through the centre of the circle.

77. Through any three points not in the same straight line, a circumference can be drawn. Let A , B , and C , Fig. 71, be any three points. Join A and B , also B and C , by straight lines. From the centre of AB draw KH perpendicular to AB ; from

the centre of BC draw FE perpendicular to BC . These two perpendiculars intersect at O . With O as a centre and OB , OA , or OC as a radius, describe a circle; it will pass through A , B , and C . This method is applied also in finding the centre of a circle already drawn, any three points A , B , C on the circumference being taken.

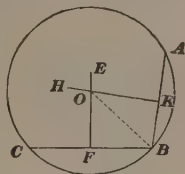


FIG. 71

78. A tangent to a circle is a straight line that touches the circle at one point only; it is always perpendicular to a radius drawn to that point. Thus, AB , Fig. 72, is a tangent to the circle; it touches the circle at E and is perpendicular to the radius OE .

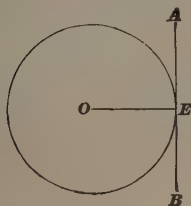


FIG. 72

79. If two circles intersect each other, the line joining their centres bisects at right angles the line joining the two points of intersection. Thus, if the two circles whose centres are O and P , Fig. 73, intersect at A and B , the line OP bisects at right angles the line AB ; or $AC = BC$.

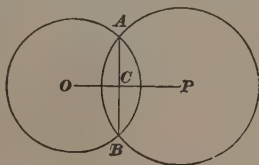


FIG. 73

80. One circle is said to be tangent to another circle when it touches the other at one point only, as in either of the cases shown in Fig. 74. This point is called the point of contact, or the point of tangency.



FIG. 74

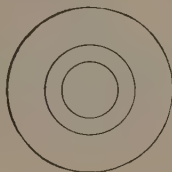


FIG. 75

81. When two or more circles are described from the same centre as in Fig. 75, they are called concentric circles.

82. If two chords of a circle intersect, the product of the segments (parts) thus formed of one of the chords is equal

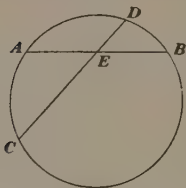


FIG. 76

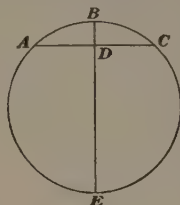


FIG. 77

to the product of the segments of the other. Thus, AB and CD , Fig. 76, are chords that intersect at E . Then,

$$AE \times EB = CE \times ED$$

EXAMPLE.—In Fig. 77, the diameter BE of the circle is 29 feet, and the distance BD , 4 feet; what is the length of the chord AC that passes through D and is perpendicular to BE ?

SOLUTION.—Since AC is perpendicular to the diameter BE , BE bisects AC , and $AD = DC$ (Art. 76); $DE = 29 - 4 = 25$ ft.

The products of the segments of intersecting chords are equal; the diameter BE being considered a chord, $AD \times DC = BD \times DE$. But, $AD = DC$; therefore,

$$\overline{AD}^2 = BD \times DE = 4 \times 25 = 100;$$

$$AD = \sqrt{100} = 10 \text{ ft.}$$

and $AC = 2 \times AD = 2 \times 10 = 20 \text{ ft.}$ Ans.

83. Since $\overline{AD}^2 = BD \times DE$, in Fig. 77, the following principle may be deduced from the preceding solution: When a chord is drawn perpendicular to a diameter of a circle, the square of one-half the chord is equal to the product of the parts of the diameter formed by the intersection of the chord.

EXAMPLE.—If, in Fig. 77, BD is 3 feet and DE is 9 feet, find the length of the half-chord AD .

SOLUTION.— $\overline{AD}^2 = BD \times DE = 3 \times 9 = 27$. Since the square of $AD = 27$,

$$AD = \sqrt{27} = 5.20 \text{ ft.} \text{ Ans.}$$

84. In the circle $ABCE$, Fig. 78, AC is the chord of the segment ABC . BD is the height of the segment; BC , the

chord of one-half the arc; and BE , the diameter perpendicular to the chord. Then, from Art. 83,

$$\overline{DC}^2 = BD \times DE; \text{ but } \overline{DC}^2 = \overline{BC}^2 - \overline{BD}^2; \text{ hence, } \overline{BC}^2 - \overline{BD}^2 = BD \times DE;$$

from which, $\overline{BC}^2 = BD (BD + DE)$.

$$\text{Then, } \overline{BC}^2 = BD \times BE, \text{ and } BE = \frac{\overline{BC}^2}{BD}$$

$$\text{or the radius } BO = \frac{\overline{BC}^2}{2BD}$$

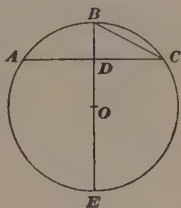


FIG. 78

Rule.—*The radius of a circle is equal to the square of the chord of one-half of a given arc divided by twice the height of the segment.*

EXAMPLE.—Referring to Fig. 78, $BC = 3$ feet 6 inches, and $BD = 1$ foot 7 inches; find the radius BO .

SOLUTION.— $BC = 42$ in.; $BD = 19$ in. According to the rule, the radius $BO = \frac{\overline{BC}^2}{2BD} = \frac{42^2}{2 \times 19} = 46.42$ in. Ans.

85. Where the chord of the arc and the height of the segment are known, the following rule may be applied:

Rule.—*To find the radius of the circle, add the square of one-half the chord to the square of the height of the segment and divide the sum by twice the height of the segment.*

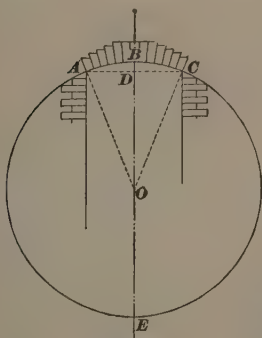


FIG. 79

Let r = radius;

c = chord of arc;

h = height of segment.

$$\text{Then, } r = \frac{\left(\frac{c}{2}\right)^2 + h^2}{2h}$$

which may be reduced to

$$r = \frac{c^2 + 4h^2}{8h}$$

EXAMPLE.—In arches, the *span* is the distance across the opening, as AC , Fig. 79, measured from the ends of the arch, as A and C . The *rise* DB is the perpendicular distance from AC to the highest point B

measured on the centre line OB .

The span AC , Fig. 79, of an arch is 6 feet, and the rise DB is 8 inches; what is the length of the radius OB ?

SOLUTION.—The span AC is the chord of the arc ABC , and BD is the height of the segment. AD , one-half of the chord, is $\frac{1}{2} \times 72$ in. = 36 in.

$$OB = \frac{\overline{AD}^2 + \overline{BD}^2}{2BD} = \frac{36^2 + 8^2}{2 \times 8} = \frac{1360}{16} = 85 \text{ in.}$$

Therefore, $OB = 85$ in. = 7 ft. 1 in. Ans.

INSCRIBED AND CIRCUMSCRIBED POLYGONS

86. An inscribed polygon is one whose vertices lie on the circumference of a circle and whose sides are chords, as

$KLMNPQ$, Fig. 80. A circumscribed polygon is one whose sides are tangent to a circle, as $ABCDEF$, Fig. 80.

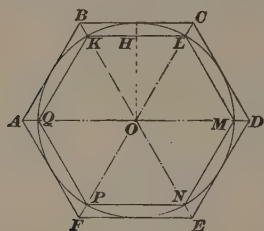


FIG. 80

Circles may be inscribed in and circumscribed about any regular polygon. The radius of the circumscribed circle is the distance from the centre to an angle of the inscribed

polygon; the radius of the inscribed circle is the perpendicular distance from the centre to the side of the circumscribed polygon, as shown in Fig. 80.

87. If lines are drawn from the centre to the angles of a regular polygon, they will make equal angles with the sides of the polygon and will form isosceles triangles. As the sum of all the angles formed about a point by lines drawn from it is equal to 360° , the angle at the centre of a regular polygon between two radii equals 360° divided by the number of sides. Thus, the angle between two radii drawn to the ends of a side of a regular hexagon is $360^\circ \div 6 = 60^\circ$. The sum of the angles of a triangle is two right angles, or 180° ; hence, since the triangles formed by drawing radii to the angles of a regular polygon are isosceles, the angles OQK and OKQ , Fig. 80, are each equal to one-half the difference between 180° and the central angle. Thus, each angle is $\frac{1}{2} \times (180^\circ - 60^\circ) = 60^\circ$, and the

triangle OKQ is equilateral. In a regular hexagon, therefore, the sides are equal to the radius of the circumscribed circle.

A line drawn from the centre of a regular polygon to an angle bisects the angle; thus OA , Fig. 80, bisects the angle BAF , which, therefore, is equal to $2 \times OAB$, or $2 \times OAF$.

88. The principles of Art. 87 are employed, in practice, to determine the various bevels used in the construction of regular polygons.

EXAMPLE 1.—A joiner marks oblique lines on his material by means of a bevel, which is an instrument consisting of two straight pieces of wood or metal so jointed that any angle may be formed by the pieces, as shown at D , Fig. 81. At what angle should a bevel be set to mark the mitres of a moulded skirting which is to be placed round an octagonal room of equal sides, as in Fig. 81?

SOLUTION.—Draw AO , BO , etc., to the centre O , forming triangles AOB , BOC , etc. Then each of the angles at O is one-eighth of $360^\circ = 45^\circ$. The angles OAB and OBA are each

$$\frac{1}{2} \times (180^\circ - 45^\circ) = 67\frac{1}{2}^\circ$$

If the blade of the bevel is set to this angle, as at D , and the ends of the pieces of skirting, etc., are cut to it, they will fit together properly. Ans.

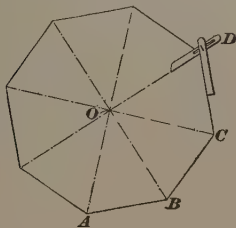


FIG. 81

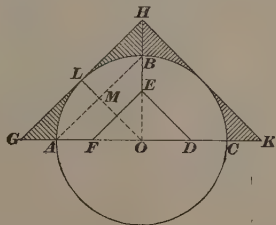


FIG. 82

EXAMPLE 2.—In Fig. 82, $GHEF$ and $HKDE$ represent two boards forming a centre for supporting a brick arch during building, the shaded parts being cut off, leaving a curve, as $ALBC$. The radius OA of the arch is 2 feet. (a) What is the length of each board, and (b) the angle of bevel GHO where they meet?

SOLUTION.—(a) Draw AB parallel to GH and meeting OG and OH ; also draw OL perpendicular to AB ; then $AM = MB$, Art. 76, and by similar triangles, $LG = LH$. OL bisects the right angle AOB ; hence, AOM is one-half of 90° , or 45° . Then the other acute angle in the right triangle OMA is $90^\circ - 45^\circ = 45^\circ$ also. The angle LGO is equal to the angle MAO , or 45° , and the right triangle OLG having two angles equal, the

opposite sides are equal also, and since $OL =$ the radius, or 2 ft., LG is also 2 ft. and GH is $2 \times 2 = 4$ ft., the required length. Ans.

(b) Angle $OHG =$ angle $OGH = 45^\circ$, the angle at which to set the bevel. Ans.

89. The following principle may also be conveniently used in many cases to find the interior angles of a regular polygon. In any polygon, the sum of all the interior angles equals 180° multiplied by a number that is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a regular polygon, multiply 180° by the number of sides less two, and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle. For example, each interior angle of a regular octagon, as Fig. 81, is $[180^\circ \times (8 - 2)] \div 8 = 135^\circ$.

EXAMPLES FOR PRACTICE

90. 1. The span of an arch is 7 feet, and the rise is 1 inch for each foot of span; what is the radius of the arch?

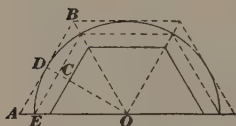


FIG. 83

Ans. 10 ft. $9\frac{1}{2}$ in.

2. The centring for an arch, Fig. 83, 8 feet in diameter, is made of three pieces of equal length. Knowing that the side of a regular inscribed hexagon is equal to the radius, what should be the length, as AB , of each piece?

Ans. 4.62 ft., or 4 ft. $7\frac{1}{2}$ in., nearly

SUGGESTION.—Find the length of OC in the right triangle OCE ; then, by the similar triangles OEC and OAD , find the length of AO . The proportion will be stated: $OC : OD = OE : OA$. AB will equal AO .

3. In the preceding example, what is: (a) the bevel angle ABO ?
(b) the angle between any two pieces?

Ans. $\begin{cases} (a) 60^\circ \\ (b) 120^\circ \end{cases}$

4. The flooring in a regular octagonal room, Fig. 84, is laid parallel with the side AB , which is 6 feet long; what is the angle of bevel for cutting the ends along AC ?

Ans. 135°

5. What is the length of the flooring between CD and HK , Fig. 84, knowing that CF and AF are equal and that angle CFA is a right angle? See Art. 62.

Ans. 14.48 ft., or 14 ft. $5\frac{3}{4}$ in., nearly

6. Referring to Fig. 77, if the diameter of the circle is 10 feet and DE is 8 feet 6 inches, find the distance DC . Ans. 3 ft. $6\frac{7}{8}$ in., nearly

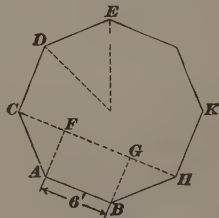


FIG. 84

MENSURATION OF SURFACES AND SOLIDS

PRELIMINARY EXPLANATIONS

READING DRAWINGS

91. The practical application of mensuration is the computation of lengths, areas of surfaces, or volumes of solids. The dimensions that furnish the data required are usually obtained from **working drawings**, or **plans**; therefore, before considering the methods of calculating areas and volumes, it is necessary to explain how drawings and plans are made, and how the dimensions are taken from them. The statements apply as well to the mensuration of lines which has preceded.

92. Working drawings are generally made to **scale**; that is, the lines on the drawings have a certain ratio to the corresponding dimensions of the full-size object that the drawing represents; this ratio is called the **scale**. For example, if the scale of a drawing is $\frac{1}{2}$ inch to 1 foot, it means that each $\frac{1}{2}$ inch on the drawing represents 1 foot on the object, and the drawing is $\frac{1}{2} \div 12 = \frac{1}{24}$ of full size.

93. To make drawings and to obtain measurements from them, divided rules called **scales** are used. These consist of strips or pieces of wood, metal, ivory, cardboard, etc., 1 or 2 feet long, generally having the edges bevelled, and on which are engraved or printed the graduations for the different scales, as 1 inch to 1 foot, $\frac{1}{4}$ inch to 1 foot, etc. The marks are numbered in order from the end, the space next to the end being subdivided into twelfths, representing inches, and in the larger scales these spaces are again subdivided for fractions of an inch.

There are usually two scales marked on each edge, and in one the divisions are twice as long as in the other, and are numbered from the other end.

To measure a line on a drawing, place the scale with the proper edge along the line, and move it until one of the foot-marks is opposite one end of the line, as at *b*, Fig. 85, while the other end of the line is at the 0 mark on the scale, or opposite

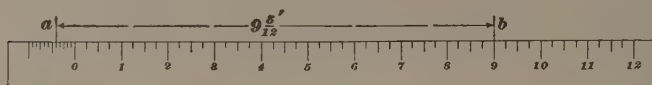


FIG. 85

one of the small graduations, as at *a*. Then read the number of large divisions from the 0 mark to the far end of the line, and also the number of small divisions from 0 to the near end of the line. Thus, in Fig. 85, the line *ab* extends over nine large spaces and five small ones, or five-twelfths of a large one, so that the length of the line represented by *ab* is 9 feet 5 inches.

94. Working drawings usually have the principal dimensions marked on them, as shown in Fig. 86, which represents a side view and an end view of a cast-iron lintel. The length from *e* to *f*

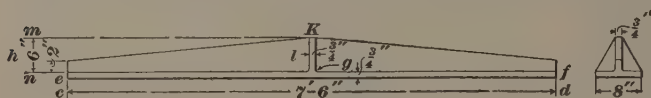


FIG. 86

is 7 feet 6 inches, as indicated on the broken line *cd* ending in the arrowheads; *ec* and *fd* are drawn perpendicular to *ef*. At *h* is shown that the height from *g* to *K* is 6 inches, being the distance between the parallel lines *mK* and *ng*. At *l* are

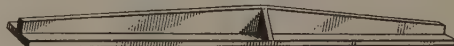


FIG. 87

shown two arrowheads pointed toward each other; these are often used where the space is too small to insert the arrowheads and figures in the ordinary manner. In this case, it means that

the thickness of the rib is $\frac{3}{4}$ inch, as shown by the near-by figures. Fig. 87 is a perspective view of the lintel.

Having read or scaled all the necessary dimensions from the drawing, the proper formulæ or rules may be applied to obtain the required length, area, or contents.

CONVERSION TABLES

95. Conversion tables are useful in changing inches or fractions of an inch to decimals of a foot or inch, and vice versa. In Table I are given the values of inches and common fractions of an inch corresponding to various decimal parts of a foot, while in Table II are given the decimal equivalents of the various common fractions of an inch.

TABLE I
INCHES TO DECIMALS OF A FOOT

Inches or Fractions	Decimal of Foot	Approximate Decimal	Inches	Decimal of Foot	Approximate Decimal
$\frac{1}{16}$	·0052	·005	3	·2500	·25
$\frac{1}{8}$	·0104	·010	4	·3333	·33
$\frac{1}{4}$	·0208	·020	5	·4167	·42
$\frac{3}{8}$	·0313	·030	6	·5000	·50
$\frac{1}{2}$	·0417	·040	7	·5833	·58
$\frac{5}{8}$	·0521	·050	8	·6667	·67
$\frac{3}{4}$	·0625	·060	9	·7500	·75
$\frac{7}{8}$	·0729	·070	10	·8333	·83
1	·0833	·080	11	·9167	·92
2	·1667	·170	12	1·0000	1·00

EXAMPLE 1.—Express 7 feet $8\frac{7}{8}$ inches in feet and a decimal of a foot.

SOLUTION.—From Table I, 8 in. = ·67 ft., and $\frac{7}{8}$ in. = ·07 ft.; hence, 7 ft. $8\frac{7}{8}$ in. = $7 + \cdot67 + \cdot07 = 7\cdot74$ ft. Ans.

EXAMPLE 2.—Express 18·19 feet in feet and inches.

SOLUTION.—By consulting the column of approximate decimals, Table I, it will be observed that ·19 ft. is greater than ·17 ft., or 2 in., and is less

than $\cdot 25$ ft., or 3 in.; $\cdot 19$ being $\cdot 02$ greater than $\cdot 17$, $\cdot 19$ ft. equals 2 in. and $\cdot 02$ ft. more. According to the table, $\cdot 02$ ft. = $\frac{1}{4}$ in.; hence, $\cdot 19$ ft. = $2\frac{1}{4}$ in., and $18\cdot 19$ ft. = 18 ft. $2\frac{1}{4}$ in. Ans.

Table II is used in a similar manner. Thus, $9\cdot 26$ in. = $9\frac{1}{4}$ in., approximately.

TABLE II
FRACTIONS OF AN INCH TO DECIMALS OF AN INCH

Fraction	Exact Decimal	Approximate Decimal	Fraction	Exact Decimal	Approximate Decimal
$\frac{1}{32}$	$\cdot 03125$	$\cdot 03$	$\frac{9}{16}$	$\cdot 5625$	$\cdot 56$
$\frac{1}{16}$	$\cdot 06250$	$\cdot 06$	$\frac{5}{8}$	$\cdot 6250$	$\cdot 63$
$\frac{1}{8}$	$\cdot 12500$	$\cdot 13$	$\frac{11}{16}$	$\cdot 6875$	$\cdot 69$
$\frac{3}{16}$	$\cdot 18750$	$\cdot 19$	$\frac{3}{4}$	$\cdot 7500$	$\cdot 75$
$\frac{1}{4}$	$\cdot 25000$	$\cdot 25$	$\frac{13}{16}$	$\cdot 8125$	$\cdot 81$
$\frac{5}{16}$	$\cdot 31250$	$\cdot 31$	$\frac{7}{8}$	$\cdot 8750$	$\cdot 88$
$\frac{3}{8}$	$\cdot 37500$	$\cdot 38$	$\frac{15}{16}$	$\cdot 9375$	$\cdot 94$
$\frac{7}{16}$	$\cdot 43750$	$\cdot 44$	1	$1\cdot 0000$	$1\cdot 00$
$\frac{1}{2}$	$\cdot 50000$	$\cdot 50$			

PLANE SURFACES

96. The area of a figure means the extent of the surface included within the bounding lines of the figure, and is expressed by the number of unit squares it will contain.

A unit square is the square having the unit for its side. For example, if the unit is 1 inch, the unit square is the square whose side measures 1 inch in length, and the area will be expressed by the number of square inches that the surface contains. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area would be the number of square feet that the surface contains, etc. The square that measures 1 inch on a side is called a **square inch**, and the one that measures 1 foot on a side is called a **square foot**. Square inch and square foot are abbreviated to sq. in. and sq. ft., or are indicated by $\square''$ and $\square'$.

QUADRILATERALS

97. A **parallelogram** is a quadrilateral whose opposite sides are parallel. There are four kinds of parallelograms: the *square*, the *rectangle*, the *rhombus*, and the *rhomboid*.

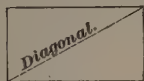


FIG. 88

98. A **rectangle**, Fig. 88, is a parallelogram whose angles are all right angles.



FIG. 89

99. A **square**, Fig. 89, is a rectangle, all of whose sides are equal.

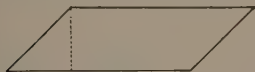


FIG. 90

100. A **rhomboid**, Fig. 90, is a parallelogram whose opposite sides only are equal, and whose angles are not right angles.

101. A **rhombus**, Fig. 91, is a parallelogram having equal sides, and whose angles are not right angles.



FIG. 91

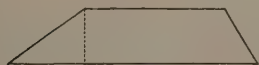


FIG. 92

102. A **trapezoid**, Fig. 92, is a quadrilateral that has only two of its sides parallel.

103. A **trapezium**, Fig. 93, is a quadrilateral not having any two sides parallel.

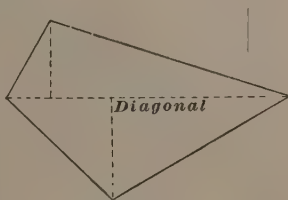


FIG. 93

104. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides.

105. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides the quadrilateral into two triangles. A diagonal divides a parallelogram into two *equal* and *similar* triangles.

106. The area of any parallelogram may be found by the following rule :

Rule.—To find the area of any parallelogram, multiply the base by the altitude.

Let b = base ;
 h = altitude ;
 A = area.
 Then, $A = b h$

EXAMPLE 1.—What is the area of the rectangle $A B C D$, Fig. 94 ?

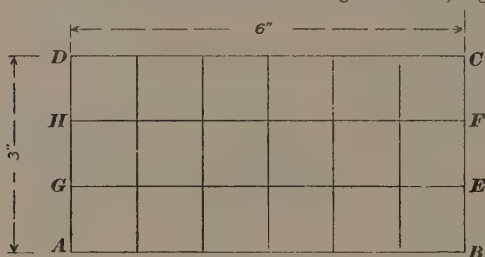


FIG. 94

SOLUTION.—The area of the rectangle equals the product of the base, which is 6 inches, and the altitude, which is 3 inches. Hence, the area is
 $6 \times 3 = 18$ sq. in. Ans.

By dividing Fig. 94 into squares as shown, it is seen that, if the figure is drawn full size, it will require 18 sq. in. to cover it.

EXAMPLE 2.—A plot is 4 poles wide and 8 poles long : (a) How many square feet does it contain ? (b) What part of an acre is it ?

SOLUTION.—(a) Reducing poles to feet, 4 poles = 66 ft., and 8 poles = 132 ft. Either side may be taken as the base. Applying the formula,

$$A = b h = 66 \times 132 = 8,712 \text{ sq. ft. Ans.}$$

(b) As there are 43,560 sq. ft. in an acre, $8,712 \text{ sq. ft.} = 8,712 \div 43,560 = .2 = \frac{1}{5} \text{ A. Ans.}$

107. The area of a trapezoid may be found as follows :

Rule.—To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.

Let a and b = lengths of parallel sides ;
 h = altitude.

Then,
$$A = h \left(\frac{a + b}{2} \right)$$

EXAMPLE.—How many square feet of floor space are there in a five-story warehouse having the dimensions given in Fig. 95 ?

SOLUTION.—As either of the parallel sides may be called a , let it be the shorter ; then, applying the formula,

$$A = h \left(\frac{a+b}{2} \right) = 27.6 \left(\frac{49.5 + 54.3}{2} \right) = 27.6 \times 51.9 = 1,432.44 \text{ sq. ft.}$$

Therefore, for five stories, there are

$$1,432.44 \text{ sq. ft.} \times 5 = 7,162.2 \text{ sq. ft. of floor space. Ans.}$$

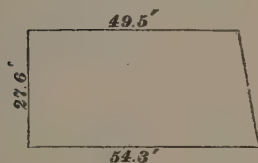


FIG. 95

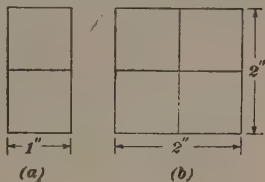


FIG. 96

108. It should be observed that 2 square inches does not mean the same as 2 inches square ; 3 square inches the same as 3 inches square, etc. Fig. 96 (a) shows a rectangle 2 inches by 1 inch, and its area is 2 square inches. Fig. 96 (b) shows a square 2 inches by 2 inches ; that is, a figure 2 inches square, and its area is 4 square inches.

TRIANGLES

109. The following rule may be employed in finding the area of a triangle :

Rule.—To find the area of a triangle, multiply the base by the altitude and divide the product by 2 ; or, in other words, multiply the base by half the altitude.

Let b = base ;
 h = altitude ;
 A = area.

Then, $A = \frac{bh}{2}$

EXAMPLE.—How many square feet of 1-inch boards will be needed for the two gables of a house having a half-pitch roof and a width of 24 feet ?

SOLUTION.—In half-pitch roofs, the rise equals one-half the span ; hence, $h = \frac{1}{2}$ of 24 = 12 ft., and $b = 24$ ft. Applying the formula,

$A = \frac{1}{2} b h = \frac{1}{2} \times 24 \times 12 = 144$ sq. ft. for each gable, or $144 \times 2 = 288$ sq. ft. of boards for the two gables. Ans.

110. If the triangle is a right triangle, one of the short sides may be taken as the base, and the other short side as the altitude ; hence :

Rule.—*To find the area of a right triangle, multiply one short side by the other and divide the product by 2.*

111. The area of any triangle may be found, when the length of each side is known, by the following rule :

Rule.—*From half the sum of the three sides, subtract each side separately. Multiply together the half sum and the three remainders, and extract the square root of the product. The result will be the area.*

Let $a, b, c =$ lengths of sides ;

$$s = \text{half sum of lengths} = \frac{a + b + c}{2} ;$$

$A =$ area of triangle.

$$\text{Then, } A = \sqrt{s(s-a)(s-b)(s-c)}$$

EXAMPLE.—What is the area of a triangular plot of ground, the sides of the plot being 60, 55, and 35 feet, respectively ?

SOLUTION.—It does not matter which side is called a, b , or c . Letting $a = 60, b = 55$, and $c = 35$,

$$s = \frac{60 + 55 + 35}{2} = 75$$

$$s - a = 75 - 60 = 15$$

$$s - b = 75 - 55 = 20$$

$$s - c = 75 - 35 = 40$$

Then,

$$A = \sqrt{75 \times 15 \times 20 \times 40} = \sqrt{900,000} = 948.68 \text{ sq. ft. Ans.}$$

THE CIRCLE

112. The ratio of the circumference of a circle to its diameter is an incommensurable number, and is usually denoted by the Greek letter π , pronounced *pi*. The approximate value of the ratio, correct to four decimal places, is 3.1416 ; hence, approximately, $\pi = 3.1416$. For off-hand calculations, the ratio is

frequently taken as $3\frac{1}{7}$; that is, the circumference is $3\frac{1}{7}$ times the diameter.

113. The following rules are used to find the circumference and the diameter of a circle :

Rule I.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

Rule II.—*To find the diameter of a circle, divide the circumference by 3.1416.*

Let, c = circumference ;
 d = diameter ;
 r = radius.

Then, $c = \pi d = 2 \pi r$ (1)

$$d = \frac{c}{\pi} \quad (2)$$

$$r = \frac{c}{2 \pi} \quad (3)$$

EXAMPLE.—A wheelwright wishes to cut a length of tire iron long enough to go round a wagon wheel $4\frac{1}{2}$ feet in diameter ; how long should he cut it, allowing $1\frac{1}{2}$ inches for welding ?

SOLUTION.—Here $d = 4\frac{1}{2}$ ft. Applying formula 1,

$$c = \pi d = 3.1416 \times 4\frac{1}{2} = 14.14 \text{ ft.} = 14 \text{ ft. } 1\frac{3}{4} \text{ in., nearly}$$

Adding the $1\frac{1}{2}$ in., the length required is 14 ft. $3\frac{1}{4}$ in. Ans.

Had the ratio been taken as $3\frac{1}{7}$, instead of at 3.1416, the result would have been $\frac{1}{200}$ in. greater, an amount too small to affect the accuracy of such work.

114. As the circle is divided into 360° , and the length of the circumference is $2 \pi r$, the length of 1° is $\frac{2 \pi r}{360}$; or, dividing

both terms by 2π ($= 6.2832$), this reduces to $\frac{r}{57.3}$, very closely.

If the angle is 57.3° , the length of the arc is equal to the radius ; this angle is called a **radian**.

115. The length of an arc of a circle may be found by the following rule :

Rule.—*To find the length of an arc of a circle, multiply the number of degrees in the arc by the radius, and divide by 57.3, or by 57.296 if greater precision is required.*

Let l = length of arc ;
 n = number of degrees in arc ;
 r = radius of arc.

Then,
$$l = \frac{r \times n}{57.3}$$

If the angle contains degrees, minutes, and seconds, reduce the minutes and seconds to decimals of a degree, thus :

$$\begin{aligned} 37^\circ 30' 15'' &= 37^\circ + \frac{30}{60}^\circ + \frac{15}{3600}^\circ \\ &= 37^\circ + .5^\circ + .004^\circ = 37.504^\circ \end{aligned}$$

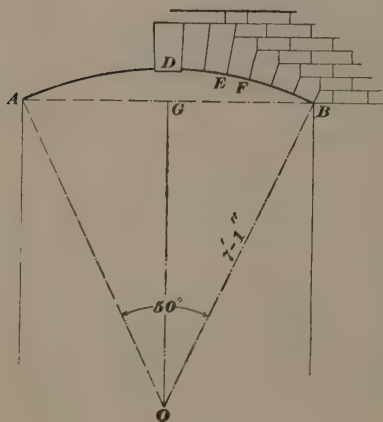


FIG. 97

EXAMPLE.—Fig. 97 shows a segmental stone arch having a radius of 7 feet 1 inch = 85 inches. The angle $A O B$ is 50° . If there are thirteen ring stones in the arch, what will be the width, as $E F$, of each stone ?

SOLUTION.—Here $A D B$ = l ; r = 85 in., and $n = 50^\circ$.

Applying the formula,

$$l = \frac{r n}{57.3} = \frac{85 \times 50}{57.3} = 74.17 \text{ in.}$$

Dividing by the number of stones,

$$\begin{aligned} 74.17 \div 13 &= 5.70 \text{ in.} \\ &= 5\frac{1}{16} \text{ in., nearly. Ans.} \end{aligned}$$

116. When only the chord $A B$, Fig. 98, of an arc and the height, or rise, $C D$ of the segment are known, the following approximate method gives the length of an arc less than one-sixth of a circumference correct to four figures, and the length of an arc less than one-third of a circumference correct to three figures.

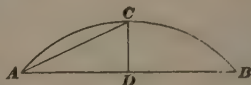


FIG. 98

$A C$, the chord of half the arc, has the value

$$A C = \sqrt{A D^2 + C D^2} = \sqrt{\left(\frac{A B}{2}\right)^2 + C D^2}$$

Rule.—To find the length of the arc : From eight times the chord of one-half the arc subtract the chord of the whole arc, and divide the remainder by 3.

That is, $\text{arc } ACB = \frac{8 \times AC - AB}{3}$

Let c = chord of whole arc ;
 h = height of segment ;
 l = length of arc.

Then, $AC = \sqrt{\frac{c^2}{4} + h^2} = \frac{1}{2} \sqrt{c^2 + 4h^2}$

and $l = \frac{4 \sqrt{c^2 + 4h^2} - c}{3}$

Either the rule or the formula may be used, whichever is the more convenient. In using the formula, it must be carefully noted that the radical sign does not extend over the last c .

EXAMPLE.—Find the length of the arc ACB , Fig. 99.

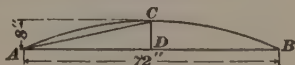


FIG. 99

SOLUTION.— AD , which is one-half of AB , = $\frac{1}{2} \times 72 = 36$ in., and $DC = 8$ in.

$$AC = \sqrt{AD^2 + DC^2} = \sqrt{36^2 + 8^2} = 36.88 \text{ in.}$$

Hence, length of arc ACB is

$$\frac{8 \times AC - AB}{3} = \frac{8 \times 36.88 - 72}{3} = 74.35 \text{ in. Ans.}$$

117. When the results that may be obtained by the method given in the preceding article are not considered sufficiently accurate, the length of one-half the arc may be found in a similar manner and this multiplied by 2. The following example explains the method of procedure :

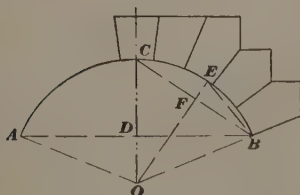


FIG. 100

EXAMPLE.—The segmental arch, Fig. 100, has a span of 60 inches and a rise of 25 inches ; what is the length of its intrados ACB ?

SOLUTION.—Let O represent the centre of the arch, and let F be the middle point of the chord CB .

$$CB^2 = CD^2 + DB^2 = 25^2 + (2^0)^2 = 1,525$$

$$CB = \sqrt{1,525} = 39.051 \text{ in.}$$

$$CF \text{ or } FB = CB \div 2 = 39.051 \div 2 = 19.526 \text{ in.}$$

According to Art. 84, the radius

$$CO = \frac{CB^2}{2CD} = \frac{1,525}{2 \times 25} = 30.5 \text{ in.}$$

In the right triangle CFO ,

$$OF = \sqrt{OC^2 - CF^2} = \sqrt{30.5^2 - 19.526^2} = 23.431 \text{ in.}$$

$$FE = OE - OF = 30.5 - 23.431 = 7.069 \text{ in.}$$

In the right triangle EFB ,

$$EB = \sqrt{EF^2 + FB^2} = \sqrt{7.069^2 + 19.526^2} = 20.766 \text{ in.}$$

According to the rule of Art. 116,

$$\text{arc } CEB = \frac{8EB - CB}{3} = \frac{8 \times 20.766 - 39.051}{3} = 42.359 \text{ in.}$$

The whole arc ACB is therefore

$$2 \times 42.359 = 84.72 \text{ in. Ans.}$$

Applying the rule of Art. 116, directly,

$$\text{arc } ACB = \frac{8 \times CB - AB}{3} = \frac{8 \times 39.051 - 60}{3} = 84.13 \text{ in. Ans.}$$

118. The following rule is employed to find the area of a circle :

Rule.—To find the area of a circle, square the diameter and multiply by .7854; or square the radius and multiply by 3.1416.

Let d = diameter of circle ;

r = radius of circle ;

A = area of circle.

$$\text{Then, } A = \frac{1}{4} \pi d^2 = .7854 d^2 \quad (1)$$

$$A = \pi r^2 = 3.1416 r^2 \quad (2)$$

EXAMPLE 1.—What is the area of a circle whose radius is 14 inches ?

SOLUTION.—Applying formula 2,

$$A = 3.1416 \times 14^2 = 3.1416 \times 196 = 615.75 \text{ sq. in. Ans.}$$

EXAMPLE 2.—The contract for a brick warehouse provides that all openings over 4 feet wide shall be deducted; what must be deducted for a semicircular arched opening 4 feet 8 inches in diameter ?

SOLUTION.—Here $d = 4 \text{ ft. } 8 \text{ in.} = 4\frac{2}{3} \text{ ft.}$ Applying formula 1,

$$A = .7854 d^2 = .7854 \times (4\frac{2}{3})^2 = .7854 \times 21.78 = 17.11 \text{ sq. ft.}$$

The area of the half-circle is

$$17.11 \div 2 = 8.55 \text{ sq. ft., practically } 8\frac{1}{2} \text{ sq. ft. Ans.}$$

119. When the area of a circle is given, the diameter may be found by the following rule :

Rule.—*To find the diameter of a circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{4A}{\pi}}$$

EXAMPLE.—To supply a certain quantity of water, it is necessary to have a pipe with an area of 28 square inches ; what will be the diameter of the pipe ?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{28}{.7854}} = \sqrt{35.65} = 5.97 \text{ in.}$$

The nearest commercial size of pipe being 6 in., a pipe of that diameter would be chosen. Ans.

120. The area of a flat circular ring, Fig. 101, may be found as follows:

Rule.—*To find the area of a flat circular ring, subtract the area of the smaller circle from that of the larger.*

Let d = outside diameter;

d_1 = inside diameter;

A = area of ring.

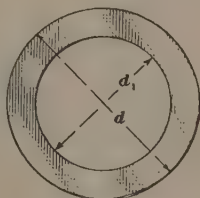


FIG. 101

Then, $A = .7854 d^2 - .7854 d_1^2 = .7854 (d^2 - d_1^2)$

EXAMPLE.—What is the sectional area of a brick chimney stack whose external and internal diameters are 6 feet 6 inches and 4 feet, respectively ?

SOLUTION.—Here $d = 6.5$ ft., and $d_1 = 4$ ft. Applying the formula,
 $A = .7854 \times (6.5^2 - 4^2) = .7854 \times 26.25 = 20.62 \text{ sq. ft.}$ Ans.

121. If one diameter and the area of the ring are known, the other diameter may be found by the following rule :

Rule.—*Compute the area of the circle whose diameter is given and add to or subtract from this result the area of the ring. The sum or difference thus found will be the area of the circle whose diameter is required.*

EXAMPLE.—The sectional area of a flat circular ring is 9.27 square inches, and the outside diameter is $4\frac{1}{4}$ inches. What is the inside diameter ?

SOLUTION.—The area of a circle $4\frac{1}{4}$ in. in diameter is $(4\frac{1}{4})^2 \times .7854 = 14.18$ sq. in. Subtracting the sectional area of the ring gives $14.18 - 9.27 = 4.91$ sq. in., as the area of the circle whose diameter is required. Applying the formula of Art. 119,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{4.91}{.7854}} = 2.5 \text{ in. Ans.}$$

EXAMPLES FOR PRACTICE

122. 1. Find the area of a square whose side is 5 feet 9 inches.

Ans. 33.062 sq. ft.

2. Find the area of a rhombus whose side is 12.5 feet, and whose altitude is 9.25 feet.

Ans. 115.62 sq. ft.

3. How many square feet are there in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end?

Ans. 15 sq. ft.

4. Find the distance round the inside of a circular brick well whose outside diameter is 24 feet 8 inches; the sides are 1 foot thick.

Ans. 71.21 ft.

5. The wheel of a carriage is recorded to have made 375 turns in going from one place to another; the diameter of the wheel is 3.5 feet; what is the distance between the two places?

Ans. 4,123.4 ft.

6. A circular column measures 45.5 inches round the outside; what is its diameter?

Ans. 14.48 in.

7. A belt covers an arc of 50° on a pulley whose diameter is 5 feet; what length of the belt is in contact with the pulley?

Ans. 2.18 ft.

8. Find the area of a triangle whose three sides are 13, 14, and 15 feet.

Ans. 84 sq. ft.

9. Find the area of a right triangle whose hypotenuse is 50 feet and one of whose sides is 40 feet.

Ans. 600 sq. ft.

10. The parallel sides of a trapezoid are 321.51 and 214.24 feet, and the perpendicular distance between them is 171.16 feet; what is the area of the trapezoid?

Ans. 45,849 sq. ft.

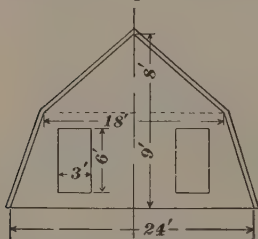


FIG. 102

11. One side of a room is 16 feet long; if the floor contains 240 square feet, what is the width of the room?

Ans. 15 ft.

12. (a) How many squares (100 square feet) of plain tiling, and (b) how many tiles, each $6\frac{1}{2}$ inches wide, and exposed 4 inches, will be required to cover the gable wall shown in Fig. 102, adding 10 per cent. for waste?

Ans. $\left\{ \begin{array}{l} (a) \ 2\frac{1}{4} \text{ squares} \\ (b) \ 1,372 \text{ tiles} \end{array} \right.$

SUGGESTION.—Gable consists of trapezoid and triangle. Find exposed area of each tile and number required per square of 14,400 square inches.

13. The cable of a suspension bridge measures 40 inches in circumference; find: (a) the diameter of the cable; (b) the area of the cross-section.

$$\text{Ans. } \begin{cases} (a) & 12.73 \text{ in.} \\ (b) & 127.27 \text{ sq. in.} \end{cases}$$

14. Referring to Fig. 98, if $AB = 40$ inches and $CD = 7\frac{1}{2}$ inches, find: (a) the length of the arc ACB ; (b) the radius of the arc.

$$\text{Ans. } \begin{cases} (a) & 43.63 \text{ in.} \\ (b) & 30.42 \text{ in.} \end{cases}$$

GEOMETRY AND MENSURATION

(PART 2)

MENSURATION OF SURFACES AND SOLIDS

PLANE SURFACES

(Continued)

SECTORS AND SEGMENTS OF CIRCLES

1. The area of a sector of a circle may be found by the following rule :

Rule.—To find the area of a sector of a circle, divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.

Let n = number of degrees in arc ;
 A = area of circle ;
 r = radius of circle ;
 a = area of sector.

$$\text{Then, } a = \frac{nA}{360} = .0087267 n r^2$$

EXAMPLE.—A circular window 4 feet in diameter is divided by radial ribs into twelve equal sections ; what is the area of each space ?

SOLUTION.—Here $n = \frac{360^\circ}{12} = 30^\circ$. Applying the formula,

$$a = \frac{nA}{360} = \frac{30 \times .7854 \times 4^2}{360} = 1.05 \text{ sq. ft. } \text{Ans.}$$

$$\text{Or, } a = .0087267 n r^2 = .0087267 \times 30 \times 4 = 1.05 \text{ sq. ft. } \text{Ans.}$$

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2. The area may also be found as follows:

Rule.—To find the area of a sector, multiply one-half of the length of the arc by the radius.

Let l = length of arc;
 r = radius of arc;
 a = area of sector.

Then, $a = \frac{1}{2} l r$

EXAMPLE.—In the sector $OACB$, Fig. 1, the radius $OA = OC$ of the arc is 6 inches, and the length of the chord AB is 7 inches; what is the area of the sector $OACB$?

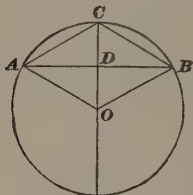


FIG. 1

SOLUTION.—In order to find the area, it will first be necessary to find the length of the arc, by applying the rule given in *Geometry and Mensuration*, Part 1; but before applying this rule, the height CD of the segment must be found. Since $AD = \frac{1}{2} AB = \frac{1}{2} \times 7 = 3.5$,

$$OD = \sqrt{OA^2 - AD^2} = \sqrt{6^2 - 3.5^2} = 4.87 \text{ in.}$$

Then, $CD = OC - OD = 6 - 4.87 = 1.13 \text{ in.}$

$$CB = \sqrt{CD^2 + DB^2} = \sqrt{1.13^2 + 3.5^2} = 3.68 \text{ in.};$$

$$\text{and arc } ACB = \frac{8 \times CB - AB}{3} = \frac{8 \times 3.68 - 7}{3} = 7.48 \text{ in.}$$

Now, applying the formula just given,

$$a = \frac{1}{2} l r = \frac{1}{2} \times 7.48 \times 6 = 22.44 \text{ sq. in. Ans.}$$

3. The area of a segment of a circle may be found by the following approximate rule, which is sufficiently accurate in most ordinary calculations:

Rule.—To two-thirds of the product of the height of the segment and the chord add the quotient obtained by dividing the cube of the height of the segment by twice the chord.

Let c = chord;
 h = height of segment;
 A = area of segment.

Then, $A = \frac{2}{3} c h + \frac{h^3}{2c}$

EXAMPLE.—What is the area of the upper pane of glass in the window shown in Fig. 2?

SOLUTION.—The glass may be divided into a rectangle 36 in. $\times$ 3 ft. 6 in. and a segment of a circle, the length of its chord being 3 ft. 6 in. and the height of segment 6.75 in. 3 ft. 6 in. = 42 in. The area of the rectangle is, therefore, $42 \times 36 = 1,512$ sq. in. Referring to the segment, $c = 3$ ft. 6 in. = 42 in., $h = 6.75$ in. Substituting in the formula,

$$\begin{aligned} A &= \frac{2}{3} \times 42 \times 6.75 + \frac{6.75^3}{2 \times 42} \\ &= 189 + \frac{307.55}{84} = 189 + 3.66 \\ &= 192.66 \text{ sq. in.} \end{aligned}$$

The entire area of the glass is

$$1,512 + 192.66 = 1,704.66 \text{ sq. in. Ans.}$$

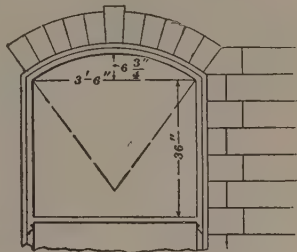


FIG. 2

4. The exact area of a segment of a circle may be found by the following rule :

Rule.—To find the area of a segment of a circle, find the area of the sector of which the segment is a part, and from this area subtract the area of the triangle formed by drawing radii to the extremities of the chord of the segment. The result is the area of the segment.

EXAMPLE.—What is the area of the segment ACB , Fig. 1 ?

SOLUTION.—The area of the segment is equal to the difference between the areas of the sector $OACB$ and the triangle OAB . The area of $OACB$ has been found, in the solution to the example in Art. 2, to be 22.44 sq. in. Area of triangle OAB is

$$\frac{1}{2} \times OD \times AB = \frac{1}{2} \times 4.87 \times 7 = 17.05 \text{ sq. in.}$$

Area of segment ACB is, therefore,

$$22.44 - 17.05 = 5.39 \text{ sq. in. Ans.}$$

THE ELLIPSE

5. An ellipse is a plane figure bounded by a curved line, to any point of which the sum of the distances from two fixed points within, called the *foci*, is equal to the sum of the distances from the foci to any other point on the perimeter.

In Fig. 3, let A and B be the foci, and let C and D be any two points on the perimeter. Then, according to the definition just stated,

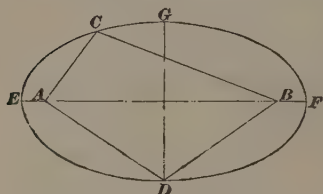


FIG. 3

$$AC + CB = AD + DB$$

Each of these sums is also equal to the **major axis**, this being the long diameter EF .

The short diameter GD is the

minor axis. The foci may be located from D or G as a centre, and radius $DA = DB = \frac{1}{2} EF$.

6. There is no simple and exact method of finding the perimeter of an ellipse. The following formula gives values very nearly exact:

Let C = perimeter;
 a = half the major axis;
 b = half the minor axis;

$$D = \text{the fraction } \frac{a - b}{a + b}$$

$$\text{Then, } C = \pi (a + b) \frac{64 - 3 D^4}{64 - 16 D^2}$$

EXAMPLE.—What is the perimeter of an ellipse whose axes are 12 inches and 8 inches long?

$$\text{SOLUTION.}—\text{In this example, } a = 6, b = 4, D = \frac{6 - 4}{6 + 4} = \frac{2}{10} = .2.$$

$$\text{Then, } C = \pi (6 + 4) \frac{64 - 3 (.2)^4}{64 - 16 (.2)^2}$$

The only difficulty in simplifying this expression will be in reducing the fraction $\frac{64 - 3 (.2)^4}{64 - 16 (.2)^2}$ to its simplest form. The numerator is simplified by raising .2 to the fourth power and subtracting three times the power from 64. Thus,

$$.2^4 = .2 \times .2 \times .2 \times .2 = .0016; 3 \times .0016 = .0048$$

Then, $64 - .0048 = 63.9952$. The denominator is simplified by multiplying the second power of .2 by 16 and subtracting the product from 64. Thus,

$$.2^2 = .2 \times .2 = .04; 16 \times .04 = .64; 64 - .64 = 63.36$$

The fraction therefore equals $\frac{63.9952}{63.36} = 1.01$. Consequently,

$$C = 3.1416 \times 10 \times 1.01 = 31.73 \text{ in. Ans.}$$

7. The following rule gives the method of finding the area of an ellipse:

Rule.—To find the area of an ellipse, multiply the product of its two diameters by $\cdot 7854$.

Let A = area of ellipse;
 D = long diameter;
 d = short diameter.

Then, $A = \frac{1}{4} \pi d D = \cdot 7854 d D$

EXAMPLE.—What is the area of the elliptic top of a table whose long and short diameters are 4 feet and 3 feet, respectively?

SOLUTION.—Here $D = 4$ ft. and $d = 3$ ft. Applying the formula,

$$A = \cdot 7854 D d = \cdot 7854 \times 4 \times 3 = 9.42 \text{ sq. ft. Ans.}$$

AREA OF ANY PLANE FIGURE

8. The area of any plane figure may be found as follows:

Rule.—To find the area of any plane figure, divide it into triangles, quadrilaterals, circles, or parts of circles, and ellipses; find the area of each part separately and add the partial areas.

EXAMPLE 1.—Fig. 4 (a) shows the cross-section of an I beam; find the area of this section, neglecting the rounding of the corners.

SOLUTION.—This section can be divided into a rectangle and four equal trapezoids, as shown in Fig. 4 (b). The area of the rectangle is $24 \times \cdot 50 = 12$ sq. in. The area of each of the trapezoids is

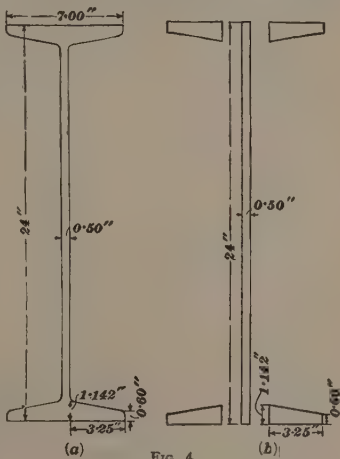
$$\frac{1}{2} (1.142 + \cdot 60) \times 3.25 = \left(\frac{1}{2} \times 1.742 \times 3.25\right) \text{ sq. in.}$$

Therefore, the sum of the areas of the four trapezoids is

$$4 \times \frac{1}{2} \times 1.742 \times 3.25 = 1.742 \times 6.5 = 11.323 \text{ sq. in.}$$

Hence, the area of the whole section is

$$12 \text{ sq. in.} + 11.323 \text{ sq. in.} = 23.323 \text{ sq. in. Ans.}$$



EXAMPLE 2.—Find the sectional area of the half arch and abutment shown in Fig. 5.

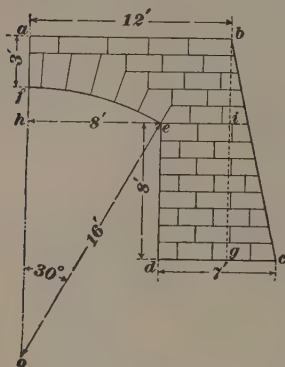


FIG. 5

SOLUTION.—Divide the section into parts, as *abih*, *eigd*, and *bgc*. Find

$$fh = fo - ho = 16 - \sqrt{16^2 - 8^2} = 2.14 \text{ ft.};$$

then, area of *abih* is

$$12 \times 5.14 = 61.68 \text{ sq. ft.};$$

area of *eigd* is

$$8 \times 4 = 32 \text{ sq. ft.};$$

area of *bgc* is

$$\frac{3 \times 13.14}{2} = 19.71 \text{ sq. ft.}$$

Adding these partial areas, the sum is 113.39 sq. ft. The area of sector *foe* is

$$.0087267 nr^2 = .0087267 \times 30 \times 16^2 = 67.02 \text{ sq. ft.}$$

The area of triangle *heo* is $\frac{1}{2} ho \times eh$;

$$ho = 16 - 2.14 = 13.86 \text{ ft.}$$

The area of *heo* is $\frac{13.86 \times 8}{2} = 55.44 \text{ sq. ft.}$

The area of part *feh* is $67.02 - 55.44 = 11.58 \text{ sq. ft.}$ Deducting 11.58 sq. ft. from 113.39 sq. ft., the net area is 101.81 sq. ft. Ans.

EXAMPLE 3.—A plot has the dimensions given in Fig. 6; what is the area in square feet?

SOLUTION.—The distances marked on Fig. 6 being measured, the partial areas are found to be:

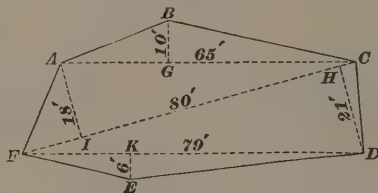


FIG. 6

$$\text{Area of } ABC = \frac{65 \times 10}{2} = 325 \text{ sq. ft.}$$

$$\text{Area of } FAC = \frac{80 \times 18}{2} = 720 \text{ sq. ft.}$$

$$\text{Area of } FCD = \frac{80 \times 21}{2} = 840 \text{ sq. ft.}$$

$$\text{Area of } FDE = \frac{79 \times 6}{2} = 237 \text{ sq. ft.}$$

The area of the plot is equal to the sum of these partial areas, or 2,122 sq. ft. Ans.

9. Area of an Irregular Figure.—If the figure is so irregular in shape that the ordinary rules cannot be easily applied, the area may be obtained as follows: Let Fig. 7 represent a drawing of such an area. Suppose it to be divided by equidistant parallel lines into strips, as ab , bc , etc.; then each of these small areas may be considered a trapezoid, and the whole area is the sum of the partial areas. From these considerations the following rule is obtained:

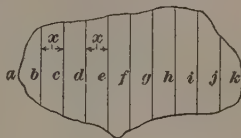


FIG. 7

Rule.—To find the area of any irregular figure, divide the figure into any number of strips by equidistant parallel lines. Measure the lengths of the lines; add together one-half the lengths of the end lines and the lengths of the intermediate lines, and multiply this sum by the distance between the lines.

Let a = length of one end line;
 k = length of other end line;
 b, c , etc. = lengths of intermediate lines;
 x = distance between parallel lines;
 A = area.

$$\text{Then, } A = \left(\frac{a + k}{2} + b + c + \text{etc.} + j \right) x$$

The shorter the distance x is, the more accurate will be the result.

EXAMPLE.—Fig. 8 is a plot of a small island whose area is to be determined. The parallel lines are 10 feet apart, and are of the lengths marked thereon. Required, the area of the island.



FIG. 8

hence, its length, or a , is 0; k is 4 ft. Applying the formula,

$$\begin{aligned} A &= \left(\frac{a + k}{2} + b + \text{etc.} \right) x \\ &= \left(\frac{0 + 4}{2} + 10 + 30 + 31 + 31 + 34 + 36 + 37 + 37 + 35 + 25 + 13 \right) \times 10 \\ &= 321 \times 10 = 3,210 \text{ sq. ft. } \text{Ans.} \end{aligned}$$

EXAMPLES FOR PRACTICE*

10. 1. How many laths 4 feet long and $1\frac{1}{2}$ inches wide, spaced $\frac{1}{4}$ inch apart, transversely, will be required for the walls of a room 14 ft. 6 in. $\times$ 10 ft. 9 in. $\times$ 8 ft. high, deducting two doors 36 in. $\times$ 7 ft. and two windows 2 ft. 6 in. $\times$ 5 ft. 6 in.? Add 5 per cent. for waste.

Ans. 601 laths

SUGGESTION.—Number of laths required for each square foot is
 $144 \div [(\text{width of 1 lath} + 1 \text{ space}) \times \text{length of lath in inches}]$

2. A hollow circular cast-iron column, having an external diameter of 8 inches, is to carry a load of 47,120 pounds, and must not be loaded more than 4,000 pounds per square inch; what will be the thickness of the metal ring?

Ans. $\frac{1}{2}$ in.

SUGGESTION.—Area of ring is total load $\div$ load per square inch. Area of smaller circle is area of 8 inch circle — area of ring; from this find smaller diameter.

3. If brickwork will safely carry 8 tons per square foot, what should be the size of the square base of the column given in the previous example?

Ans. 2.63 sq. ft., or 19.5 in. square, nearly

SUGGESTION.—Side of square is square root of area required. The weight of column is neglected.

4. A bar of iron 1 yard long and 1 inch square weighs 10 pounds; what will be the weight per yard of the I beam shown in Fig. 9? Disregard curved edges and corners. All the work should be done in fractions.

Ans. $41\frac{2}{3}$ lb. per yd.

SUGGESTION.—Note solution of example 1, Art. 8.

5. What is the length of the intrados $ABECD$ of the three-centred arch shown in Fig. 10?

Ans. 16.76 ft.

SUGGESTION.—Find lengths of arcs BEC , AB , and CD separately. $AB = CD$.

6. The space above the line AD , in Fig. 10, is to be covered with grille-work, costing 10 shillings per square foot; what will be the total cost?

Ans. £21 13s. 6d.

SUGGESTION.—Area required = area of three sectors less the area of triangle GOF . $GF = GO = FO$, because angles are all equal; then the lengths of three sides are known, to find the area of triangle.

7. The segment ABC of the window shown in Fig. 11 is to be filled with ornamental glass; what is the cost at 6 shillings per square foot?

Ans. 5s. $7\frac{1}{2}$ d.

8. What is the area of an elliptic window whose axes are 7 feet 4 inches and 4 feet?

Ans. 23.03 sq. ft.

* As stated in *Geometry and Mensuration*, Part I, two decimal places, as a general rule, are to be retained when decimal quantities occur in the solutions of examples.

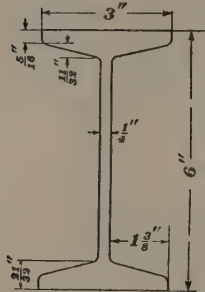


FIG. 9

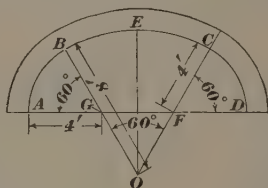


FIG. 10

9. Facing bricks are estimated at seven per square foot of external area. How many will be required for a building 40 ft. $\times$ 20 ft. and 20 feet high; with two chimneys 28 in. $\times$ 20 in. and 8 feet high; and one chimney 28 in. $\times$ 20 in. and 12 feet high? Deduct six window openings, 6 ft. 6 in. $\times$ 3 ft.; one double window, two openings, each 9 ft. $\times$ 2 ft. 8 in.; one front door, 9 ft. $\times$ 4 ft. 8 in.; one rear door, 9 ft. $\times$ 3 ft.; six window openings, 6 ft. $\times$ 3 ft.; and one double window, two openings, each 6 ft. $\times$ 2 ft. 8 in.

Ans. 15,750 bricks

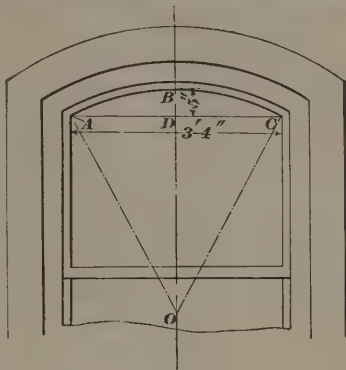


FIG. 11

10. (a) How many squares (100 square feet) of slating are there on a house 40 ft. $\times$ 25 ft. with a half-pitch roof, supposing the roof to extend beyond the walls 1 foot along the roof at eaves and ends? (b) How many slates, 10 inches wide and exposed $8\frac{1}{2}$ inches, will be required?

Ans. $\left\{ \begin{array}{l} (a) \text{ 15.69 squares} \\ (b) \text{ 2,658 slates} \end{array} \right.$

SUGGESTION.—Rise of roof is one-half width. Find exposed area of each slate and number required per square of 14,400 square inches.

REGULAR POLYGONS

11. The area of any regular polygon is found as follows:

Rule.—To find the area of a regular polygon, divide the figure into isosceles triangles, compute the area of one triangle, and multiply by the number of triangles. Or, multiply the perimeter by one-half the perpendicular distance from the centre to a side.

Let n = number of sides of polygon;

l = length of one side;

p = perimeter;

h = perpendicular distance from the centre to a side;

A = area of polygon.

$$\text{Then,} \quad A = \frac{n l h}{2}$$

$$\text{Or, since } n l = p, \quad A = \frac{p h}{2}$$

EXAMPLE.—Find the floor surface of an octagonal room, the length of whose sides is 5 feet. The distance between parallel sides is 12 feet 1 inch, nearly.

SOLUTION.—As the room is a regular octagon, $n = 8$, $l = 5$ ft., and $h = \frac{1}{2}$ of 12 ft. 1 in. $= \frac{1}{2}$ of 12.08 ft. $= 6.04$ ft. Applying the formula,

$$A = \frac{n l h}{2} = \frac{8 \times 5 \times 6.04}{2} = 120.8 \text{ sq. ft. Ans.}$$

12. It is frequently more convenient, however, in order to ascertain the areas of regular polygons, to make use of the principle that *the areas of similar figures are to each other as the squares of their like dimensions*. As regular polygons of

TABLE I
TABLE OF REGULAR POLYGONS

Number of Sides	Name	Area, When Side = 1	Radius of Circumscribed Circle, When Side = 1	Radius of Inscribed Circle, When Side = 1	One-Half of Angle at Vertex
A	B	C	D	E	F
3	Triangle	.4330	.5773	.2887	30°
4	Square	1	.7071	.5	45°
5	Pentagon	1.7205	.8507	.6882	54°
6	Hexagon	2.5981	1	.8660	60°
7	Heptagon	3.6339	1.1524	1.0383	64° 17'
8	Octagon	4.8284	1.3066	1.2071	67° 30'
9	Nonagon	6.1818	1.4619	1.3737	70°
10	Decagon	7.6942	1.6180	1.5388	72°
11	Undecagon	9.3656	1.7747	1.7028	73° 38'
12	Dodecagon	11.1962	1.9319	1.8660	75°

the same number of sides are similar figures, the area of any regular polygon may be obtained from the known area of a similar polygon by simple proportion. In Table I are given, in

column *C*, the areas of the regular polygons named in column *B*, when each side is unity or 1. Table I may also be used for finding various dimensions of regular polygons.

Rule I.—*To find the area of a regular polygon, multiply the square of the side by the number in column C standing opposite the name of the polygon in column B.*

EXAMPLE 1.—What is the area of a regular twelve-sided figure, each of its sides being 9 feet long ?

SOLUTION.—The figure is a dodecagon, and the corresponding number in column *C* is 11.1962.

$$\text{Area} = 9 \times 9 \times 11.1962 = 906.89 \text{ sq. ft.} \quad \text{Ans.}$$

Rule II.—*To find the side of a regular polygon when the area is known, divide the area by the number in column C corresponding to the name of the polygon and extract the square root of the result.*

EXAMPLE 2.—What is the side of a regular octagon that contains an area of 2,000 square feet ?

SOLUTION.—The number in column *C* to be used for an octagon is 4.8284. $2,000 \div 4.8284 = 414.22$. The square root of 414.22 is 20.35. Hence, 20.35 ft. is the length of the side. Ans.

13. In column *D* are given the radii of the circles that may be described about the regular polygons named, when each side is of unit length. In column *E* are given the radii of the circles that may be inscribed in polygons when the length of each side is 1. In column *F* are found the values of one-half the angle at each vertex, or the angles that should be used in making the bevel cuts for mouldings or material laid parallel to the sides, and jointed. The use of columns *D* and *E* will be made clear by the following rules and illustrative examples :

Rule I.—*To find the radius of the circumscribed circle when the side of a regular polygon is known, multiply the side by the number in column D opposite the name of the polygon.*

EXAMPLE 1.—It is required to set out a regular seven-sided figure, each side being 2 feet ; what is the radius of the circle whose circumference will pass through each corner of the figure ?

SOLUTION.—The number in column *D* opposite heptagon is 1.1524. Radius of circumscribed circle is

$$2 \times 1.1524 = 2.3048, \text{ or } 2.30 \text{ ft.} \quad \text{Ans.}$$

Rule II.—*To find the length of the side of a regular polygon that may be inscribed in a circle of known radius, divide the known radius by the number in column D opposite the name of the polygon.*

EXAMPLE 2.—What is the length of the side of an octagon that may be inscribed in a circle 12 feet in diameter?

SOLUTION.—Radius is $12 \div 2 = 6$ ft. Number in column D opposite octagon is 1.3066. Length of side is

$$6 \div 1.3066 = 4.592, \text{ or } 4.59 \text{ ft. Ans.}$$

Rule III.—*To find the radius of the inscribed circle when the side of a regular polygon is known, multiply the side by the number in column E opposite the name of the polygon.*

EXAMPLE 3.—It is desired to determine the perpendicular distance from the centre of a regular nonagon to each of its sides, when the length of each side is 5 feet.

SOLUTION.—The perpendicular distance from the centre of the polygon to its sides is equal to the radius of the circle that may be inscribed in the polygon. The number in column E opposite nonagon is 1.3737; hence, perpendicular distance is

$$5 \times 1.3737 = 6.8685, \text{ or } 6.87 \text{ ft. Ans.}$$

Rule IV.—*To find the side of a regular polygon when the radius of the inscribed circle is known, divide the radius by the corresponding number in column E.*

EXAMPLE 4.—What is the length of side of a regular octagon if the perpendicular distance between parallel sides is 8 feet?

SOLUTION.—The perpendicular distance between the parallel sides is twice the radius of the inscribed circle. Hence, radius $= 8 \div 2 = 4$. The number in column E opposite octagon is 1.2071. Length of side is

$$4 \div 1.2071 = 3.31 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

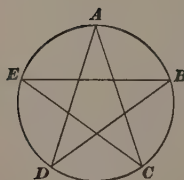


FIG. 12

14. 1. A five-pointed star is inscribed in the circle $ABCDE$, Fig. 12. The diameter of the circle is 4 feet 6 inches. What is the distance between consecutive points of the star measured on a straight line?

Ans. 2.65 ft.

2. Each side of an octagonal-shaped room is 12 feet 3 inches; find the number of square feet of flooring required for the room, not allowing for waste.

Ans. 724.56 sq. ft.

3. What is the weight for each foot in length of a hexagonal wrought-iron rod, the distance between parallel sides of the rod being 2 inches and the weight of a cubic inch of wrought iron being .28 pound?

Ans. 11.64 lb.

SUGGESTION.—To find volume, multiply area of end, in square inches, by 12.

4. In Fig. 13, $ABCD$ represents the end section of a piece of timber 6 inches square.

(a) What will be the side of a regular octagon that may be formed from the piece of timber, if the corners are cut off as shown? (b) What is the distance BE ?

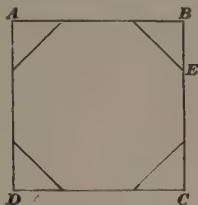


FIG. 13

Ans. $\begin{cases} (a) & 2.49 \text{ in.} \\ (b) & 1.76 \text{ in.} \end{cases}$

SOLIDS

DEFINITIONS

15. A **solid**, or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called the **faces**, and their intersections are called **edges**.

16. The **entire surface** of a solid is the area of the whole outside of the solid, including the ends. The **convex**, or **lateral surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

17. The **volume** of a solid is the space included between its bounding surfaces, and is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubic content** or **cubic contents** is frequently used.

THE PRISM AND CYLINDER

18. A **prism** is a solid whose ends are equal parallel polygons and whose sides are parallelograms. Prisms take their names from their bases. Thus, a triangular prism is one whose bases are triangles; a pentagonal prism is one whose bases are pentagons, etc.

19. A **parallelopiped** is a prism whose bases (ends) are parallelograms. See Fig. 14.

20. A cube is a parallelopiped whose faces and ends are squares. See Fig. 15. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the volume of a solid. Thus, if the unit of length is 1 inch, the unit of volume will be 1 *cubic inch*, this being the volume of a cube whose edges measure 1 inch.

If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard may be abbreviated to cu. in., cu. ft., and cu. yd., respectively.



FIG. 14

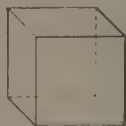


FIG. 15

21. A cylinder is a solid whose ends are parallel and equal *curved* figures, and whose cross-section is uniform throughout its length. A *circular cylinder* is one any section of which, perpendicular to the axis, is a circle. See Fig. 16. Unless otherwise expressed, the word *cylinder* always means a circular cylinder.



FIG. 16

22. A *right prism*, or *right cylinder*, is one whose centre line (axis) is perpendicular to its base.

23. The *altitude* of a prism or cylinder is the perpendicular distance between its two ends.

24. The area of the convex surface of any right prism or right cylinder may be found according to the following rule:

Rule.—To find the area of the convex surface of any right prism or right cylinder, multiply the perimeter of the base by the altitude.

Let p = perimeter of base;
 h = altitude;
 S = convex surface.

Then, $S = ph$

To find the entire area, add the areas of the two ends to the convex area.

EXAMPLE 1.—What quantity of 1-inch boarding is required for covering a wood-framed building 28 ft. $\times$ 20 ft. $\times$ 18 ft. high, including gables under a half-pitch roof, not allowing for openings, waste, or overlapping?

Express the result in square feet or feet superficial, which is commonly abbreviated to ft. super., or ft. sup.

SOLUTION.—The perimeter p of the house is $28 \times 2 + 20 \times 2 = 96$ ft. $h = 18$ ft.; hence, $S = 96 \times 18 = 1,728$ ft. super. Adding two gables, 20 ft. wide at base and (since the roof is half-pitch) 10 ft. high, or an area of 200 ft. super., the total is 1,928 ft. super. Ans.

EXAMPLE 2.—A circular iron chimney, 7 feet in diameter and 85 feet high, is to be painted; what will the painting cost at 3 shillings per super. yard (square yard) ?

SOLUTION.—The perimeter p of the base is $\pi d = 21.99$ ft., and $h = 85$ ft. The surface of the chimney is $21.99 \times 85 = 1,869.2$ ft. super. or 207.7 yd. super. The cost of painting at 3s. per yd. super. is therefore $207.7 \times 3 = 623.1$ s. = £31 3s., nearly. Ans.

EXAMPLE 3.—How many square feet of zinc will be required to line a refrigerator having interior dimensions of 4 ft. $\times$ 6 ft. $\times$ 8 ft. high ?

SOLUTION.—The area of the sides is

$$(4 \times 2 + 6 \times 2) \times 8 = 160 \text{ sq. ft.}$$

Add for floor and ceiling

$$(4 \times 6) \times 2 = 48 \text{ sq. ft.}; \text{ the total is } 208 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLE 4.—What will be the cost, at 18 pence per square yard, of plastering the walls and ceiling of an octagonal room, having sides 5 feet long and 11 feet high, the distance between the parallel sides being 12 feet ?

SOLUTION.—The area of the sides is $8 (5 \times 11) = 440$ sq. ft.; the area of the ceiling is one-half the perimeter multiplied by one-half the distance between the parallel sides, or

$$\frac{8 \times 5 \times 6}{2}$$

120 sq. ft.; the total area

is 560 sq. ft., or 62.22 sq. yd. At 1s. 6d. per sq. yd., the cost is £4 13s. 4d.

Ans.

25. The volume of a rectangular solid may be found as follows :

Rule.—To find the volume of a rectangular solid, multiply together the length, breadth, and height.

Let l = length;
 b = breadth;
 h = height;
 V = volume.

Then, $V = lbh$

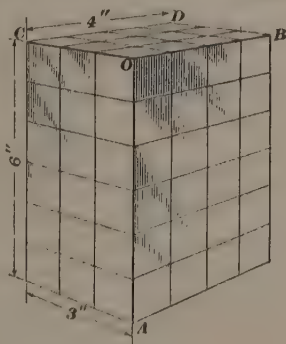


FIG. 17

EXAMPLE.—Find the volume of the rectangular block shown in Fig. 17.

SOLUTION.—The length is 4 in., the breadth is 3 in., and the height is 6 in. The volume therefore equals

$$4 \times 3 \times 6 = 72 \text{ cu. in.} \quad \text{Ans.}$$

The block may be divided into six pieces, each 4 in. long, 3 in. wide, and 1 in. thick. Each of these pieces may be cut up into twelve 1-in. cubes. There will, therefore, be $6 \times 12 = 72$ one-inch cubes, or 72 cu. in. in the entire block.

26. The volume of a right prism, or cylinder, may be found by the following rule:

Rule.—*To find the volume of a right prism, or cylinder, multiply the area of the base by the altitude.*

Let a = area of base;
 h = altitude;
 V = volume.

Then, $V = ah$

If the volume and area of the base are given, the altitude is $\frac{V}{a}$. If the cylinder, or prism, is hollow, the base is equal to the area of the cross-section of the material. If the given prism is a cube, the three dimensions are all equal, and the volume is equal to the cube of one of the edges.

EXAMPLE 1.—If brickwork averages 21 bricks per cubic foot, how many bricks will there be in a pier 18 inches square and 6 feet high?

SOLUTION.—The sectional area or a is $1.5 \times 1.5 = 2.25$ sq. ft.; $h = 6$ ft.; hence,

$$V = ah = 2.25 \times 6 = 13.5 \text{ cu. ft.}$$

At 21 bricks per cu. ft., the number of bricks in the pier is

$$13.5 \times 21 = 283.5. \quad \text{Ans.}$$

EXAMPLE 2.—If cast iron weighs .26 pound per cubic inch, what length of sash weight will be necessary to balance a window weighing 8 pounds, the diameter of the cylindrical weight being $1\frac{1}{2}$ inches?

SOLUTION.—As there are two weights, the weight of each is 4 lb. The number of cubic inches required, or V , is $4 \div .26 = 15.38$. The area a of a $1\frac{1}{2}$ -in. circle is 1.77 sq. in.; hence, h , the length, is

$$\frac{V}{a} = \frac{15.38}{1.77} = 8.69 \text{ in., practically } 8\frac{3}{4} \text{ in.} \quad \text{Ans.}$$

27. Measurement of Timber.—In measuring sawn timber, a general unit of measurement is the **St. Petersburg**, or **Petersburg**, standard of 120 pieces, each 12 feet by 11 inches by $1\frac{1}{2}$ inches, or 165 cubic feet.

Rule.—*To ascertain the number of running feet of any scantling required to make a St. Petersburg standard, multiply 165 by 144 and divide the product by the sectional area of the scantling, the quotient will then be the number of feet required.*

EXAMPLE.—How many running feet of scantling 11 in. $\times$ $2\frac{1}{2}$ in. are contained in a St. Petersburg standard?

SOLUTION.—Area of cross-section = 11 in. $\times$ $2\frac{1}{2}$ in. = $27\frac{1}{2}$ sq. in. = 27.5 sq. in. The number of running feet will be
 $(165 \times 144 = 23,760) \div 27.5 = 864$ running feet. Ans.

28. Board Measure.—In Canada and America sawn timber is termed **lumber**, and the unit of measurement is the **board foot**, which is equal to the contents of a board 1 foot square and 1 inch thick. A board foot is, therefore, equal to one-twelfth of a cubic foot. Boards less than 1 inch thick are usually reckoned as though the thickness were 1 inch.

Rule I.—*To find the number of board feet in a piece of lumber, multiply together the length, in feet, the width, in feet, and the thickness, in inches. Or,*

Rule II.—*Multiply together the length, in feet, the width, in inches, and the thickness, in inches, and divide the product by 12.*

EXAMPLE 1.—How many board feet are there in a plank 10 feet 6 inches long, 15 inches wide, and 3 inches thick?

SOLUTION.—Length = 10 ft. 6 in. = $10\frac{6}{12}$ ft. = $10\frac{1}{2}$ ft. = 10.5 ft., width = $\frac{15}{12}$ ft. = 1.25 ft. The number of feet equals
 $10.5 \times 1.25 \times 3 = 39.375$, or 39.4

The result is, therefore, 39.4 ft. B.M. (board measure). Ans.

EXAMPLE 2.—Find the number of board feet in twenty pieces $\frac{1}{2}$ inch thick, $5\frac{1}{2}$ inches wide, and 10 feet 9 inches long.

SOLUTION.—Since the thickness is less than 1 in., the boards are considered to be 1 in. thick when finding the amount of material in them. The calculation can be more easily performed in this case by using the second rule. Length = $10\frac{9}{12}$ ft. = 10.75 ft., width = $5\frac{1}{2}$ in. = 5.5 in.

$$\frac{20 \times 10.75 \times 5.5 \times 1}{12} = \frac{1,182.5}{12} = 98.54 \text{ ft. B.M. Ans.}$$

THE PYRAMID AND CONE

29. A pyramid is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex**. See Fig. 18.

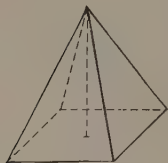


FIG. 18

30. A cone is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the *vertex*. See Fig. 19.

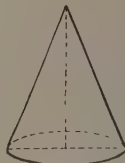


FIG. 19

31. A right pyramid or cone is one whose axis is perpendicular to the base.

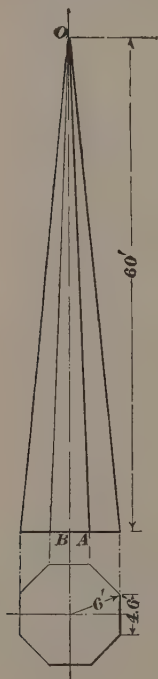


FIG. 20

32. The altitude of a pyramid or cone is the perpendicular distance from the vertex to the base.

33. The slant height of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

34. The convex area of a right pyramid or cone may be found as follows:

Rule.—To find the convex area of a right pyramid or cone, multiply the perimeter of the base by one-half of the slant height.

Let p = perimeter of base;
 s = slant height;
 A = convex area.

$$\text{Then, } A = \frac{ps}{2}$$

EXAMPLE.—An octagonal church steeple is 60 feet high. The outer radius of the base (see Fig. 20) is 6 feet, and the length of one of the sides of the base is 4.6 feet. Compute the convex surface.

SOLUTION.—To find the slant height, the length OA along the hips is first found. This is the hypotenuse

of a right triangle of which the altitude is the height of the tower, or 60 ft.; and the base is the radius, or 6 ft. Hence, $OA = \sqrt{60^2 + 6^2} = 60.3$ ft. Now, in the triangle OAB , OA is 60.3 ft. and AB is $4.6 \div 2 = 2.3$ ft. The slant height OB is, therefore, $\sqrt{60.3^2 - 2.3^2} = 60.3$ ft., nearly. The perimeter of the base is $4.6 \times 8 = 36.8$ ft. Applying the formula,

$$A = \frac{ps}{2} = \frac{36.8 \times 60.3}{2} = 1,109.5 \text{ sq. ft. Ans.}$$

35. The following rule states how to find the volume of a pyramid or cone :

Rule.—To find the volume of a pyramid or cone, multiply the area of the base by one-third of the altitude.

Let $a =$ area of base ;
 $h =$ altitude ;
 $V =$ volume.

Then, $V = \frac{a h}{3}$

EXAMPLE 1.—The granite capstone of a monument is a square pyramid having a base 4 feet square, and is 6 feet high. If granite weighs 170 pounds per cubic foot, what will be the cost of carriage on the piece at 6 pence per hundredweight ?

SOLUTION.—The area a of the base is $4 \times 4 = 16$ sq. ft., and the altitude h is 6 ft. Applying the formula,

$$V = \frac{a h}{3} = \frac{16 \times 6}{3} = 32 \text{ cu. ft.}$$

At 170 lb. per cu. ft., the weight is $32 \times 170 = 5,440$ lb. = 48.57 cwt. The cost of carriage is, therefore,

$$48.57 \times \frac{1}{2} = 24.29 \text{ shillings} = \text{£}1 \text{ 4s. } 3\frac{1}{2}\text{d. Ans.}$$

EXAMPLE 2.—If in the preceding example the capstone were copical, having a base 4 feet in diameter and a height of 6 feet, how much less would it weigh than the pyramidal stone ?

SOLUTION.—The area of the base is $.7854 d^2 = 12.57$ sq. ft. Using the formula, the volume is

$$\frac{12.57 \times 6}{3} = 25.14 \text{ cu. ft.}$$

and the weight at 170 lb. per cu. ft. is 4,274 lb., nearly. As the pyramidal capstone weighs 5,440 lb., the difference in weight is

$$5,440 - 4,274 = 1,166 \text{ lb. Ans.}$$

36. Frustum of Pyramid or Cone.—If a pyramid or cone is cut by a plane parallel to the base, so as to form two parts, the

lower part is called the frustum of the pyramid or cone. See Figs. 21 and 22.

The upper end of the frustum of a pyramid or cone is called the *upper base*, and the lower end the *lower base*. The altitude of a frustum is the perpendicular distance between the bases.

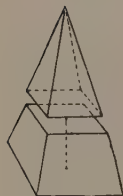


FIG. 21

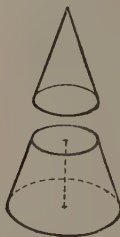


FIG. 22

37. The convex area of a frustum of a right pyramid or right cone is found as follows :

Rule.—*To find the convex area of a frustum of a right pyramid or right cone, multiply one-half the sum of the perimeters of the bases by the slant height.*

Let P = perimeter of lower base ;
 p = perimeter of upper base ;
 s = slant height ;
 A = convex area.

Then,
$$A = \frac{(P + p)s}{2}$$

To find the entire area, add to the convex area the areas of the two bases.

EXAMPLE.—A square mansard-tower roof has the dimensions shown in Fig. 23. To find the area of slating required to cover the roof, the convex area is required. What is this convex area ?

SOLUTION.—The tower has the form of a frustum of a pyramid, 6 ft. 6 in. square at the base and 3 ft. 6 in. square at the top, 6 ft. being the altitude. 6 ft. 6 in. = $6\frac{1}{2}$ ft. and 3 ft. 6 in. = $3\frac{1}{2}$ ft. The perimeter of the lower base is $4 \times 6\frac{1}{2} = 26$ ft., and that of the upper base is $4 \times 3\frac{1}{2} = 14$ ft. The slant height of the frustum is the hypotenuse of the right triangle whose height is equal to the altitude of the frustum AC , or 6 ft., and whose base is equal to BC , or $\frac{1}{2}(6\frac{1}{2} - 3\frac{1}{2}) = 1\frac{1}{2}$ ft.

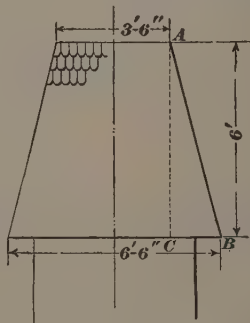


FIG. 23

The slant height is, therefore,

$$\sqrt{(1\frac{1}{2})^2 + 6^2} = 6.18 \text{ ft.}$$

Applying the formula,

$$A = \frac{(P + p)s}{2} = \frac{(26 + 14) \times 6.18}{2} = 123.6 \text{ sq. ft. Ans.}$$

38. The following rule gives a method of finding the volume of the frustum of a pyramid or cone :

Rule.—To find the volume of the frustum of a pyramid or cone, add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases ; multiply this sum by one-third of the altitude.

Let A = area of lower base ;
 a = area of upper base ;
 h = altitude ;
 V = volume.

Then,
$$V = \frac{h}{3} (A + a + \sqrt{A \times a})$$

EXAMPLE.—A marble monument is $2\frac{1}{2}$ feet square at the base, 1 foot square at the top, and 16 feet high ; if marble weighs 160 pounds per cubic foot, what is the weight of the stone ?

SOLUTION.—The area of the lower base is $2\frac{1}{2} \times 2\frac{1}{2} = 6.25$ sq. ft. ; that of the upper base is 1 sq. ft. Applying the formula,

$$V = \frac{h}{3} (A + a + \sqrt{A \times a}) = \frac{16}{3} (6.25 + 1 + \sqrt{6.25 \times 1}) = 51.97 \text{ cu. ft.}$$

At 160 lb. per cu. ft., the weight is $51.97 \times 160 = 8,315$ lb. Ans.

39. When the bases of the frustum are square, let d and D represent the sides of the bases, respectively, and V the volume of the frustum ; then,

$$V = \frac{h}{3} (d^2 + dD + D^2)$$

When the bases are circular, let d and D represent the respective diameters ; then,

$$V = \frac{h}{3} (d^2 + dD + D^2) \times \frac{\pi}{4}$$

EXAMPLE.—Find the volume of a pail having the form of a frustum of a cone, the upper and lower diameters being 10 and 8 inches, respectively, and the height of pail $9\frac{1}{2}$ inches.

SOLUTION.—

$$\begin{aligned} V &= \frac{9.5}{3} (8^2 + 8 \times 10 + 10^2) \times .7854 \\ &= \frac{9.5}{3} \times 244 \times .7854 = 606.85 \text{ cu. in. Ans.} \end{aligned}$$

THE WEDGE

40. A wedge is a solid having plane surfaces, of which the base is a parallelogram, the ends are triangles, and the sides are quadrilaterals meeting in a line parallel to the sides of the base. See Fig. 24.

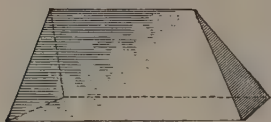


FIG. 24

41. The rule for finding the volume of a wedge is as follows :

Rule.—To find the volume of a wedge, multiply together the width of the base, the perpendicular distance from the base to the edge, and the sum of the lengths of the three parallel edges ; divide the product by 6.

Let w = width of base ;

h = perpendicular distance from base to edge ;

s = sum of lengths of the three parallel edges ;

V = volume.

Then,
$$V = \frac{w h s}{6}$$

If the base is a rectangle, and the triangular ends are parallel, the wedge becomes a triangular prism, and the rule for prisms may be used.

EXAMPLE.—(a) What is the volume of a steel wedge 8 inches long and 3 in. $\times$ $1\frac{1}{2}$ in. at the head, and (b) assuming that steel weighs .28 pound per cubic inch, what is its weight ?

SOLUTION.—(a) Here, w is $1\frac{1}{2}$ in., h is 8 in., s is $3 + 3 + 3 = 9$ in.

Then,
$$V = \frac{1.5 \times 8 \times 9}{6} = 18 \text{ cu. in.}$$

Or, as this wedge is a prism, the volume = area of end $\times$ length of edge; area of triangle is $\frac{1.5 \times 8}{2} = 6$ sq. in. Multiplying by the length of the edge, 3 in., the volume is 18 cu. in. Ans.

(b) The weight at .28 lb. per cu. in. is

$$.28 \times 18 = 5.04 \text{ lb. Ans.}$$

THE PRISMOID

42. If any solid is cut into two parts, so that the surfaces thus formed are flat, such a surface is called a **plane section** of the solid. Plane sections are divided into three classes: *longitudinal sections*, *cross-sections*, and *right sections*. A **longitudinal section** is any plane section taken lengthwise through the solid; any other plane section is called a **cross-section**. If a plane section of a solid is perpendicular to the centre line of the solid, the section is called a **right section**. Any longitudinal section of a cylinder is a rectangle. A right section of a cube is a square; of a cylinder or cone, a circle; an oblique cross-section of a cylinder is an ellipse.

43. A **prismoid** is a solid whose bases are parallel and whose faces are triangles or quadrilaterals joining the bases. Thus, the solid shown in Fig. 25 is a prismoid; its bases are the pentagon $ABCDE$ and the quadrilateral $Fghi$, which are parallel; and its faces are the triangle GBC and the quadrilaterals $GCDH$, $HDEI$, $IEAF$, and $FABG$.

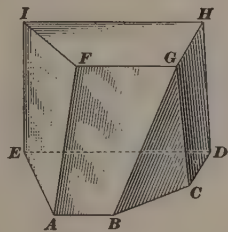


FIG. 25

44. The **altitude** of a prismoid is the perpendicular distance between the bases or parallel faces, which are commonly termed the **end sections**.

45. A prismoid is also defined as a solid having two parallel end faces, and composed of any combination of prisms, wedges, and pyramids, whose common altitude is the perpendicular distance between the parallel faces.

46. The middle section of a prismoid is the polygon formed by a plane parallel to the bases, and cutting the prismoid at equal distances from the two bases or end sections. Thus, polygon $PQRS$ is the middle section of the prismoid shown in Fig. 26.

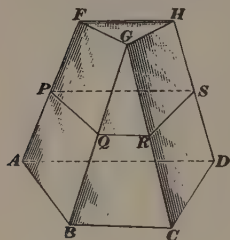


FIG. 26

Any dimension of the middle section of a prismoid may be taken equal to one-half the sum of the corresponding dimensions of the two end sections or bases. Thus, in Fig. 26, $PQ = \frac{1}{2}(AB + FG)$, $QR = \frac{1}{2}BC$, $RS = \frac{1}{2}(GH + CD)$, and $SP = \frac{1}{2}(HF + DA)$.

47. The area of the middle section of a prismoid may be measured directly, or calculated from its dimensions as determined from the dimensions of the end sections. It is *not*, in general, equal to one-half the sum of the areas of the bases.

48. The volume of a prismoid may be found by the following rule :

Rule.—*To find the volume of a prismoid, add together the areas of the two parallel ends, and four times the area of a parallel section midway between them ; multiply the sum by one-sixth of the perpendicular distance between the parallel ends.*

Let A = area of one end ;
 a = area of other end ;
 M = area of middle section :
 h = distance between ends ;
 V = volume.

Then, $V = \frac{h}{6} (A + a + 4M)$

EXAMPLE.—What is the volume, in cubic yards, of a masonry pier, uniformly tapered, 30 feet high, the upper and lower rectangular bases being, respectively, 24 ft. $\times$ 7 ft. and 30 ft. $\times$ 11 ft. ?

SOLUTION.—The area of the upper base is $24 \times 7 = 168$ sq. ft. The area of the lower base is $30 \times 11 = 330$ sq. ft. The dimensions of the

middle section are the half sums of the corresponding dimensions of the bases; therefore, the length is

$$\frac{30 + 24}{2} = 27 \text{ ft.},$$

and the width is

$$\frac{11 + 7}{2} = 9 \text{ ft.}$$

The area of the middle section is $27 \times 9 = 243$ sq. ft. Applying the prismatical formula, the volume is

$$V = \frac{30}{6} (330 + 168 + 4 \times 243) = 7,350 \text{ cu. ft.} = (7,350 \div 27) \text{ cu. yd.} \\ = 272\frac{2}{3} \text{ cu. yd. Ans.}$$

49. This formula, which is called the **prismoidal formula**, is of very extended application and may be used to calculate the volume of a prism, cylinder, pyramid, cone, frustum of pyramid or cone, wedge, sphere, or segment of a sphere, in addition to the irregularly shaped bodies to which it is usually applied. One end area of a pyramid, cone, or wedge is 0; and the end areas of a sphere are both 0. The area of the middle section of a prism is the same as the area of either base; the area of the middle section of a wedge, when the three parallel edges are of the same length, is equal to one-half the area of the base; the area of the middle section of a pyramid is equal to one-fourth the area of the base.

EXAMPLE.—The upper and lower bases of the frustum of a right cone are respectively 30 inches and 14 inches in diameter, and the perpendicular distance between them is 36 inches; required, the volume of the conical frustum.

SOLUTION.—The diameter of the middle section is the mean, or average, of the diameters of the ends, or

$$\frac{30 + 14}{2} = 22 \text{ in.}$$

Area of lower base is

$$A = .7854 \times 30^2 = 706.86 \text{ sq. in.}$$

Area of upper base is

$$a = .7854 \times 14^2 = 153.94 \text{ sq. in.}$$

Area of middle section is

$$M = .7854 \times 22^2 = 380.13 \text{ sq. in.}$$

Using the prismoidal formula,

$$V = \frac{36}{6} (706.86 + 153.94 + 4 \times 380.13) \\ = 14,287.92 \text{ cu. in.} = 8.27 \text{ cu. ft. Ans.}$$

THE SPHERE

50. A sphere, or ball, Fig. 27, is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within called the *centre*.

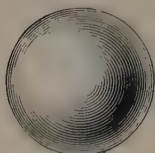


FIG. 27

51. The following rule states the method of finding the area of the surface of a sphere:

Rule.—*To find the area of the surface of a sphere, multiply the square of the diameter by 3.1416.*

Let d = diameter of sphere;
 S = surface.

Then,
$$S = \pi d^2 = 3.1416 d^2$$

EXAMPLE.—A ball on a flagstaff is 10 inches in diameter and is to be gilded; deducting 20 square inches for space covered by attachment to pole, how many books of gold leaf, each containing 25 leaves $3\frac{3}{8}$ inches square, will be required for gilding it, not allowing for waste?

SOLUTION.—Applying the formula, $S = \pi d^2 = 3.1416 \times 10 \times 10 = 314.16$ sq. in. Deducting 20 sq. in., the net surface is 294.16 sq. in. Each gold leaf has an area of $3\frac{3}{8}$ in. $\times$ $3\frac{3}{8}$ in., or 11.4 sq. in., nearly; hence, for 294.16 sq. in., there will be needed $294.16 \div 11.4 = 25.8$ leaves, or 1 book and 1 leaf. Ans.

52. The volume of a sphere may be found when the diameter is known by applying the following rule:

Rule.—*To find the volume of a sphere, multiply the cube of the diameter by .5236.*

Let d = diameter;
 V = volume.

Then,
$$V = \frac{\pi}{6} d^3 = .5236 d^3$$

Since $\frac{\pi d^3}{6} = \frac{1}{6} \times \pi d^2 \times d$, the volume of a sphere is equal

to one-sixth of the surface multiplied by the diameter.

EXAMPLE.—What is the weight of a metal ball 6 inches in diameter, if the metal weighs .26 pound per cubic inch?

SOLUTION.—Applying the formula,

$$V = .5236 d^3 = .5236 \times 216 = 113.1 \text{ cu. in.}$$

The weight is therefore $113.1 \times .26 = 29.41$ lb. Ans.

53. The volume of a spherical shell is equal to the difference in volume between two spheres having respectively the outer and inner diameters of the shell.

54. The diameter of a sphere of known volume may be found by the following rule :

Rule.—To find the diameter of a sphere of known volume, divide the volume by $\cdot 5236$ and extract the cube root of the quotient.

$$d = \sqrt[3]{\frac{V}{\cdot 5236}}$$

EXAMPLE.—A metal ball weighing 25 pounds is required for a certain purpose ; if the metal used weighs $\cdot 26$ pound per cubic inch, what will be the diameter of the ball ?

SOLUTION.—The volume of the ball will be $25 \div \cdot 26 = 96\cdot 1$ cu. in. Using the formula,

$$d = \sqrt[3]{\frac{V}{\cdot 5236}} = \sqrt[3]{\frac{96\cdot 1}{\cdot 5236}} = \sqrt[3]{183\cdot 5} = 5\cdot 68 \text{ in.} = 5\frac{11}{16} \text{ in., nearly.}$$

Ans.

SYMMETRICAL AND SIMILAR FIGURES

55. An axis of symmetry is any line so drawn that, if the part of the figure on one side of the line is folded over on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 28, if the upper semicircle is folded over on the diameter CD , it will coincide exactly with the lower semicircle ; also, if the part on the right of the diameter AB is folded over on AB , it will coincide exactly with the part on the left of this line.

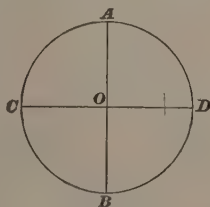


FIG. 28



FIG. 29

It is evident from what has just been stated that a circle may have any number of axes of symmetry. In certain cases, however, a figure may be symmetrical with regard to only one axis. Thus, the isosceles triangle ABC , Fig. 29, is symmetrical with regard to the axis BD , because the part BCD would coincide with the part BAD if folded over on the line

BD ; but no other axis of symmetry could be drawn. A rectangle has two axes of symmetry at right angles to each other. A regular hexagon has six axes of symmetry.

56. Similar figures are those that are alike in form. As in the case of polygons, which have been considered, two figures, to be similar, must have their corresponding sides in proportion, and the angles of one equal to the corresponding angles of the other. Circles are, of course, similar figures.

57. The areas of similar figures are to each other as the *squares* of their corresponding dimensions. Thus, if an area contains, for example, 5 square feet, a similar area with dimensions twice as great will contain 2^2 , or 4, times 5 square feet. This principle has already been considered in connection with the areas of regular polygons. Its use often saves considerable labour in determining the areas of other similar figures, as, for instance, the end areas of frustums of pyramids, cones, etc.

EXAMPLE.—Required, the volume of the frustum of a pyramid, the bases of which are equilateral triangles, one with sides 6 feet in length, the other with sides 2 feet in length. The altitude of the frustum is 8 feet.

SOLUTION.—The area of the lower triangular base may be found from the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{6+6+6}{2} = 9$$

where

and the factors $(s-a)$, etc., are each $9-6=3$. Hence,

$$A = \sqrt{9 \times 3 \times 3 \times 3} = 15.59 \text{ sq. ft.}$$

Since the ends are similar triangles, their areas are proportional to the squares of the sides; or, denoting the area of the upper base by a ,

$$a : 15.59 = 2^2 : 6^2$$

$$\text{Hence, } a = 15.59 \times \frac{2^2}{6^2} = 15.59 \times \frac{1}{9} = 1.73 \text{ sq. ft.}$$

A section midway between the bases is evidently also an equilateral triangle with sides $\frac{6+2}{2} = 4$ ft. in length; hence, its area M may be obtained from the proportion

$$M : 15.59 = 4^2 : 6^2$$

$$\text{or } M = 15.59 \times \frac{4^2}{6^2} = 6.93 \text{ sq. ft.}$$

Now, using the prismoidal formula,

$$V = \frac{8}{3} (15.59 + 1.73 + 4 \times 6.93) = 60.05 \text{ cu. ft. Ans.}$$

NOTE.—If a table of areas of regular polygons is at hand, the areas in this example can be more readily calculated by its use than by the method here explained.

58. The cubical contents and weights of similar solids are to each other as the *cubes* of any one dimension.

EXAMPLE.—If a cast-iron ball 9 inches in diameter weighs 100 pounds, what would a ball 15 inches in diameter weigh?

SOLUTION.— $100 : x = 9^3 : 15^3$, or

$$x = \frac{100 \times 3,375}{729} = 462.96 \text{ lb.}$$

The weight of the larger ball is, therefore, 462.96 lb. Ans.

EXAMPLES FOR PRACTICE

59. 1. What is the weight of the cast-iron lintel shown in Fig. 30, estimating iron at .26 pound per cubic inch? Ans. 201.08 lb.

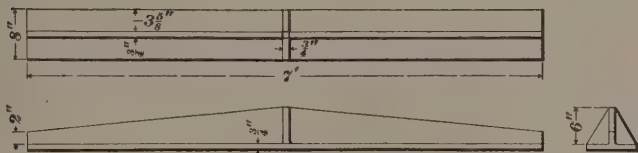


FIG. 30

SUGGESTION.—The lintel consists of a rectangular base, a longitudinal rib, and two triangular cross-ribs.

2. Calculate (a) the number of cubic feet of timber in the following bill of material; (b) the cost at £15 per St. Petersburg standard of 165 cubic feet.

- 23 pieces 10 in. $\times$ 3 in. $\times$ 18 ft. 6 in. long
- 3 pieces 10 in. $\times$ 3 in. $\times$ 15 ft. 0 in. long
- 3 pieces 10 in. $\times$ 3 in. $\times$ 12 ft. 0 in. long
- 34 pieces 6 in. $\times$ 3 in. $\times$ 14 ft. 0 in. long
- 4 pieces 6 in. $\times$ 3 in. $\times$ 6 ft. 0 in. long
- 38 pieces 4 in. $\times$ 3 in. $\times$ 5 ft. 6 in. long
- 2 pieces 6 in. $\times$ 3 in. $\times$ 10 ft. 6 in. long
- 5 pieces 8 in. $\times$ 3 in. $\times$ 18 ft. 6 in. long
- 56 pieces 4 in. $\times$ 2 in. $\times$ 9 ft. 9 in. long
- 81 pieces 4 in. $\times$ 2 in. $\times$ 9 ft. 3 in. long
- 42 pieces 4 in. $\times$ 2 in. $\times$ 1 ft. 2 in. long

Ans. $\left\{ \begin{array}{l} (a) \text{ 278.16 cu. ft.} \\ (b) \text{ £25 5s. 8}\frac{1}{2}\text{d.} \end{array} \right.$

3. What is the weight of the boiler front shown in Fig. 31 if the metal is $1\frac{1}{4}$ inches thick, and weighs .26 pound per cubic inch? Ans. 1,638 lb.

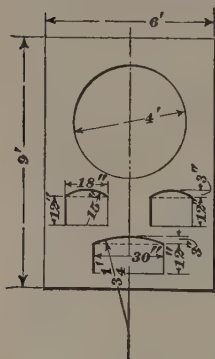


FIG. 31

SUGGESTION.—First find net area of surface; door openings consist of rectangular portion + circular segment whose chord and height are known.

4. Calculate the cost of digging and lining a well 24 feet deep and $3\frac{3}{4}$ feet clear diameter, the lining to be one brick, or 9 inches, thick, laid without mortar. The excavation is to be $5\frac{1}{4}$ feet in diameter. The digging is to cost 2s. 6d. per cubic yard, and the brickwork $10\frac{1}{2}$ d. per cubic foot provided and laid.

Ans. £13 10s. 9d.

5. Find the diameter of the balls of a 10-pound dumbbell, the cylindrical handle of which is $4\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches in diameter. Cast iron weighs, say, .26 pound per cubic inch. Disregard the small volumes at ends of handle, which occur twice.

Ans. 3.16 in. = $3\frac{1}{8}$ + in.

SUGGESTION.—Find contents of handle; one-half of remainder is volume of each ball.

6. A window weighing 10 pounds is to be balanced by two $1\frac{1}{2}$ -inch (inside diameter) lead-pipe weights. If the metal is $\frac{1}{4}$ inch thick, and lead weighs .41 pound per cubic inch, what will be the length of each piece? Ans. 8.9 in., say 9 in.

SUGGESTION.—Find number of cubic inches in each weight. Length required = volume ÷ area of ring.

7. A rectangular shop 40 ft. long × 28 ft. wide and 18 feet high, with a gable at each end 8 feet 8 inches high, is to be built of brick. The lower walls are $13\frac{1}{2}$ inches, and the gables 9 inches, in thickness. Deduct twelve windows each 6 ft. × 4 ft., and four doors each 7 ft. × 4 ft., all in $13\frac{1}{2}$ -inch wall. Estimating seven bricks to each square foot of wall $4\frac{1}{2}$ inches thick, and counting the four quoins or corners of the building twice, how many bricks will be required, with 5 per cent. allowance for waste, etc.?

Ans. 48,726 bricks

SUGGESTION.—Find areas and not volumes.

8. (a) What will be the weight of the square granite shaft shown in Fig. 32, estimating the stone at 170 pounds per cubic foot? (b) Allowing that the soil on which it is to be placed will support a load of 4,000 pounds per square foot, what will be the size of foundation required?

Ans. (a) 168,640 lb. = 75.3 T.
(b) 6.5 ft. square, nearly



FIG. 32

QUERY.—What two solids make up the monument?

9. What is the weight per foot of the **Z**-bar stanchion shown in Fig. 33, taking the weight of steel at .283 pound per cubic inch? Neglect the rounding of the edges of the **Z** bars, but allow for the fillets, as at *a*. Check the result by multiplying the area of the section

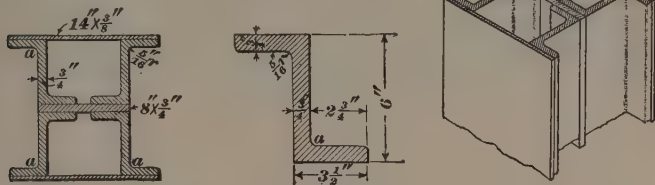


FIG. 33

by 3.4, which is the weight per foot of steel for each square inch of section. The illustration on the right shows how the stanchion is formed.

$$\text{Ans. } \begin{cases} 173.67 \text{ lb.} \\ 173.88 \text{ lb.} \end{cases}$$

SUGGESTION.—Area of a fillet is one-fourth the difference between square of twice the radius, and area of circle.

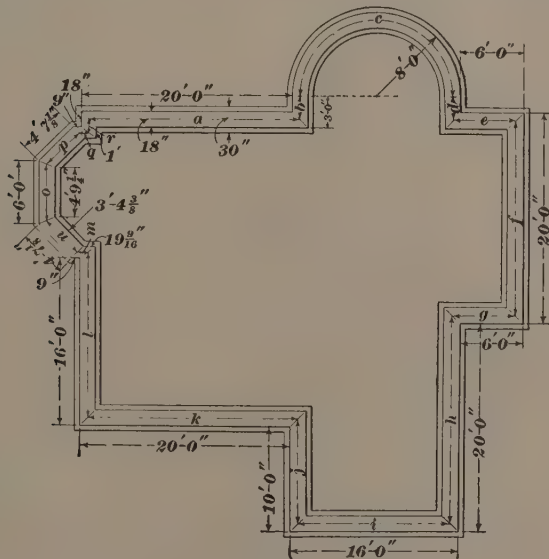


FIG. 34

10. Find the number of cubic yards of brickwork in the foundation walls shown in Fig. 34, the wall being 18 inches thick and 9 feet high, and the footing course 2 feet 6 inches wide and 6 inches deep, projecting 6 inches each side of the foundation wall. Ans. 95.73 cu. yd., nearly

SUGGESTION.—Find length of centre line of wall, reducing all dimensions to feet and decimals of a foot; thus, $a = 20$ ft., $b = 3$ ft., less one-half width of wall $= 3$ ft. $- 9$ in. $= 2$ ft. 3 in., or 2.25 ft. c is a semi-circumference whose radius is 8 ft. $- 9$ in. $= 7.25$ ft. $f = 20$ ft. $-$ width of wall. m is one-half of the sum of inside and outside measurements. $19\frac{9}{16}$ in. $= 1.63$ ft.; 9 in. $= .75$ ft. $m = (1.63 + .75) \div 2 = 1.19$ ft. $n =$ one-half sum of 4 ft. $7\frac{3}{8}$ in. and 3 ft. $4\frac{3}{8}$ in., which is 3.98 ft. $p = n$, etc. The length of centre line will be found to be 175.26 ft., with a possible difference in the last figure owing to the different manner in which the quantities may be taken off. The same centre line applies to both footings and wall.

PRELIMINARY BUILDING OPERATIONS

EXCAVATING

DATUM LINES

1. **Levels.**—When preparing a site for the erection of a building, the elevation of the structure above some imaginary reference level, called the **datum line** or simply the **datum**, must be predetermined. The datum line of railway structures, ordinarily, is the level of the tracks, which, to a certain extent, governs the floor level of the buildings. On the seashore, or on streams affected by tide water, either the mean high tide or the mean low tide is taken as a datum. On inland streams subject to freshets and floods, the datum line is generally assumed at high-water mark.

2. **Formal Datum Line.**—The **formal datum line** in the British Isles is that adopted by the Government Ordnance Survey Department in its detailed survey of Great Britain. This line is taken to be at the mean level of the sea at Liverpool. On the ordnance maps, all levels are given in feet above this line, the place at which the level was taken being shown by a **bench mark** , which indicates that the level was taken on the centre line of the horizontal bar over the broad arrow. This mark is cut in a kerbstone or a wall, or in some other suitable place as near as possible to the position shown on the ordnance map. Some of the ordnance levels were taken 50 years ago, and in parts of the country must now be regarded only as approximate, owing to the various geological movements in land surfaces

which are constantly taking place. The levels shown on the centre of a road on ordnance maps, but without bench marks, are, of course, not so accurate as those indicated by bench marks, as a road surface in a very short time may vary considerably.

3. Where a Government ordnance survey has not been made, a local system of bench marks often exists. These marks are made on the foundation wall of some enduring structure or on monuments partly buried in the earth. Records of bench marks, with their proper levels, are kept at the municipal offices, where the officials will indicate the position of such marks on request. When setting out the site of a building, a level should be run from the nearest bench mark to the point of excavation, especially in new sections of a town where the streets have not been formed and paved. In this way, a building can be erected at such an elevation that a subsequent extension of the street to that point will not necessitate raising or lowering the structure. Ordinarily, where the streets are formed and paved, a point on the kerb is taken as datum, and the various levels of a building are measured from that point. In the case of buildings erected in the country, and engineering works constructed in isolated places, an arbitrary datum is established, depending on the topography of the land. Before work is commenced on the structure, a monument is usually set up to mark the level of the datum.

STAKING OUT EXCAVATIONS

4. **Boundaries of a Building.**—In addition to the level of a building, its exact position on a plot must be determined. Too great care cannot be exercised in staking out a building excavation, particularly when the walls are to be carried up close to, or on, the boundary line. If an architect is not engaged in connection with the erection of the proposed structure, a surveyor should be employed to set out, not only the excavation boundaries, but also to stake out the corners of the plot of ground. If the structure is to be erected in a built-up district, the front of the building probably will be in line with those of the adjacent buildings, and the depth of the new building can be

measured and marked on the foundations or walls of the adjoining buildings. If, on the other hand, the building is to be detached, or if it is to be set back from the street, a surveying instrument should be used. Where this instrument cannot be procured, the corners of the plot may be staked out and the boundary lines defined by sighting with the eye from stake to stake. The distance from the street at which it is desired to make the front of the building is measured on each side line of the plot from the front corners; the front of the building will be on the line joining the points last found. By measuring from the side lines, the front corners of the building may readily be fixed.

5. The position of the rear wall of the building is found in the same manner as the front wall, and the side walls can generally be run in without any trouble if the building is rectangular. Where the building is not rectangular, it is generally necessary to run diagonal lines across the plot before all the corners of the building can be fixed. Stakes should be driven at each corner, or angle, and by measuring from stake to stake the dimensions of the building can be checked with the drawings.

6. **Line Boards.**—When excavating for foundations, the stakes driven to mark the position of corners and angles will necessarily be removed; therefore, it is advisable to set up at each corner of the excavation **line boards**. As shown in Fig. 1, these boards should be placed from 6 to 8 feet from the corners, so as to be out of the way as much as possible. The stakes *r, s, t* should be from 4 to 5 feet apart, and long enough, when firmly driven into the ground, to allow the cross-pieces *w, w* to be nailed to them, the upper edge being level with the top of the

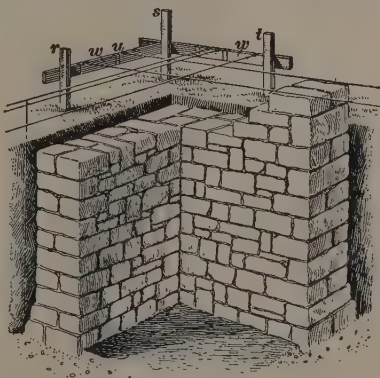


FIG. 1

foundation wall. All the cross-pieces, or line boards, should be levelled by marking the required level of the top of the foundation, or some other convenient height, on the stakes, then setting the upper edge of each cross-piece at the level of these marks, and nailing the boards firmly to the stakes.

7. Having guarded all the corners by means of line boards, the lines for the excavation may be set out. These are stretched from corner to corner of the building and fastened to the line boards at each end, as shown at *u*, Fig. 1. The bottom, or footing, courses of the foundation usually extend beyond the line of the wall; consequently, the lines which mark the boundary of the excavation are wider apart than those marking the face of the wall.

CHARACTER OF SOIL

VIRGIN AND ARTIFICIAL GROUND

8. Before plans are drawn for a structure, the nature of the soil that is to support it should be determined. If this cannot be ascertained from existing structures and wells, borings should be made. These are often holes from 5 to 10 feet apart, over the entire area, made with an auger about 2 inches in diameter and of suitable length. As the auger brings up samples of the soil, the character of the substrata is determined. When the importance of the proposed structure justifies it, trial pits are dug from 10 to 20 feet apart, especially where a shelving bed of rock or gravel exists at a comparatively short distance below the surface. It is not always safe to rely on the character of the soil in close proximity to the proposed building site, since conditions varying from a bog to a solid granite ledge are often found in the space of a few yards. The soil or strata usually encountered in building operations may be classed under three divisions: *rock*, *virgin soil*, and *made ground*.

9. **Rock.**—**Rock**, in its original geological formation, is spoken of as bed rock, or solid ledge. It forms one of the finest foundations for heavy structures, provided that it is of sufficient

area for the entire building to rest on. But it is undesirable to have a portion of the foundation rest on ledge while another portion rests on softer material, since the building will not settle uniformly and the walls will often crack at the junction of the ledge and softer material. It sometimes happens that the surface of the rock is uneven and requires considerable blasting and concrete work to secure a good foundation, which adds considerably to the cost. The sandstones and limestones are often found in strata, beds, or layers, one on another ; if these layers are not separated by clay, and the beds are even, they make a good foundation. The strata, or beds, of rock may shelf, or dip, at varying angles, especially in hilly sections. A ledge is not always reliable, and in many cases will be found to be shaly in structure, partly decomposed, or faulty in other respects.

10. Virgin Soil. — Virgin soil is either gravel, clay, loam, sand, or marshy ground in its natural condition.

1. *Gravel*, when compact and united with sharp sand into a firm, unyielding stratum, is known as *hard pan*. It makes the best foundation, excepting bed rock, and, because it is more easily levelled, is much less expensive to build on. Usually a 6- or 8-foot stratum of gravel will support any structure, although there may be softer material underneath. Gravel is considered the most healthful soil upon which to build, because it is easily drained.

2. *Clay* is the most uncertain of soils, owing to its elasticity, due to admixture with marl, etc., to its tendency to absorb moisture, and, in many cases, to the position of its bed, or stratum. In dry seasons it is very firm, but in wet seasons it is elastic and unreliable. When the layers of clay are inclined, the foundation has a tendency to slide, producing results threatening the stability of the superstructure.

3. *Loam*, or clay mixed with sand and other earthy substances, when compact and of considerable depth, is a good material to build on, provided that the structure is not extremely tall or heavy.

4. *Sand* is formed from the decomposition of the older rocks, either by the effects of the weather, the action of heavy rains,

the wearing away by running water, or the spontaneous decomposition of the rocks themselves. The particles are carried to the rivers and either deposited in their beds or borne out to the ocean. The sand usually found in excavations has its origin either by formation in the beds of ancient rivers which have long ceased to flow, and is therefore called *river sand*, or by the attrition or grinding of the rocks themselves during the geological upheavals in past ages. The latter is called *virgin*, or *pit sand*, and has never been exposed to the action of water.

5. *Quicksand* is a very fine sand, often mixed with clay or loamy material in such proportion that it will retain a large quantity of water. By confining quicksand and keeping it as nearly dry as possible, it may be excavated or built on with little more difficulty than common sand. In many cases, quicksands are mixed with a bluish or lead-coloured silt, or soapstone slime. Often, in excavating quicksand, beds of this blue marl are found, which material, when wet, is tough and hard, but when dry crumbles to powder and is utterly unfit for foundations. An attempt to excavate in quicksand, without previously getting rid of the water contained therein, is almost as useless as to dig in water itself, for the saturated sand will flow into the excavation as fast as it can be removed.

6. *Marshy soils* are formed by the decay of plants, weeds, and other vegetable matter in sluggish water, which latter, having no current, allows the plants to take root in the bed. When these plants die, others take their place each year. These successive beds of decayed matter are formed under slight pressure and have innumerable cavities between them, as would a heap of decayed hay, thus forming uncertain foundations for heavy structures. Sometimes these deposits reach such a depth that their bottoms have not been reached. Large areas of marshy land are formed in this way, by the periodical overflowing of rivers and the rise and fall of the tides along the coast. The terms *swamp* and *bog* are often used, and may be considered here as having the same meaning as marshy soil.

11. *Alluvial*.—During freshets, rivers bring down large quantities of soil held in suspension, which is deposited when the

waters subside. This formation is called *alluvial*, from the Latin word *alluvius*, meaning a washing upon. The term alluvial is often used to designate deposits which are of yearly recurrence, as in the Nile delta, although the river bottoms of many streams are of this origin. The value of alluvial soil for foundation purposes varies considerably. In many cases, it consists of a clay formation which is hard on top, especially during dry weather, but soft and unreliable underneath. Heavy buildings should not be erected on alluvial ground without a careful investigation of the subsoil by means of borings or trial pits.

12. Made, or Artificial, Ground.—**Made, or artificial, ground** may consist of various kinds of materials, such as the refuse of towns, earth and other materials removed from cellars and other excavations, the cinders, ashes, etc., from manufactories and furnaces. It should not be built on, if the structure is of importance, until the nature of its subsoil has been investigated, though for minor edifices a suitable foundation may sometimes be obtained on it. If, because of the cost, it is not desired to place the foundations, by means of deep digging, directly on the subsoil, the made ground can be levelled to an even surface and a bed or raft of Portland cement concrete, about 24 inches or 18 inches thick, laid over the whole area to be occupied by the building. The top of this raft should be made level and the building may be erected upon it. This method is suitable for such buildings as houses, shops, small halls, etc., but buildings of great weight must rest directly on the subsoil, either by excavating or by driving piles.

13. Conclusions.—From the foregoing description of the materials met with in excavations the following deductions may be made: It is generally safe to build on bed rock any structure which may be required, provided that the foundation beds are kept level. Gravel, even when mixed with small boulders, can be considered perfectly reliable for any ordinary structure, under usual conditions. Sand will carry very heavy loads, if it is confined; but great precautions must be taken to properly confine it, and also to keep water from it, especially running water, as the action of the latter will soon wash it away.

Clay, when compact and dry, will carry large loads, but water should be kept from it, both under and round the structure, the foundations of which might otherwise fail, owing to the difficulty of holding back the pasty or semi-liquid mass formed. On the other hand, clay containing moisture is by no means unsuitable as a foundation, provided that the water contained in it is retained. The sinking of wells, or the construction of sewers, in the immediate neighbourhood, may tend to drain the moisture from the clay surrounding the foundations, or the depth of the latter may be insufficient to ensure against atmospheric influence. Serious fractures in buildings have resulted from the foregoing causes, and, where all the conditions cannot be controlled, the foundations should be suitably increased in area.

A thick, hard, or compact stratum, overlying a much softer one, even if of silt or quicksand, will often carry a considerable load, the hard stratum floating on the soft as a raft floats on the water. It is usually better not to break through this hard stratum, as it serves to spread the base of the building and to distribute the pressure over a large area. The silt, slush, and decayed vegetation contained in marshy lands are not fit to build on without piling.

BEARING VALUE OF FOUNDATION SOILS

14. There is some difference of opinion regarding the safe bearing value of foundation soils, due probably to the difficulty of arriving, by experiment, at results which will have general application. Conservative practice dictates that the greatest unit pressure on the different foundation soils should not exceed the values given in Table I. Where there is any uncertainty as to classification, a simple experiment can be made by building a platform on the top of a 12" × 12" post, loading it, and noting, by readings with a spirit level, the settlement caused by the application of each ton of load. The fact that great variations in the bearing power of soils may occur within a limited area on the site of a building should, however, be borne in mind, and, wherever heavy buildings are to be erected, the experiment above advised should be carried out in several places on the site.

TABLE I

SAFE BEARING VALUES OF DIFFERENT FOUNDATION
SOILS

Material	Tons Per Square Foot
Granite formation	30
Limestone, compact beds	25
Sandstone, compact beds	20
Shale formation, or soft, friable rock	8 to 10
Gravel and sand, compact	6 to 10
Gravel, dried and coarse, packed and confined	6
Gravel and sand, mixed with dry clay	4 to 6
Clay, absolutely dry and in thick beds	4
Clay, moderately dry and in thick beds	3
Clay, soft	1 to 1½
Sand, compact, well-cemented, and confined	4
Sand, clean and dry, in natural beds, and confined	2
Earth, solid, dry, and in natural beds	4

DEPTH AND AREA OF EXCAVATIONS

15. Cellars for dwelling houses should have a depth of at least 8 feet from the under side of the ground-floor joists to the cellar floor, and a greater depth should be provided where the house is to be heated with a boiler, in order to give sufficient height above the boiler to allow a proper gradient for the pipes. For shops, office and manufacturing buildings, the depth is greater, varying according to the nature of the business carried on, or the requirements of the occupants. In some of the large office buildings in towns there are three cellars, *basement*, *sub-basement*, and *cellar*, and the excavations vary from 25 to 60 feet in depth. That portion of a building from the footing course to the top of the basement or cellar wall is known as the **substructure**, while that portion of the building above the substructure is known as the **superstructure**.

16. In excavating cellars, it is customary to leave a **runway**, that is, a part of the ground sloped down from the bank to the cellar bottom, for the more convenient removal of the excavated material. In very deep cellars covering a large area, this runway is usually built of heavy planking supported on wooden beams and posts.

17. Care should always be taken, where there is no cellar under a building, that the excavation in the trenches extends below the frost line. If the foundations are not started below the frost line, the alternate freezing and thawing of the earth tend to throw the walls out of plumb, and eventually will destroy them, besides racking the superstructure. The depth to which frost will penetrate the earth varies in different parts of the world, although 3 feet may be taken as the average limit of depth. In some parts of Canada and America the depth must be at least 4 feet for ordinary buildings, if the subsoil is satisfactory, while in the colder parts a depth of at least 5 feet is necessary. Frost will penetrate to a greater depth in hard dense ground than in soft porous earth.

18. Width of Trenches.—In excavating for concrete foundations for buildings, the trenches generally are made from 8 inches to 12 inches wider than the footings of the walls. The depth of concrete and any additional width are determined by the nature of the ground in conjunction with any exceptional weight that may have to be borne by the foundations. Where it is necessary to use wooden forms into which the concrete is deposited, allowance should be made, in the width of excavation, for these forms ; but in firm ground the forms will rarely be necessary.

19. When the plans of a building are being prepared, the character of the ground on which the building is to stand may not be accurately known, or, while the excavations are being made, an underground spring or a bed of quicksand may be encountered, although the soil had been found solid in all the places tested before the plans were made. Should such a condition necessitate changes in the foundations after the work is begun, the drawings should be made to conform to the foundations

as actually built. Then if at any future time it should become necessary to make an addition to the building, or to excavate near it, a correct drawing of the foundations will be available. In addition, the corrected plans will be valuable in calculating the payments to be made to the contractors for additional excavation and foundations.

REMOVAL OF MATERIAL

20. **Shoring Trenches.**—It is frequently necessary in excavating for foundations or drain

TABLE II
SAFE DEPTHS FOR
EXCAVATIONS

Material	Feet
Clay	9 to 12
Drained loam . . .	5 to 8
Ordinary earth . .	2 to 3
Dry sand or gravel .	1 to 2

trenches to support the sides by means of timber in order to prevent disturbance of the bank and to protect the workmen from the possible fall of the ground. The method of putting in the timber supports varies considerably, on account of the differing nature of the ground, and also the size of the trench. Ordinarily,

excavations may be made to the depths given in Table II before the material will fall in.

In a trench, in firm ground, not more than 5 feet deep and 6 feet wide, the timbering used, as shown in Fig. 2, consists merely of pairs of *boling boards* *a* with single *struts* *b*. The struts are made nearly as long as the width of the trench and are

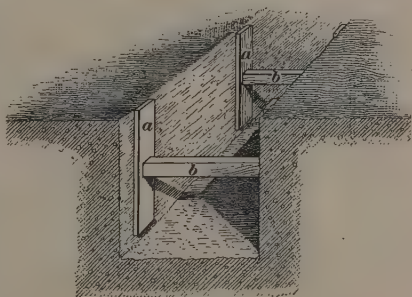


FIG. 2

driven tightly in place. They should be spaced horizontally 6 feet apart, for convenience of working.

21. In a trench of the same size as that just described, but in moderately firm ground, the timbering used, as shown in Fig. 3,

consists of poling boards *a* supported by horizontal timbers *b*,

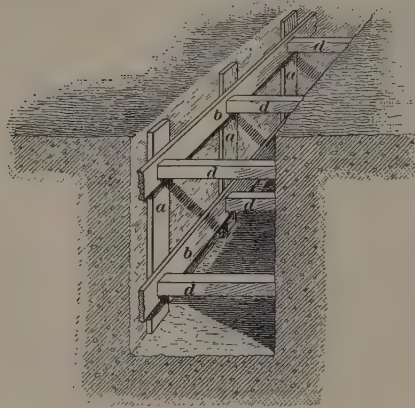


FIG. 3

called *walings*, which are kept in position by means of single, or double, rows of struts *d*. By means of the walings, the struts are kept at the necessary distance apart for convenience in digging. In loose ground, the trench would probably require the close timbering shown in Fig. 4. Horizontal boards *a*, called *sheeting*, are used, which make it necessary to

excavate only to the depth of one board without support.

Temporary strutting *b* is then used until several boards are in position, after which the vertical poling boards *c* are introduced, and maintained by struts *d*. Where the depth of the trench is over 5 feet, a platform *f* is laid on the lower struts, so that the excavated material may be deposited thereon, and reshovelled to the surface. In wet ground, if the sides of the trenches are cut to a

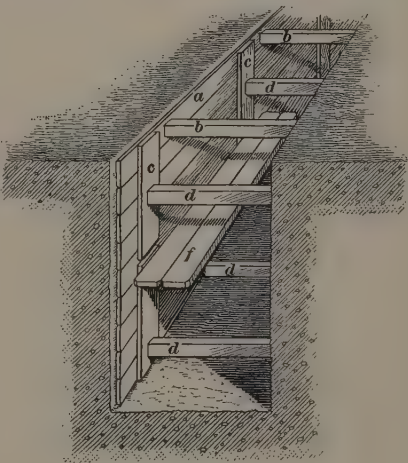


FIG. 4

slight slope, or batter, the struts are prevented from falling in consequence of any shrinkage of the earth.

Timbering for excavations is usually of red or white fir. The sheeting is from 9 inches by $1\frac{1}{2}$ inches to 9 inches by 2 inches, and from 8 to 14 feet long. The walings are from 7 inches by 2 inches to 11 inches by 4 inches, according to conditions, and up to 14 feet long. The struts are from 4 inches by 4 inches to 12 inches by 12 inches, and the poling boards are made from scrap ends, varying from 5 inches by 1 inch to 9 inches by $1\frac{1}{2}$ inches, and up to 4 feet 6 inches long. The timbering is of course removed as the walls are built, or the drain trenches refilled, but if there is likelihood of danger to adjoining buildings in consequence of the removal of the timbering, it is best to leave it in place and to pay the contractor for it.

22. In very loose soil or in bad ground, and where the excavation is deep and extensive, the method shown in Fig. 5 may be adopted. The ground is excavated to whatever depth is considered safe without support when vertical timbers *a*, termed *guide runners*, are driven into the ground at intervals of from 7 to 10 feet, to support the sides of the trench. The walings *b* placed across the guide runners are properly supported by the struts *c*, after which guide runners are driven farther into the ground and other runners *d* are inserted between the ground *e* and the waling. These close runners are cut obliquely at the foot as shown in Fig. 6, and sharpened with an axe, or sharpened and shod with sheet iron, to

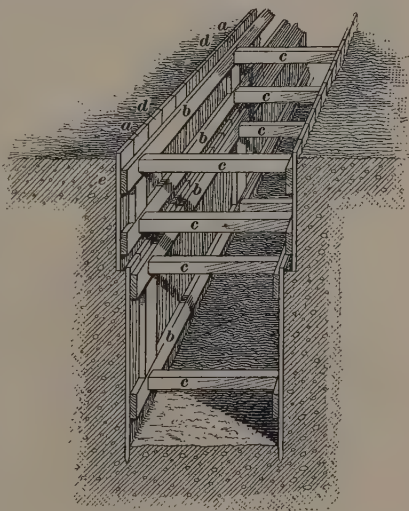


FIG. 5

facilitate driving and to force the runner against the side of the one previously driven. The earth is cleared away from the foot of the runner at intervals to facilitate driving.



FIG. 6

23. When excavating cellars, *sheet piling* is often used to protect the adjoining property, footpaths, etc., from injury by the caving in of the bank. Sheet piling, Fig. 7, consists of structural steel, reinforced concrete, or planks pointed at the lower end, placed closely together in line, and driven into the ground against the bank of the excavation so as to form

a wall, as shown in (a), which is a front elevation of plank sheet piling. At *a* is shown the piling driven into the cellar bottom *b*, against the bank of the excavation. In (b) is a section across the piling showing the brace *d* placed against the batten *e* to

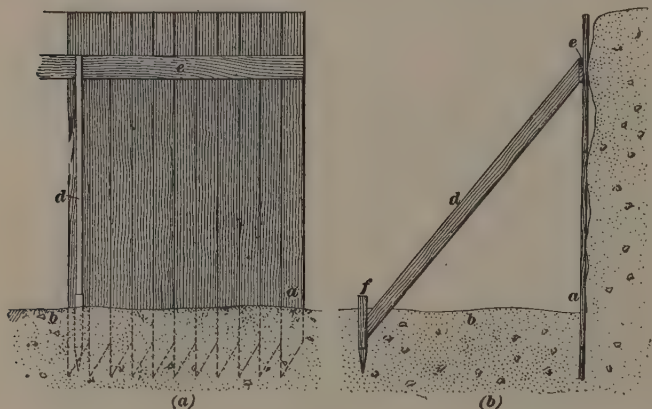


FIG. 7

retain the piling in place and to resist the pressure at the top. The stake shown at *f* is driven in to keep the brace *d* from slipping. These braces are usually spaced about 10 or 12 feet apart, and are necessary when the excavation is a deep one.

24. **Blasting.**—In many cases when rock is encountered during excavation blasting must be resorted to. This should be done only under skilful management as, apart from the danger of damaging adjoining property, the blasting has a tendency to shake and loosen the surrounding rock. The holes are drilled into the rock either by hand or power drills, and vary from $\frac{1}{2}$ inch to $2\frac{1}{2}$ inches in diameter and from a few inches to several feet in depth. Their direction varies with the dip or slope of the rockbed. In hand drilling, a steel-pointed bar, called a *borer*, or *jumper*, is held by one man while others hit it with heavy sledge hammers. To keep the hole cylindrical the jumper is turned a little after each blow. Occasionally water is poured into the hole to cool the bar and the powdered stone is removed with a scraper. When a sufficient depth is reached, the hole is dried with a rag on the end of a wire and the explosive is inserted. If the charge is to be ignited with a match, a fuse is placed in the hole, in contact with the explosive, and dry sand, clay, or some similar material is *tamped*, or packed down, round it. When dynamite is used, the end of the fuse in contact with the explosive is provided with a cap, as dynamite can be exploded only by concussion. Frequently, the charge is exploded by electricity, in which case wires must be connected with the explosive before the tamping is done. In order to confine the pieces of rock, which would otherwise be shot into the air after an explosion, and to prevent damage to adjoining property, or possible loss of life, a number of heavy logs, sometimes bound together with a chain, are placed over the rock to be blasted; these prevent the fragments of stone from flying when the charge is exploded. For extensive blasting operations rock-boring or drilling machines are used; these do the work for one-third the cost of hand drilling, and by them the drill is repeatedly driven against the rock, either by compressed air or by steam, and made to rotate slightly at each blow.

25. **Wedging.**—Where rock must be taken out so close to existing walls that injury will result from blasting, the operation of *wedging* is resorted to. This consists of breaking up the rock with steel wedges about 8 inches long and having wire wound

about them to form a handle. A workman holds the point of the wedge against the rock while another man strikes it with a sledge-hammer until the rock is split or broken. When the rock breaks easily and is formed in layers with defined seams between them, large quantities may be cheaply removed by this means, but it is a slow and expensive process if the rock is hard and lies in large compact masses.

MEASUREMENT OF EXCAVATION

26. Excavation is usually measured by the cubic yard, or 27 cubic feet, and the price paid varies according to locality, nature of the soil, distance excavated material must be hauled, and the disposition to be made of it. Twenty-two cubic feet of sand, 33 cubic feet of earth, or 28 cubic feet of clay, when comparatively dry, will weigh a ton. One cubic yard of earth in place will, after removal, occupy about $1\frac{1}{2}$ cubic yards when dug, and contains 21 struck bushels, and is considered a single cartload. When the site of a building is level or slopes regularly, and when the bottom of the excavation is to be level, it is an easy matter to calculate the number of cubic yards of excavation required. If, however, the surface of the ground is very irregular, the amount of excavation is obtained by means of a levelling instrument. When no instrument is available, the measurement may be obtained by driving several stakes into and flush with the ground. Then, when excavating, the piers containing these stakes are left untouched until the remaining excavation is completed. By measuring the heights of these piers before they are removed, the depth of the excavation can be quite accurately obtained. But the levelling instrument should be used whenever possible, as it permits an accurate calculation to be made before the building is commenced. Any variation during the progress of the work, necessitated by unforeseen circumstances, should be indicated carefully on the drawings, as it is desirable that they should constitute a true record of the work as actually carried out.

SHORING, NEEDLING, UNDERPINNING, AND BRACING BUILDINGS

SHORING

27. When the foundations of a building, whether new work or an alteration, extend below the foundations of adjoining property, *shoring*, *needling*, *underpinning*, and *bracing* must be resorted to, in order to protect the walls of the existing structure. The contractor is usually held responsible for the successful carrying out of this part of the work, and, consequently, there are, in all the large towns, firms or individuals who make a speciality of shoring and underpinning. The architect, however, should be familiar with the methods used, and should see that due precautions for safety are taken, and that all the beams, shores, braces, and posts are sufficiently strong to carry safely the loads imposed on them.

28. **Definition.**—**Shoring** is a method of temporarily supporting the walls of a building by means of posts or struts set at an angle to maintain an old wall which threatens to collapse, or to prevent a wall from failure when the adjoining buildings are removed, or during the underpinning and removal of the lower portion of the wall itself. It is often used to support the corner of a building, when it is desired to build a pier, or to set a column under it. When a girder is placed under the upper part of a wall, needling is necessary, being attended with less risk.

29. The method of strengthening a wall from 40 to 50 feet high by means of a **raking shore** is shown in Figs. 8 to 11, the reference letters to the principal parts being common to all. Where the wall is higher, the cross-sectional area of the main timbers should be 12 inches by 9 inches. Fig. 8 shows four *raking shores* *a*, 9 inches square, placed where the thrust of the

various floors occur. The feet of the raking shores rest on a

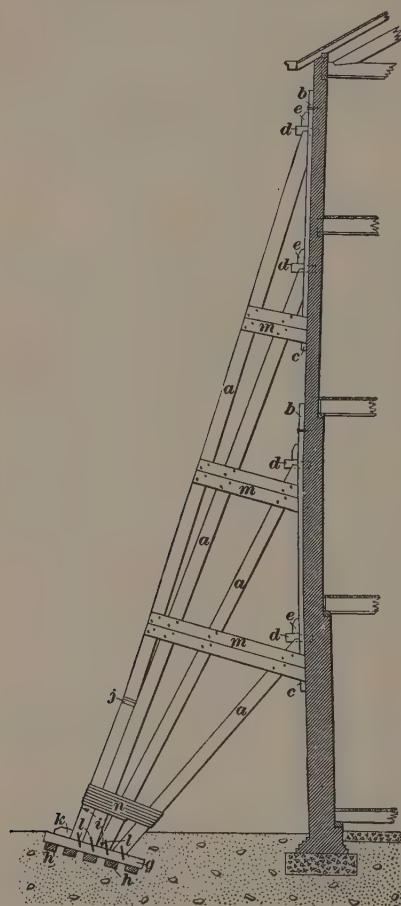


FIG. 8

sole piece, or *footing block*, *g*, which on soft ground is set on transverse timbers *h* long enough to distribute the pressure over a safe area. The heads of the raking shores rest on *wall pieces b c*, and are held in position by *needles d* placed in holes $4\frac{1}{2}$ inches square cut into the wall from $4\frac{1}{2}$ to 6 inches.

These needles are strengthened by *cleats e* nailed to the wall piece. The wall piece is 9 inches wide by 3 inches thick, and long enough to take the thrust of the raking shore and give a safe bearing to the needle placed near its end. In shoring, the wall piece is first fastened to the wall by hooks *f*, Fig. 9 (*a*), then the needles are passed through it into the wall. The shortest raking shore is then set in place and the others in the order of their length. The sole piece

is not placed at right angles to the shores, but has the end toward the wall slightly elevated, making the angle *i* somewhat less than a right angle, so that the raking shores, on being gently moved by means of a crowbar along the sole piece *g* toward

the wall, as shown in Fig. 9 (c), are tightened and take the weight of the wall. The raking shores are held in place by a cleat *k* nailed to the upper side of the sole piece or by *iron dogs l*, shown in detail at (d), driven into both sides of each raking shore and the sole piece.

The outer raking shore, which is nearly at right angles to the sole piece, is often made in two lengths for convenience of handling, as illustrated in Fig. 8, the upper length being known as the *rider* and lower length or supporting shore as the *back*

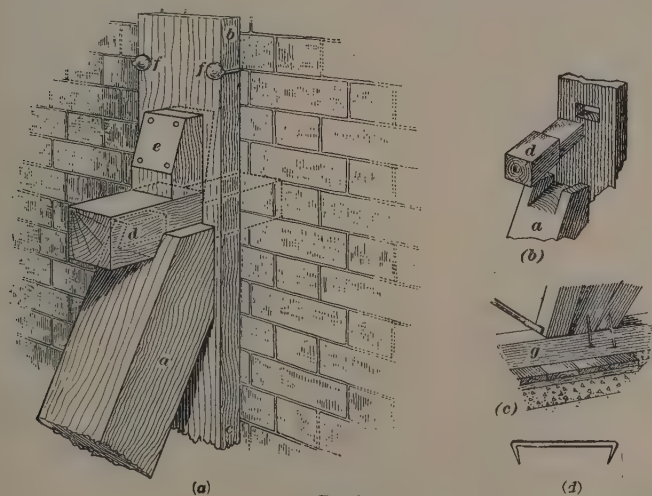


FIG. 9

shore. The rider is usually made to take its bearing by means of a pair of folding wedges *j* introduced between the ends of the two raking shores. The shores are tied together by planks spiked to their sides as at *m*, and, when fixed in roadways or public places, several turns of hoop iron *n* are usually bound round the lower end and nailed with clout nails. Sometimes the raking shores are tightened by driving wedges between the feet of the raking shores and the sole piece, but this method should not be used in dealing with dangerous structures, as the hammer blows in driving the wedges jar the wall.

30. In Fig. 10 is shown a perspective view of part of the shore shown in Fig. 8, and gives a clearer idea of the method of construction. At *a, a, a* are shown the raking shores in two cases resting against the wall piece *b c*. At *d, d* are shown the needles inserted in the wall piece and brickwork behind, strengthened by

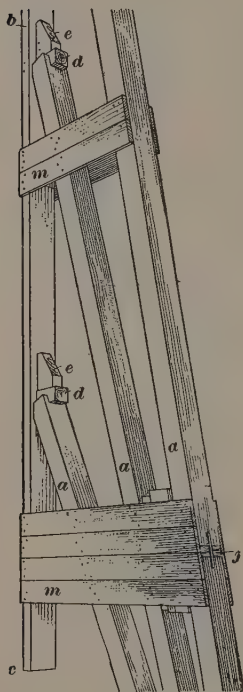


FIG. 10

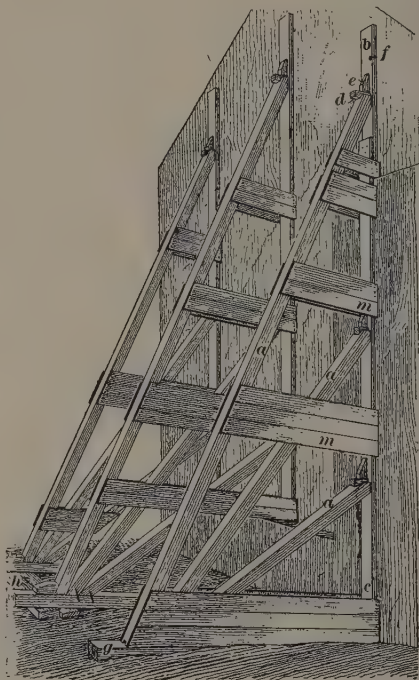


FIG. 11

the cleats *e, e*, against which the needles *d, d* are pressed by the head of the shores *a, a*. The heads of the raking shores are cut to fit the needles, as shown in Fig. 9 (*b*), and the raking shores themselves are held together by planks spiked to their sides, as shown at *m, m*. At *j* are shown the wedges between the rider and the back shore.

31. If the building to be shored is an independent structure, the principal shores should be placed at the external angles of the front and return walls and in the centre of the length of each wall. Other shores should be placed, where possible, not more than from 10 to 15 feet apart; in most cases the position of window and other openings will be the determining factor, but, as absolute security from settlement of the sole is required, great care must be exercised that no position is selected under which are vaults, cellars, or unsuitable ground. Fig. 11 shows a system of three shores, from an actual example in practice, differing only in minor details from those already described. In one case it will be noticed that the sole piece *g* is

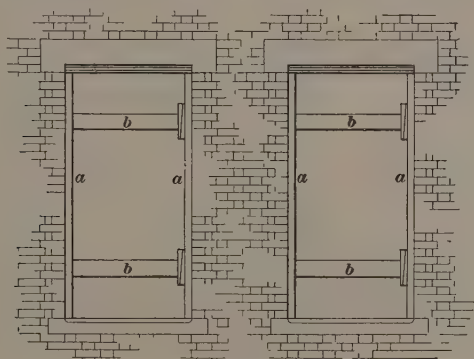


FIG. 12

simply bedded in the ground, while in the other the sole pieces *h* are resting on timbers laid transversely, with the idea of raising the end of the sole piece and minimizing the liability of slipping. The wall piece *b c* is shown to overlap. This is due to an offset in the wall and is not objectionable if the lengths are well spiked together. The heads of the raking shores *a, a, a* are in each case held in position by the needles *d* and the cleats *e*. At *f* is shown the hook which holds the wall piece *b* in position until the needle and shore are fixed. Planks nailed to the foot of the raking shore prevent slipping on the sole piece *g*, so no dogs are necessary. A cleat would be fixed on the sole piece before the raking shore was complete, but this has been omitted in the

illustration for the sake of clearness and to show the hole cut in the foot of the shore to admit the crowbar, which is used to move the shores along the sole piece and thus tighten up the heads.

32. Strutting of Openings.—Openings in the walls which are affected by shoring operations must be securely strutted, as shown in Fig. 12. Two uprights *a* are cut to fit tightly between head, or arch, and sill, and are kept in position by strainers *b*, cut roughly to the required width, and fastened in place with folding wedges. All the timbers are then securely nailed to prevent movement caused by shrinkage of the material.

BRACING

33. Horizontal or Flying Shoring.—When adjoining buildings

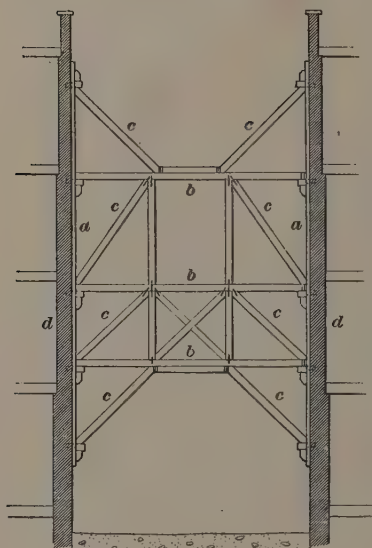


FIG. 13

have been built originally with party walls, or walls supporting the floorbeams of two buildings, and one of these buildings is to be pulled down, the adjacent walls should be guarded against falling by raking or flying shores. When there are buildings on each side of the site on which the new building is to be erected, the walls of these structures may be supported by horizontal or flying shores. When the distance is not more than 25 feet, the shores or braces may be arranged as shown in Fig. 13. Here *a, a* are 12" × 6" uprights placed against the walls *d, d*, to

distribute the bearing of the shores; *b, b* are the horizontal

struts; and *c, c* the angle braces, all of which may be 8" $\times$ 8" timbers. When the old buildings are from 40 to 50 feet apart, the horizontal struts should be trussed, as shown in Fig. 14. The 12" $\times$ 6" uprights are shown at *a, a* against the walls *e, e*; the 10" $\times$ 10" horizontal struts are shown at *b, b*; the 8" $\times$ 8" struts and braces at *c, c*; and the vertical iron or steel ties at *d, d*. It is preferable to use iron or steel rods for these vertical ties, as they can be readily screwed up, thus obviating

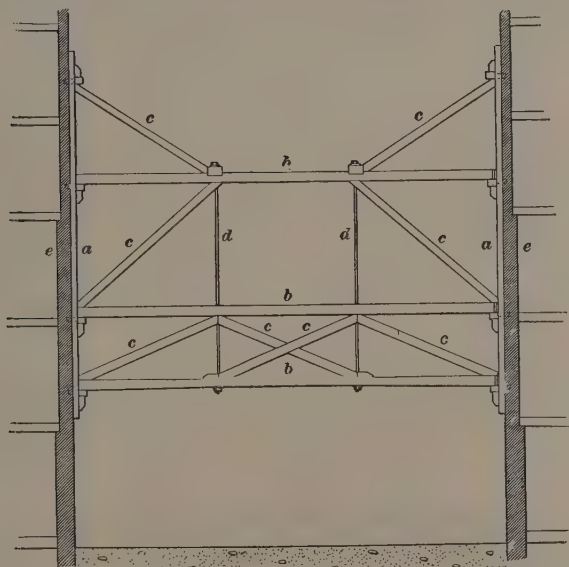


FIG. 14

any sagging that might occur in the joints of the truss. One truss should always be placed in the front, another in the rear, of the building; an intermediate one about every 25 feet will then generally be found sufficient.

34. A pair of horizontal, or flying, shores for a span of about 55 feet, with transverse trussing, is shown in Fig. 15. Owing to the great span, it is almost impossible to obtain sound timbers of the correct dimensions in one piece, so they are built up of

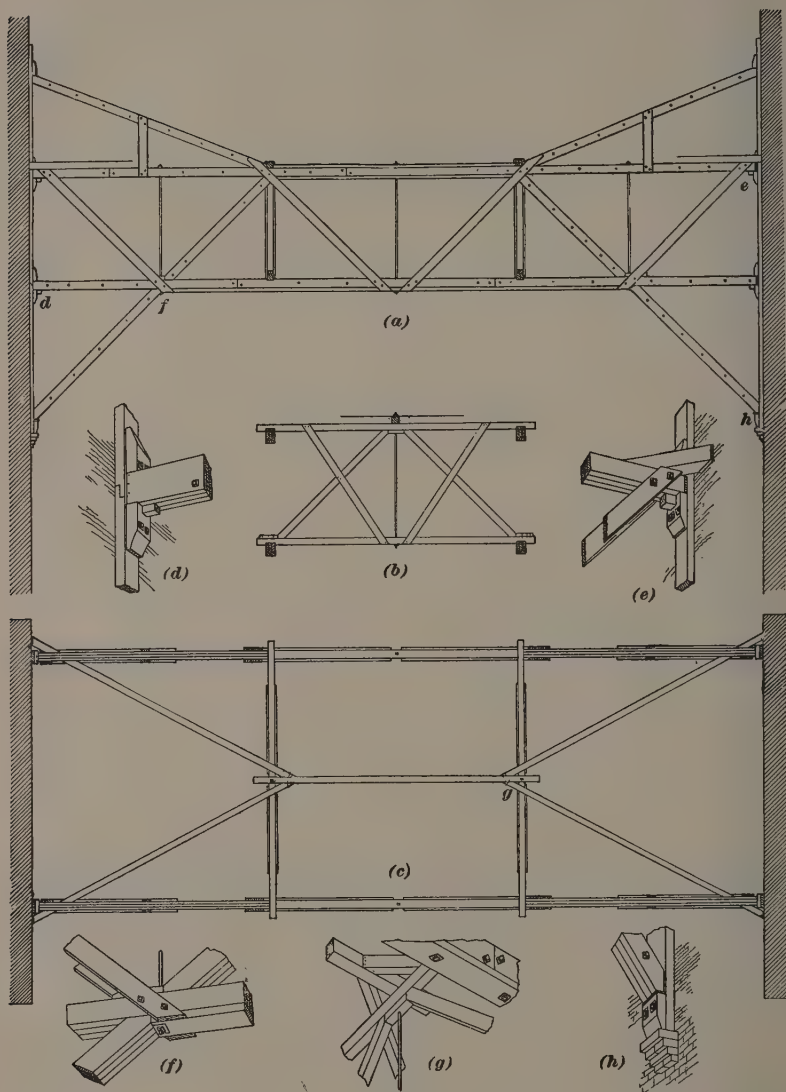


FIG. 15

several thicknesses, as shown in the plan (*c*). These pieces are stiffened by being bolted right through, at frequent intervals, with bolts, nuts, and washers, as shown in the elevation (*a*). Timbers built in this way are said to be **laminated**. In (*b*) is shown the transverse trussing, two sets of which run between the flying shores, helping to stiffen them, and keeping them from swaying in the wind. The flying shores are constructed very much like a raking shore: wall pieces are first fastened to the wall to take the thrust of the shores, which are then erected. In some cases the wall pieces rest on corbels *h*, which give them support and prevent the shore from slipping down the face of the wall. From the foot of the wall piece just above the corbel a raking timber runs to the point where the transverse trussing meets the shore. A perspective detail of the foot of the wall piece resting on the brick corbelling, with a cleat bolted through to the base of the wall piece, helping to support the laminated raking strut, is shown in (*h*). In (*d*) is shown a detail of the joints at *d* in (*a*). In this instance the wall piece *d* is in two pieces, joined with a halved joint; it is held in position by cleats on the top and bottom of the laminated horizontal strut. A detail of the connections at *e* in the elevation (*a*) is shown in (*e*), where the horizontal laminated strut is held in position against the wall piece by means of double cleats and a needle. Additional strength is given by a piece of timber laid across the top of the horizontal struts and securely fixed to them and to the centre of the transverse strutting shown in (*b*). The diagonal braces are shown one on each side of the horizontal struts and bolted through to it. In (*f*) is shown a detail of the connections at *f* in (*a*), together with the method of securing the iron tension rod. In (*g*) is shown the construction of the connections at *g* in (*c*).

35. Where the wall of a building is to be maintained by a flying shore, and the opposite building from which support is to be gained is not of similar height, the type of shoring shown in Fig. 16 is more suitable than the horizontal type just described. This is composed of rakers *a, a*, horizontal struts *b*, and sheeting *c*; packing pieces *d* are used under the sheeting where necessary. The construction of the joints and connections for this type of

shore is very similar to those described, but great care must be taken to get the timbers of correct length, as the crowbar cannot

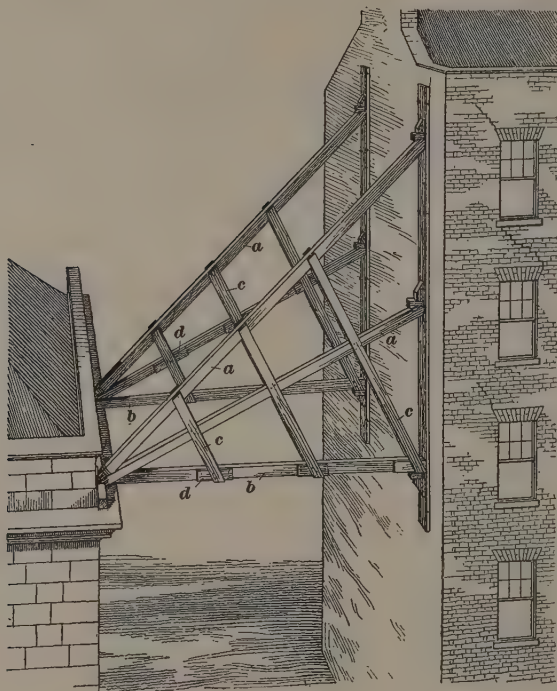


FIG. 16

be used at the foot of the rakers as it can be when the raking shores rest on a sole piece laid on the ground.

NEEDLING

36. When a wall already built is supported on beams or needles placed transversely through holes cut in the wall and supported at each end by posts or jack-screws, it is said to be *needled*, and the operation of preparing it is called **needling**. Fig. 17 shows a wall supported by needles in order that a new

wall may be built under it, or columns and girders placed to carry it. Holes *a, a* are cut in the wall *c* at intervals of from 5 to 7 feet from each other to receive the needles *b, b*, which consist of heavy timbers. Small cross-beams *d, d* are laid on the upper sides of the needles, where they pass through the holes, and are wedged in with oak or iron wedges *e* in order to secure a more even bearing for the wall. At the inner and outer ends of

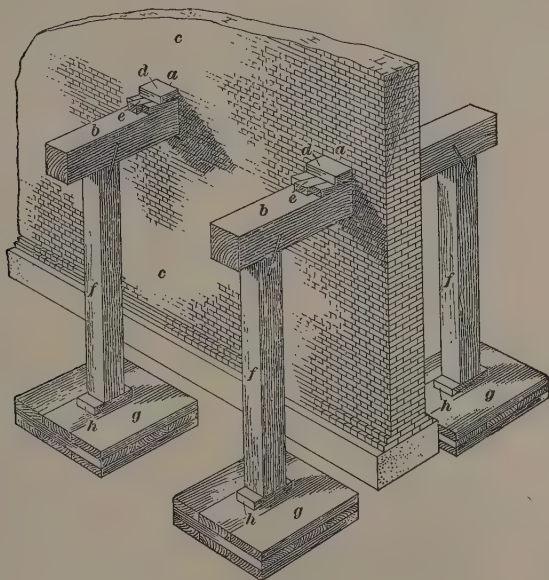


FIG. 17

the needles, heavy perpendicular timbers *f, f, f* are placed to support the needles and to carry the weight of the wall. The foot or ground bearing of these timbers is formed by three courses of heavy planking *g, g, g* crossing one another at right angles to spread the weight over a large surface, or often a continuous sole piece is used. Wedges *h, h, h* are driven under the foot of each upright, or the sole pieces are laid at a slight inclination and the upright posts levered into position in the same way as

the feet of raking shores, forcing the ends of the needles upwards until they show a slight downward deflection, or bending, at the centre, thus indicating that the weight of the wall is being carried by the needles. As soon as the needles support the wall, the lower portions may be removed, and the columns and girders, or other substructures, are put in place, or the excavation to the lower level is begun. Where the ground is soft or compressible,

sheet-piling is placed round the feet of the uprights to hold the ground in place. Very frequently, especially in the case of heavy work, if a high wall is to be underpinned, steel beams are used for the needles, and jack-screws are placed under the feet of the uprights, or a gantry of heavy beams is built in place of the upright posts.

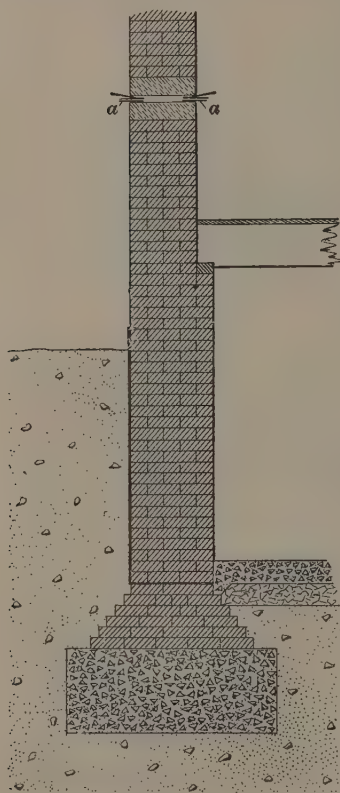


FIG. 18

UNDERPINNING

37. When a new wall is being placed under a wall already built, either to make deeper foundations or because the foundations of adjoining structures are deeper, the operation is called *underpinning*. During this operation, the old wall is generally supported either by shores or by needles. It is often necessary to remove the old wall as far up as the needles or shores. In first-class work the joint between the old and

the new work is generally made with cut stone, and wedges *a, a*, Fig. 18, of stone, slate, or iron, which are driven between the cut

stone of the old and the new work until the weight is taken off the shores or needles. The joint between the two courses of cut stone is then filled with cement mortar and the needling removed, leaving the wedges permanently in the work. The wedges must be driven in to the same degree of tightness on both sides, otherwise there is a tendency for the wall to crack from being supported only on one edge. Underpinning operations should be carefully performed. The new work should be built as quickly as possible after the shores or needles are in place, so as not to require their support for a longer time than neces-

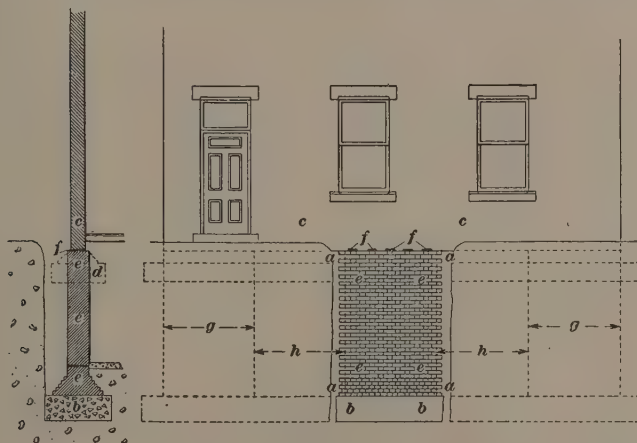


FIG. 19

sary. But the needles or shores should not be removed until the cement in the new work has had ample time to set.

38. Underpinning is also resorted to when a basement must be added to a building. In this case, the ground is excavated to the required depth under a section of the wall *a a a a*, Fig. 19, about 5 or 6 feet in width. The new wall *e* is then built upon a concrete foundation *b*, built on the solid ground underneath the old wall *c*, the footings *d* of which are usually removed. If well built, the old work will require no support during these operations. The new walling is built in cement, and carefully

wedged under the old with slate or steel wedges *f*, and is afterwards made good with neat cement under the old work. When this portion of the work has been completed, and the cement is set, similar spaces *g, g* may be cleared away, and the new work built under the walling at each side. After this is done, the spaces *h, h* may be dealt with in a similar way, care being taken to bond each portion properly into that previously built. In this manner the whole of the shallow foundations can be removed and deep ones substituted.

SCAFFOLDING

CLASSIFICATION

39. **Scaffolding** is the name given to the temporary platforms placed at various levels for the support of workmen, plant, and materials engaged in the erection of a building. The principal varieties are *pole*, *framed*, *trestle*, and *suspended scaffolding*, and *stagings* and *gantries*. Pole and framed scaffolds are of two types, *attached* and *independent*. The former is attached to and gains support from the wall; the latter is self-supporting. In both types the standards are frequently formed of pairs of poles. The scaffold is increased in height by lashing other standards to those already in position, a long and a short pole being used alternately to break joint. Continuous **guard rails** *e* and **guard boards** *f*, Fig. 20, are fixed to the outer edge of all platforms which are 10 feet or more above the ground. The former is placed at a height of 3 feet 6 inches above the platform, and has removable portions for access of workmen or materials. The guard boards should rise above the platform not less than 7 inches.

40. **Systems.**—There is a marked difference between the system of scaffolding used in the North of England and Scotland and that in use in the South of England, and the systems have been called, respectively, *northern* and *southern*, although the difference is not strictly a geographical one, exceptions being found in certain towns and districts, and combinations of the

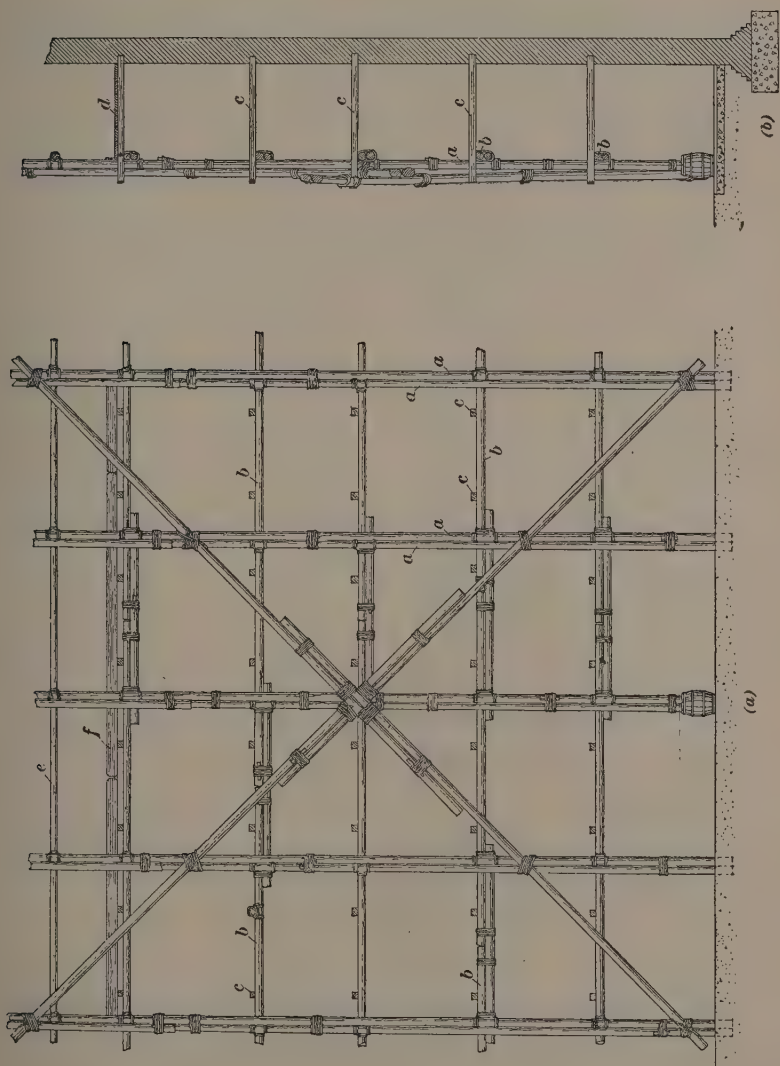


FIG. 20

systems being also adopted on occasion. Framed and trestle scaffolds are characteristic of the northern system, and pole scaffolds of the southern. The remaining varieties are now so frequently combined with either as to have lost all purely local application.

POLE SCAFFOLDING

41. Attached pole scaffolds, called bricklayers' scaffolds, Fig. 20 (a) and (b), consist of a framing of straight tapering

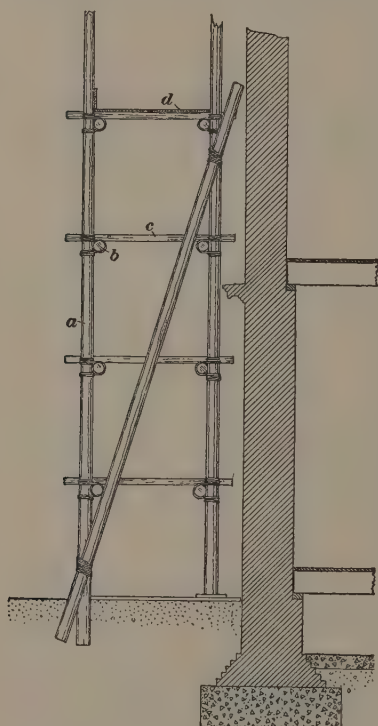


FIG. 21

spruce poles, from 15 to 30 feet long and from 4 to 6 inches in diameter. The vertical members *a*, called standards, are spaced about 8 feet apart in a row, parallel with, and about $4\frac{1}{2}$ feet from, the face of the wall. When possible, the lower ends of these standards are embedded in the earth, but when the surface cannot be disturbed, the ends are placed in barrels filled with sand or on planks provided with pockets by means of wood fillets, to prevent lateral movement of the feet of the standards. To the inside face of the standards, horizontal members *b*, called ledgers, are lashed at vertical intervals of about 5 feet. The vertical framework thus formed

is then adequately struted and braced by means of poles run diagonally across it and securely lashed at intervals as shown in Fig. 20 (a). The ledgers form the outer bearing for the putlogs *c*,

which span the distance between the walling and the framing of the scaffold. They are generally of birch and are about 6 feet long and 3 inches square. One end is firmly wedged in holes left or formed in the walling; the other is lashed to the ledgers. The putlogs carry the platform *d*; this is formed of spruce boards from 8 to 12 feet long, 9 inches wide, and $1\frac{1}{2}$ inches thick, trimmed as shown at *e*, Fig. 41 (*b*), and are bound with hoop iron to prevent them from splitting.

42. Independent pole scaffolds, called masons' scaffolds, Fig. 21, are similar in construction to the bricklayers' scaffold,

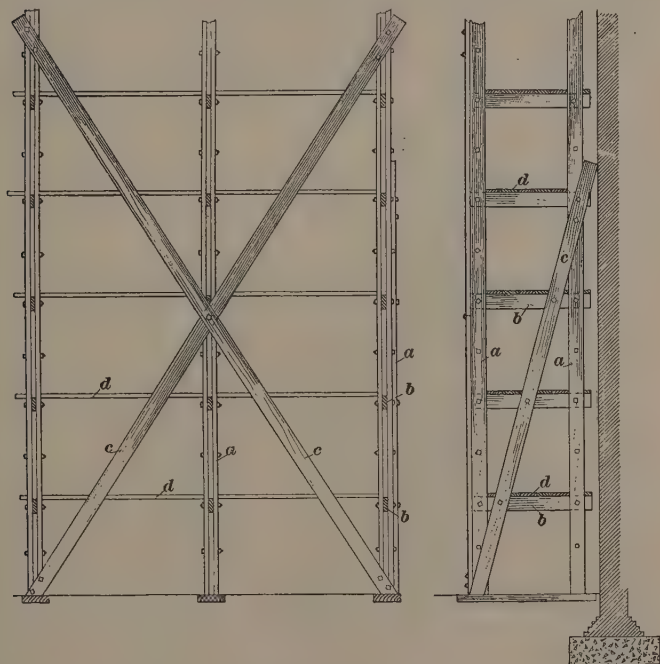


FIG. 22

with the exception that the putlogs are carried by a second framing of standards and ledgers, which adjoins the wall.

This is designed to prevent the disfigurement which, in the case of walls faced with wrought stone, would be caused by the putlog holes. Masons' scaffolds, although of the same general form as bricklayers' scaffolds, are stronger throughout in consequence of the greater weight of material to be carried, and the standards are spaced at smaller intervals. In Figs. 20 and 21 the reference letters designate the same parts.

FRAMED SCAFFOLDING

43. In framed scaffolding the members are formed of sawn timbers in short lengths, bolted together, the standards being usually of two or more thicknesses of 7" $\times$ 3" battens. Ledgers are not used, as the putlogs, or needles as they are called, are

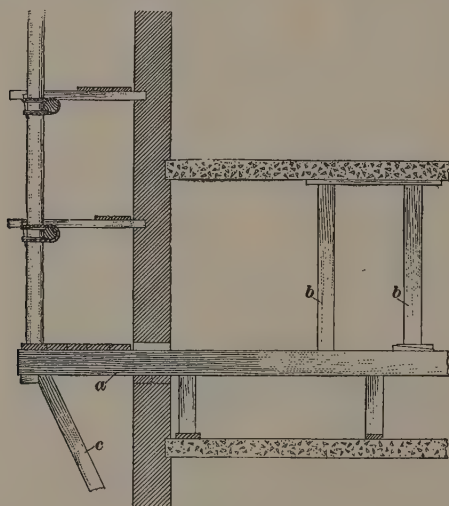


FIG. 23

bolted directly to the standards. This involves the use of thicker scaffold boards, since these must span the distance from standard to standard. Cross-bracing is used, and is generally about half the thickness of the material for standards. Fig. 22

shows a framed scaffold, also known as a *gabbard scaffold*, in which *a, a* are the standards; *b, b*, the needles; *c, c*, the cross braces; *d, d*, the scaffold boards, 12 feet long, 9 inches wide, and 3 inches thick.

NEEDLE SCAFFOLDING

44. Needle scaffolding is utilized where it is undesirable to build the scaffold on the usual solid base. The needles are stout horizontal timbers *a*, Fig. 23, projected from the wall to support a scaffold of the ordinary type. They are usually passed through a window or other opening in the wall and are tailed down by struts *b, b* wedged to the under side of the floor above. Where this does not provide sufficient weight, the inner end is tied down with cords or chains and the outer end is supported by struts *c*. The lower end of these struts rests on a cleat securely fixed to the wall, or in a hole cut in the wall.

TRESTLE SCAFFOLDING

45. Trestles are used very extensively in *overhand work*, that is, in walling, etc., erected entirely from the inside. As shown in

Fig. 24, where (*a*) is the front and (*b*) the side elevation, stout trestles, similar to, but larger than, those used by carpenters, are placed on temporary boarding, laid on the floor joists, which are put in position for the purpose as the work proceeds. The trestles

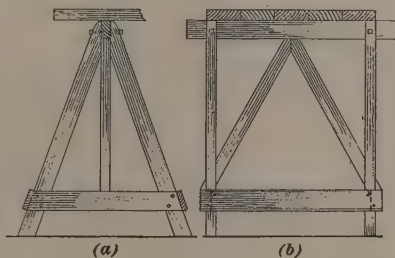


FIG. 24

are from 4 to 6 feet high, are spaced about 5 to 10 feet apart, and are covered with stout boarding about 12 feet long, 9 inches wide, and 3 inches thick, which form the first working platform, about 6 feet high. Further tiers of trestles and boarding are added, as the work proceeds, until a height of about 16 feet is reached. In work above this level, if no upper floor is available, the frame or gabbard scaffold must be used

BRACKET SCAFFOLDING

46. Wall brackets, Fig. 25, are used in certain of the northern districts of England, chiefly for the purpose of pointing, etc., and for repairs. They are fixed by means of long spikes, which are driven into holes carefully prepared for their reception, and are used in pairs from 5 to 10 feet apart for supporting a platform that is two or three boards wide.

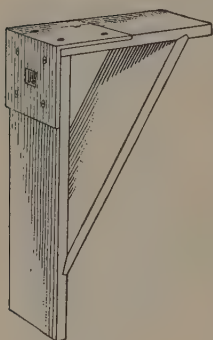


FIG. 25

47. Slaters' trusses, Fig. 26, are held in position by means of a rope attached to the eye *a*, and carried over the ridge and suitably attached or counterbalanced.

They are used by slaters when slating or repairing a roof. The sloping face of the roof affords no foothold for a workman, so these trusses are fixed on the slope of the

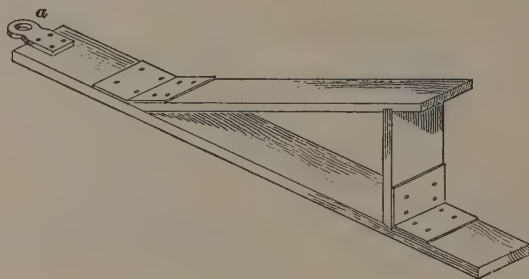


FIG. 26

roof, at suitable intervals, and boards are laid across the horizontal projecting portion, thus forming a platform for the man to work on.

48. Cripples.—Where it is necessary to work directly from a ladder, some form of bracket, to bring the platform nearer to the wall, must be used. This bracket is called a *cripple*, and is

fixed to one or more rungs of the ladder. The upper projecting surface is three boards wide, and is sometimes at a fixed angle

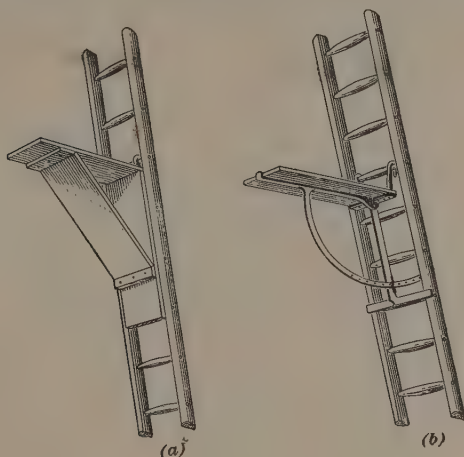


FIG. 27

in relation to the ladder, as in Fig. 27 (a), and sometimes is adjustable as in (b).

SUSPENDED SCAFFOLDING

49. Cradles, or boats, are suspended platforms used chiefly in cleaning down masonry, pointing brickwork, painting, etc., and in general repairs. As shown in Fig. 28, they consist of a platform *a* carried on bearers *b, b*, which are framed into the trussed sides *c, c* enclosing the platform on two sides. At the ends of the cradle iron straps *d, d* are securely bolted to the corner posts and main bearers, and bent to form a curved head. An eye *e* is riveted to the iron head, for the reception of the hooks on the tackle by which the cradle is suspended. The tackle by which the cradle is suspended and adjusted is attached to poles projected from the face of the walling and suitably strutted and counterbalanced.

50. The travelling cradle, Fig. 29, is an improved form of cradle to which, in addition to the vertical adjustment of the

cradle by the occupants, a horizontal movement may also be given. In this type the cradle *a* is suspended by tackle *b, b*, from a

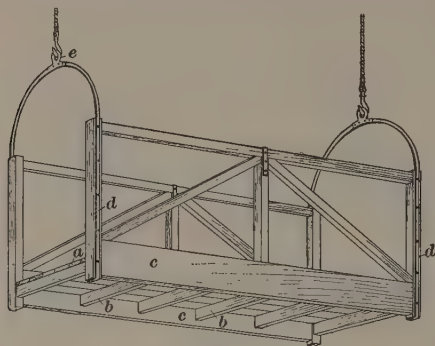


FIG. 28

steel-wire rope *c*, which is attached to poles *d, d* as in the following example. By means of the ropes running over the pulleys attached to the ends of the poles *d* the occupants of the cradle

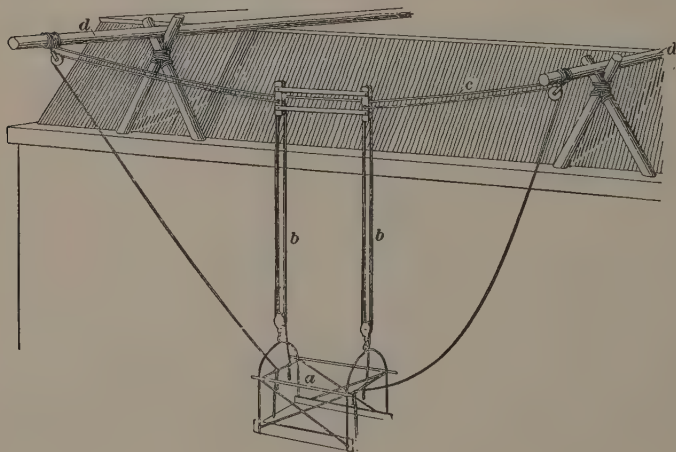


FIG. 29

can move themselves along the wire rope *c* at will, and bring themselves to any point of the wall on which they desire to work.

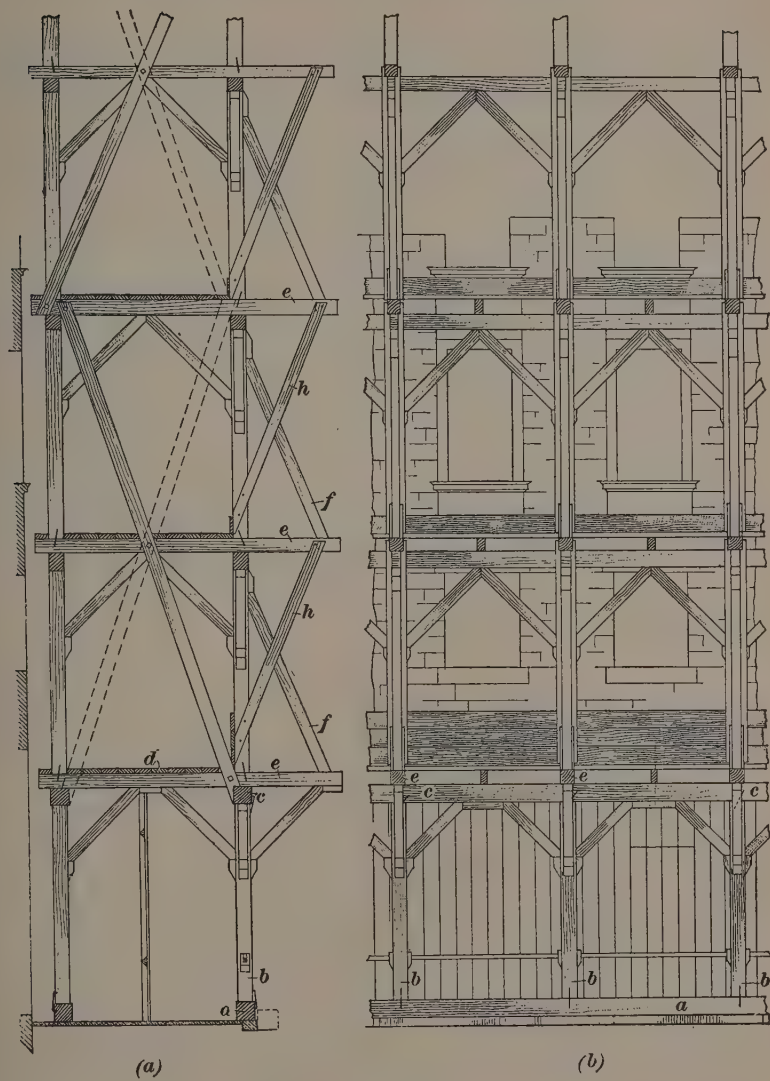


FIG. 30

STAGING

51. **Staging** is distinct from ordinary independent scaffolding, in that it is much more substantial in character, and is used chiefly in the erection of lofty and extensive frontages. The stages, generally one to each story of the building, are constructed of heavy squared timber, as required, for which reason the transverse bracing is important. In some cases this bracing can be accomplished within the staging, but more frequently the method adopted is that shown in Fig. 30, where (*a*) is a section through the staging and (*b*) a front elevation. The uprights *b* are placed, at intervals of from 7 to 10 feet, on the horizontal timber or sill *a*, laid on the pavement or earth. These uprights are securely fixed to the sill by means of iron dogs, and in turn support a head or *runner* *c*, upon which is constructed the first stage or platform *d*. In succeeding stages the standards are raised upon transverse timbers *e, e*, called *footing pieces*, which extend across the staging and project beyond it about 5 feet or more. This projection is utilized for the attachment of struts *f, f* and *h, h*, which maintain the upper standards. The struts *h, h* are arranged in pairs, and are bolted to the standards and footing pieces.

GANTRIES

52. The term *gantry* was originally applied to a species of elevated tramway for the transport, within a limited area, of material ; this type, however, is here known by the more descriptive term *traveller gantry*. The **gantry** is a single elevated platform constructed of heavy balk timbers and forms a scaffold on the base of which an ordinary scaffold is erected. It leaves a wide passage under the platform, whereas an ordinary scaffold or staging will obstruct traffic. As shown in Fig. 31, it consists of two horizontal timbers *a*, called *sole pieces*, which are placed on the ground, parallel with each other and the width of gangway apart. On these are erected vertical standards or pillars *b, b*, which carry the heads or *transoms* *c, c*, these three members being secured by iron dogs. Any joints in the head

or transom should occur only over the standards, when a short length of timber *d*, called a *head piece*, is interposed to give increased bearing. Struts *e, e* are inserted between the standards and under the heads, and are sustained by cleats *f, f* which are spiked to the standards. The cross bracing *g, g* is fixed not lower

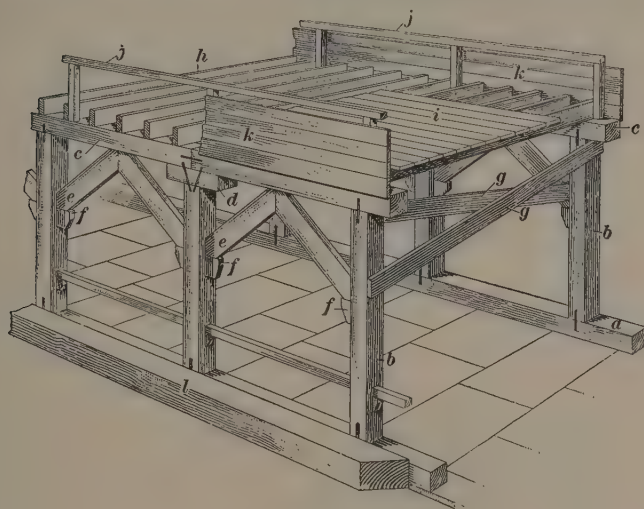


FIG. 31

than 6 feet 6 inches at the lowest point. The working platform is formed of joists *h*, laid transversely on the heads or transoms, and spaced about 2 feet on centres, and covered with stout boarding *i*; it is enclosed with a guard rail *j* and guard boards *k*. A horizontal balk of timber *l*, called a *fender*, is placed alongside the sole piece for the purpose of preventing damage to the structure by the wheels of vehicles in the street.

53. A *traveller gantry*, Fig. 32, is constructed as just described, except that, in place of the platform, steel rails are laid on the heads, forming a track for the *traveller a*, which is a stage constructed of trussed beams mounted on wheels and extending transversely across the gantry. It carries a cradle containing the mechanism for moving the traveller and the carriage, and

a double purchase crab. This carriage runs on rails, on the beams of the traveller, so that a double motion is imparted to

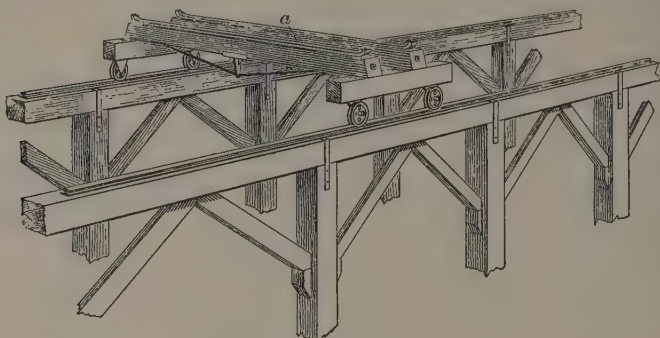


FIG. 32

the crab, which can be conveyed by the traveller in a longitudinal direction and by the cradle in a transverse direction, thus covering the entire space within the gantry. Traveller gantries

are less used in building than they are as permanent structures on the premises of contractors.

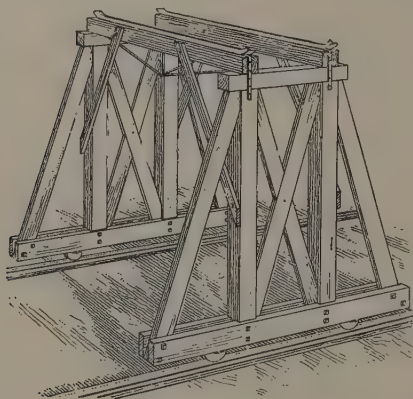
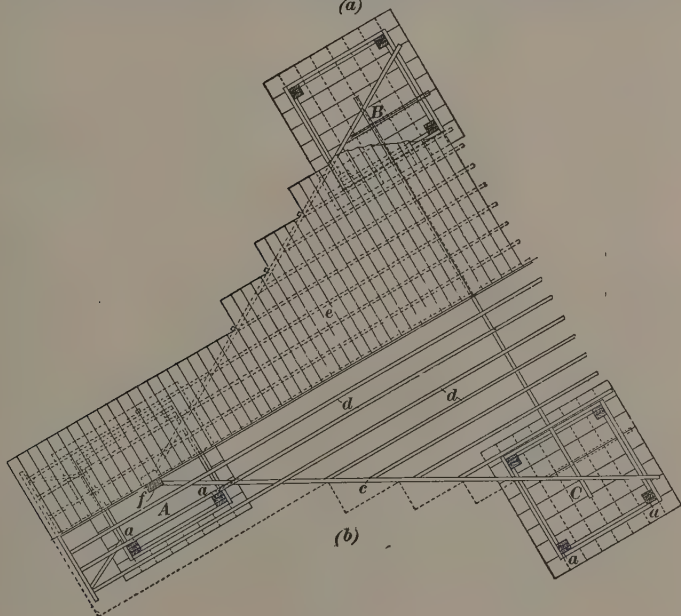
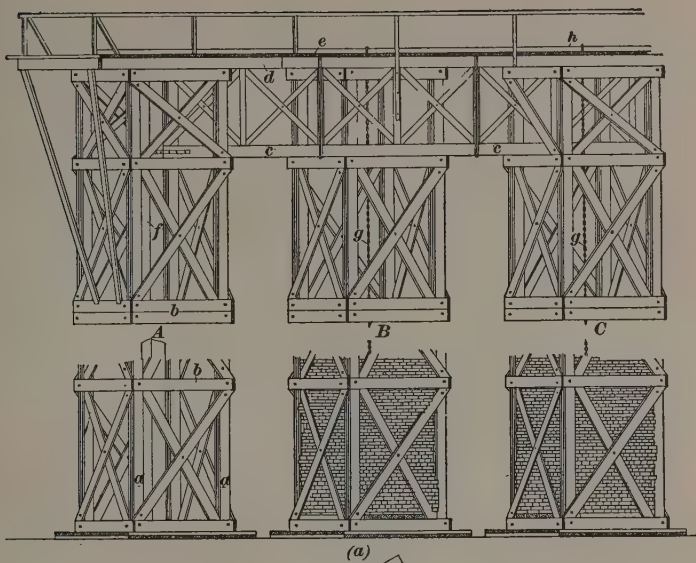


FIG. 33

54. A travelling gantry is distinct from the foregoing in that the gantry itself is mounted upon wheels and rails, as shown in Fig. 33, and can be moved at will to any desired position on the rails. Since the traveller runs in an

opposite direction on the rails at the top of the gantry, any space within the area of the rails of the travelling gantry can be commanded at will.



55. A tower gantry is a lofty platform supported on three, and sometimes four, towers or legs of framed timber. It is erected for the purpose of carrying transporting power, usually a steam jib crane, at a sufficient altitude to clear the highest adjacent parts of the proposed building. Fig. 34 shows the most usual form of tower gantry with three towers or legs *A, B, C*, arranged on a good foundation, in the form of an equilateral triangle having 30-foot sides; view (*a*) is the elevation of the staging and base of the legs, and (*b*) is the plan. These towers are skeleton framings from 5 to 10 feet square and from 50 to 125 feet high, and consist of four standards, or corner posts, *a, a*, built of whole or laminated timbers. At vertical intervals of about 10 feet, the standards are connected with horizontal braces *b, b*, and each section thus formed is struttled diagonally on each of the four sides. The towers are connected at the top by means of deep trussed girders *c, c*, which carry the joists *d* supporting the platform *e*. The tower *A* directly under the bed of the crane, or *king leg*, as it is sometimes called, has, in addition to the corner standards, a central one *f*, thoroughly struttled and braced, for the purpose of transferring the weight of the crane directly on to the subfoundation. The other towers *B, C*, called *anchor towers* or *queen legs*, contain stout adjustable chains *g, g* which are attached at the top to the horizontal crane stays or sleepers *h*, and at the bottom are heavily weighted, generally with stacks of bricks, to counterbalance the crane when loaded. The essential type of crane is one capable of hoisting, lowering, sluing or revolving, and derricking motions, the latter being its most characteristic feature, and consisting in the power of raising and lowering the jib, by means of which the load can be deposited within a very small or a very extended radius. The engine is of the vertical type, with two cylinders, and the crane, which is shown in Fig. 56, consists of a centre post or mast, single and double purchase gear stays or guys, horizontal braces or sleepers, jib, and flexible steel-wire rope or chain.

56. In Fig. 35 is shown a perspective view of a tower gantry actually erected at a large building at Westminster, London. The king leg can be seen immediately under the crane, and the



FIG. 35

two queen legs show the chains running down their centres. The long stays or guys running from the centre post of the crane to the extreme outside edge of each queen leg, and chained securely down to them, are also shown. Fig. 36 is a perspective

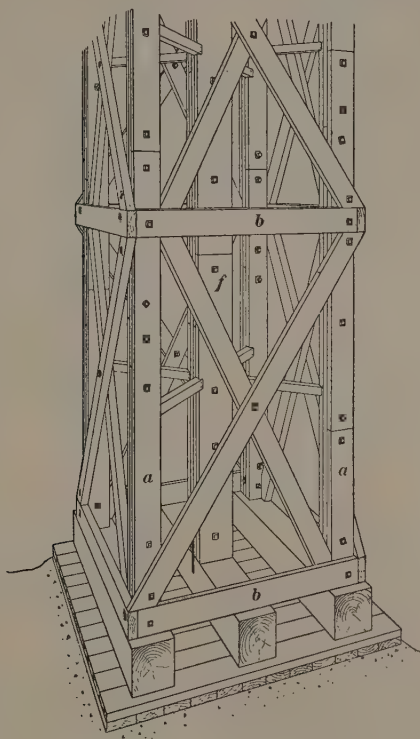


FIG. 36

detail of the base of the king leg. The corner posts or standards *a, a* are built up of several thicknesses of timber, bolted through at frequent intervals, as shown; *b, b* are the horizontal braces securely bolted through to the corner posts, the whole resting on the large timber sill pieces and crossed boarding as shown. At *f* is shown the central standard, which occurs in the

king leg only, securely dogged down to the sill piece and

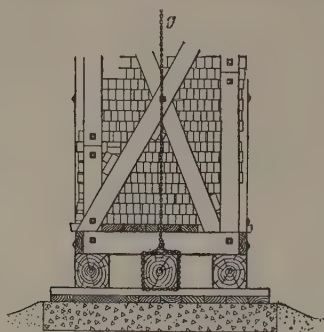


FIG. 37

strutted at frequent intervals across to the corner posts. Fig. 37 is a detail of the base of one of the queen legs. The construction of this is very similar to that shown in Fig. 36, except that instead of a centre post *f*, Fig. 36, a chain *g*, Fig. 37, is run down the centre of the leg and securely attached to the sill piece. The base of the leg is then heavily weighted by means of stacks of bricks or

other material. The bases of tower gantry legs are often made to rest on a bed of concrete to obtain a surer foundation.

SCAFFOLDING DETAILS AND ACCESSORIES

MEANS OF COMMUNICATION

57. Ladders.—The various stages of a scaffold or building are generally reached from the ground and from each other by means of ladders. These are frames consisting of a pair of sides connected with cross-pieces called rungs. The sides *a, a*, Fig. 38, are formed by cutting longitudinally in half a straight fir pole, and are holed for the rungs *b, b*. These are usually oak or ash, about $1\frac{1}{2}$ inches in diameter, tapered at each end, and are spaced about 9 inches apart; they are housed into the sides and wedged. An iron rod *c*, with nuts and washers inserted under every tenth rung, ties the sides together. The distance between the sides is tapered in the length of the

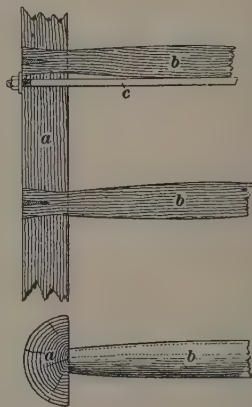


FIG. 38

ladder from 9 to 12 inches or more, depending on the length of the ladder. The use of extremely long ladders is not to be recommended, short ladders with intermediate landing stages being safer and less fatiguing. At the present time the more general use of power for transporting material has almost superseded the hand-carrier, with a great saving of labour.

58. Runs.—Gangways establishing communication between various platforms at approximately one level are called **runs**. They are formed of two or more scaffold boards, laid side by side, which should be connected

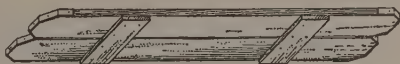
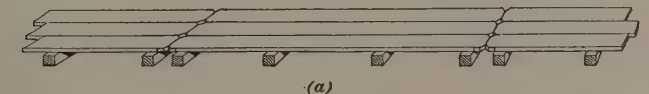
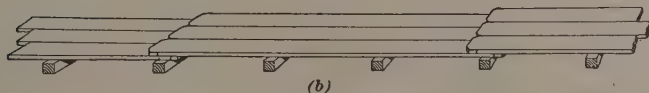


FIG. 39

together to prevent unequal sagging. This is accomplished by clinch-nailing battens to the under side, as shown in Fig. 39. The boards should be given a proper bearing on the putlogs, or other supports, one of the most fruitful sources of accident being the neglect of this



(a)



(b)

FIG. 40

precaution. Fig. 40 (a) shows a good method of supporting the ends of scaffold boards in communication runs and platforms, two putlogs being used near each other instead of the single putlog and lapped joint for the boards, as shown in (b).

CONNECTIONS

59. Scaffold Lashings.—In pole scaffolding the **lashings** are usually cords of tarred hemp, the best quality being termed Manila. They are $\frac{3}{4}$ inch thick and about 16 feet long, and are

bound or whipped with tarred twine at the ends. They should

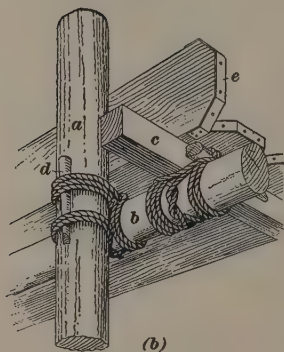
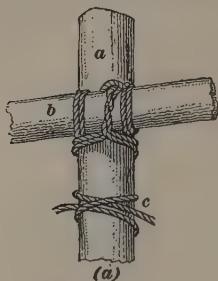


FIG. 41

be kept dry when not in use, and free from lime, which rots them. Various methods of tying are in use in securing the several parts of a scaffold and its accessories, the form of knot known as a **scaffold hitch** being the most general. Fig. 41 (a) illustrates the lashing together of a standard *a* and ledger *b*, the preliminary hitch round the standard being shown in addition at *c*. View (b) shows the lashing, from the outer side of the scaffold, of a standard *a* and ledger *b*, and an adjoining putlog *c* lashed to the ledger. The scaffold hitch is tightened by means of the scaffolder's hammer, and half-round tapering wedges *d* are inserted to increase the security of the joint. These wedges should be adjusted from time to time, and especially during and after the prevalence of damp weather. Galvanized chains and wire rope have been utilized for attaching the members of a pole scaffold, but these have not met with general adoption. Bolts, Fig. 42 (a), are used in scaffolds formed of squared or sawn timber. Iron dog hooks, Fig. 42 (b) and (c), are used to connect the members of scaffolding, etc., composed of balk timbers which are not framed together, as has been previously mentioned in connection with the description of shores and gantries.

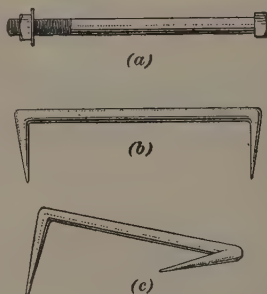


FIG. 42

EQUIPMENT

60. Sling chains are used for lifting stone slabs and similar material. They are fitted with a ring, or rings and hooks, as

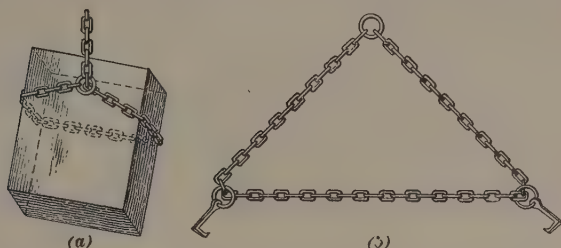


FIG. 43

shown in Fig. 43. When the chain shown in (a) is put round the material to be lifted, the chain slips in the ring and becomes tight round the material, thus enabling it to be hoisted to any desired position. To use the chain shown in (b), the hooks are placed in holes bored in opposite sides of the material to be lifted, one in each. Then, when the chain is lifted, it is drawn through the slip rings, and thus draws the hooks tight against the holes made for their reception in the material to be lifted.

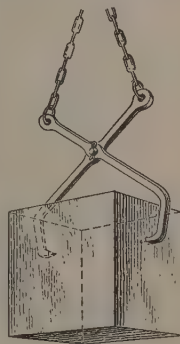


FIG. 44

61. Shears, or nippers, Fig. 44, are used for lifting squared or dressed stone blocks. Two L-shaped arms are riveted together in such a position that the attached ends of the chain will close the arms when the chain is pulled up. Holes are formed in a suitable position in the sides of the block for the reception of the pointed ends of the shears.

62. Lewises.—Lewises are contrivances for securing a hold on blocks of stone without risk of damage to the wrought faces. In Fig. 45 (a), (b), and (c), are shown three forms of lewises. In

(a) is shown a very ordinary yet very useful form. Two wedge-shaped lewises are pressed tightly against a wedge-shaped hole cut in the back or sides of the block of stone by a straight piece pressed down between them. These three pieces are held together by a pin running through a hole in their top and holding in place a horseshoe-shaped lifting piece. A chain or rope attached to this lifting piece, and operated by a crane or other lifting power, can now easily lift the piece of stone, as the lewis is of such shape that only by fracture of the stone can the lewis be detached. After the stone has been placed in the desired position, the lewis is removed by taking out the pin and the straight centre piece. In (b) is shown a form of lewis much used

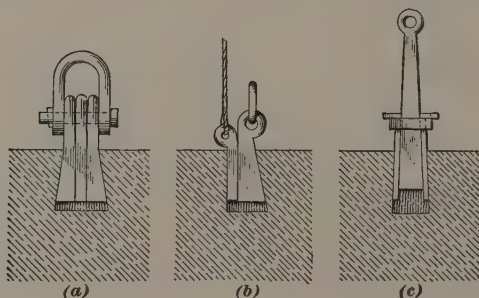


FIG. 45

for work under water. A hole is cut in the stone in a similar manner to that given in the description of view (a), but with one side vertical. The wedge-shaped lewis is then placed in position, and the straight piece, with a cord attached, is pushed in. The lifting cord from the crane or other appliance is then attached to the ring in the wedge-shaped lewis, and the whole can be lifted. When the stone is fixed in position, the lewis can be taken out by pulling away the straight piece by means of the cord attached to it, after which the wedge-shaped piece can be easily removed. In (c) is shown a combined lewis. The method of fixing this lewis to the stone is very similar to that described for the forms shown in (a) and (b).

63. Barrows, hods, buckets, baskets, skips, crates, etc., are simply items of general equipment, and do not merit special description here.

HOARDINGS AND FANS

64. Hoardings are temporary enclosures to the site of a proposed structure, erected for the protection of the public and of plant and material. In urban districts the construction of hoardings is subject to regulation by the local public

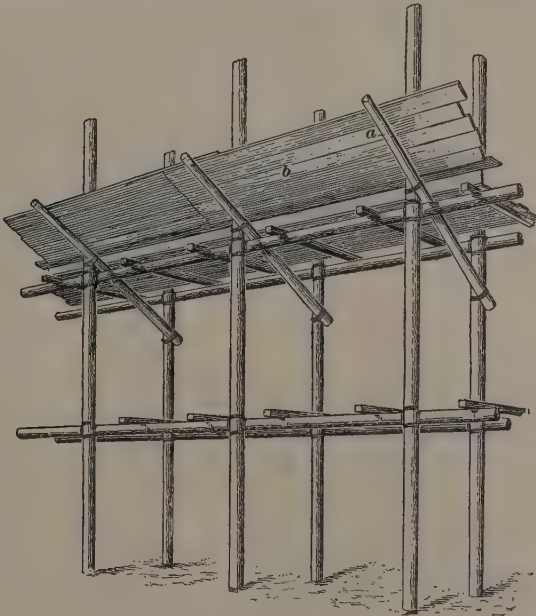


FIG. 46

authorities. The elementary forms consist of posts driven into the ground, or otherwise supported, and properly maintained with struts and braces. To these posts are attached horizontal members forming heads and sills, close or open boarding being nailed to both. It is frequently necessary, in the case of buildings

to be erected in public thoroughfares, to enclose, in addition to the actual site, a portion of the public pavement. In this case, a path at least 18 inches wide should be left unenclosed, or an external gangway should be projected over the roadway, and furnished with a guard rail, etc. Hoardings are available for future use if roughly framed and made in convenient sections, the doors being hung so that they may be lifted off their hinges, the different sections being simply bolted together or roughly nailed with long nails, which can be drawn on the completion of the building operations.

65. Fans.—When a scaffold abuts upon the pavement or other public way, it is essential to prevent material from falling outside the area of the scaffold or enclosure, for which purpose sloping guards, Fig. 46, called *fans*, have been devised. These are formed of *bearers a*, from 4 inches by 2 inches to 10 inches by 2 inches, sloping upwards and projecting 3 feet or more from the face of the scaffold and covered on top with close boarding *b*. Where it is not practicable to erect projecting fans, a portion of the scaffold is enclosed with close boarding, canvas, or other similar material.

REGULATIONS AND RECOMMENDATIONS

66. The regulations and recommendations affecting the construction of scaffolding and hoardings, and the use of plant and machinery, are framed for the protection of (*a*) the workmen, (*b*) the general public, and (*c*) the public property. The first stipulate for, or recommend, the adoption of guard rails and guard boards; specify the minimum width and construction of runs; direct the arrangement of putlogs and support of scaffold boards, the staying of ladders, etc.; and prescribe or provide for the periodical examination of machinery, and the notification of accidents. The second refer to the provision of fans and guard boards, double boarding to lower stages over passageways, etc. The regulations for the protection of public property and the maintenance of public rights or conveniences forbid or restrict the taking up or enclosure of pavements, the enclosure of street lamps, fire hydrants, and alarms, etc.

TRANSPORT OF MATERIAL

BUILDERS' HOISTING PLANT

SIMPLE LIFTING TACKLE

67. **Pulley Blocks.**—The simplest apparatus for lifting materials and plant is the ordinary pulley block, which consists of a grooved wheel, or sheave, mounted on a frame, to which is attached a hook for suspending the pulley block when in use. The single fixed pulley, Fig. 47, is the most elementary form, and affords no mechanical advantage beyond changing the direction of the force. When, however, the

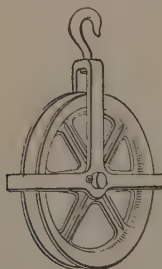


FIG. 47

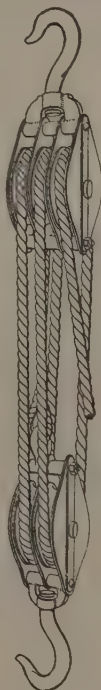


FIG. 48

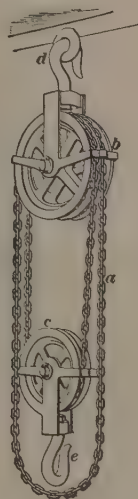


FIG. 49

fixed pulley is combined with the single movable pulley, only

about one-half the force is required to raise a weight, and this principle may be continued by mounting additional sheaves in the blocks, as shown in Fig. 48, in which case the gain is

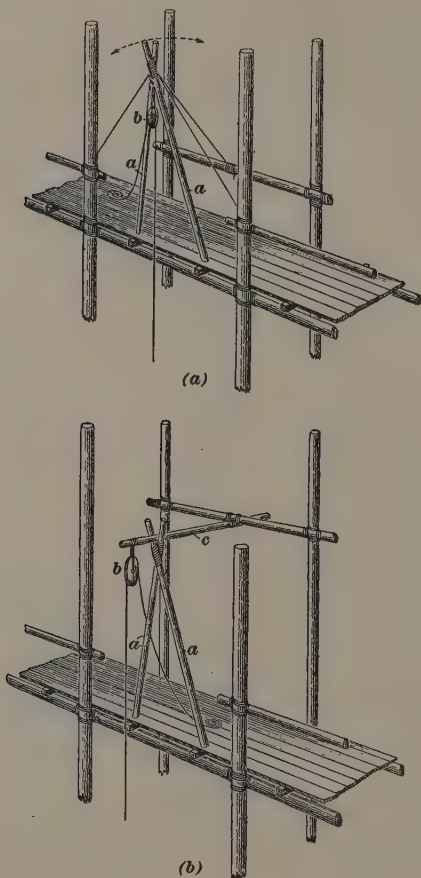


FIG. 50

estimated by dividing the weight by twice the number of pulleys in the lower block, an allowance being made for loss by friction.

68. Differential Pulleys.—In the differential pulley, Fig. 49, the two sheaves of the fixed pulley are cast in one piece, and rotate together, but they are of different diameters. An endless chain passes over each sheave, but the loop over the smaller sheave carries a movable pulley, the other loop, to which power is applied, hanging loose. Great friction is developed by this principle, and although the weight is lifted far more slowly, it enables two men to raise a load of 1 ton.

69. Shear legs is the term applied to a pair of crossed poles *a, a* lashed together as shown in Fig. 50 (*a*) and (*b*). These are used to support a pulley block *b* as in (*a*) or a jib *c* as in (*b*), to which a pulley block *b* is attached. In the first case, the shear legs are not fixed, but rotate or rock, as indicated by the dotted lines, within a limited radius, for the purpose of transferring the load

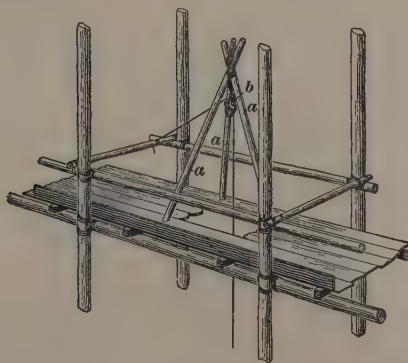


FIG. 51

when raised or lowered to a different position. A further modification is shown in Fig. 51, in which three poles *a, a, a* are lashed together to form a tripod, or *gin*, for the purpose of raising or lowering a weight by means of the pulley *b* mounted at the apex. For the purpose of allowing the material to pass, a hole or opening must be left in the run of the scaffolding at each stage, as shown.

70. Hand winches are machines by which a rope or chain is wound round a drum or axle, which is revolved by means of a

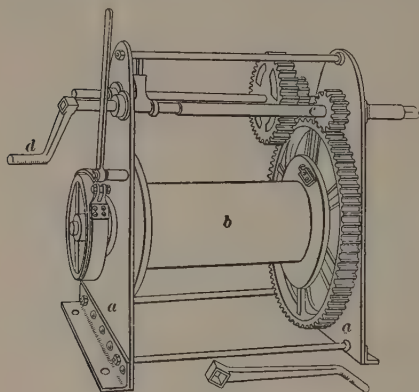


FIG. 52

cranked handle actuating one or more geared wheels. They are known as *single*, *double*, or *treble purchase winches*, in accordance with the type of gear, and their power ranges from $\frac{1}{2}$ ton to 30 tons, when used in combination with compound pulley blocks. Fig. 52 illustrates an ordinary form of hand winch,

a, a being the cheeks; *b*, the drum round which the rope or chain is wound; *c*, the geared wheels; and *d*, the cranked handle operating them. When heavy loads are to be lifted, a handle is often worked on each side of the winch.

71. Jennies and travelling crab winches are used on the traveller of a gantry for the purpose of raising and lowering loads, and transporting the same within the area occupied by the gantry. The traveller and jenny, or winch, may be actuated by hand or by power, and from below or above the gantry, the motions being operated by endless ropes or chains working on sheaves, or by gear. Fig. 53 shows a travelling jenny *a* mounted on a traveller *b*,

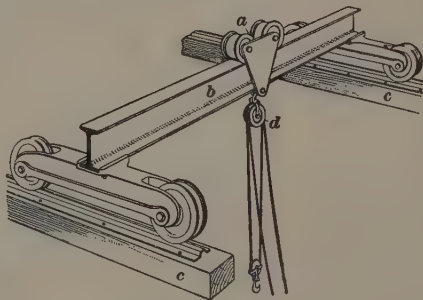


FIG. 53

which is supported by the gantry *c, c*; the jenny carries a differential pulley *d* to which the load to be transported is attached. Fig. 54 shows a travelling crab winch *A* mounted

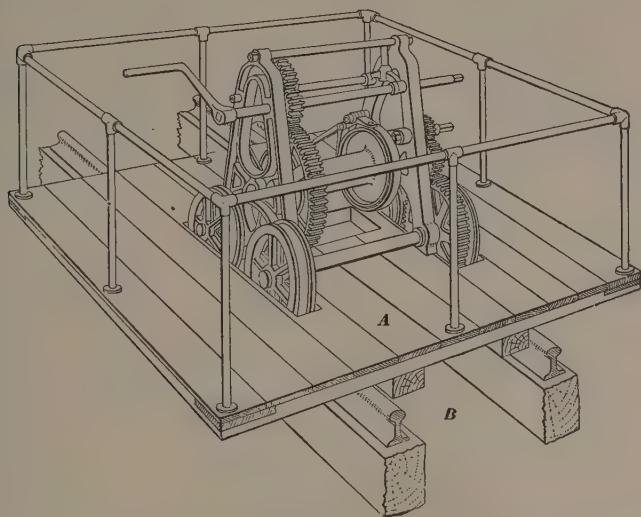


FIG. 54

on a traveller *B* supported by the gantry shown in Fig. 32. This type will take considerably heavier loads than that just described; it is operated by the handles and cog wheels by men standing on the traveller table of the travelling crab winch *A*.

CRANES

72. Portable Jib Crane.—Two types of crane are in general use by contractors for building operations; namely, the *portable jib crane* and the *fixed derrick crane*. Both are actuated by hand or power, but principally by the latter. The *portable jib crane* is used very largely in preliminary work on the site of a building in transporting material. Fig. 55 shows a contractor's portable or locomotive steam crane, which consists of a trolley *a* on which

is mounted a revolving crane *b* provided with a vertical steam boiler *c*, the power of which is available for operating the crane

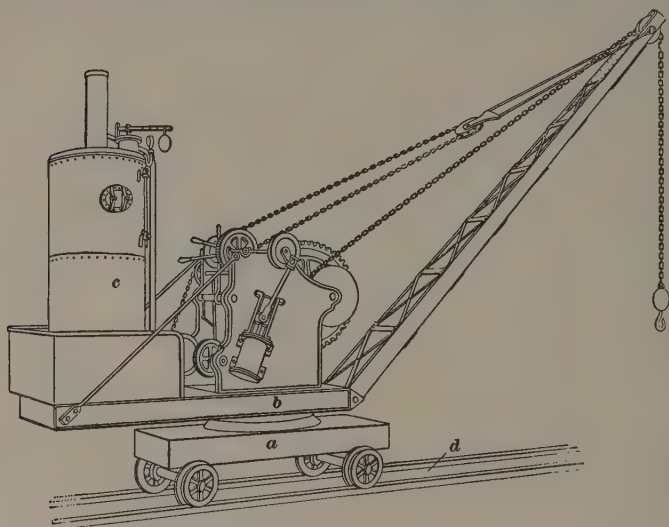


FIG. 55

and also for propelling it on a railed track *d*, by which means the crane can be transferred from one place to another with or without its load.

73. Derrick Cranes.—The derrick crane is distinguished by a compound system of gearing, by means of which a variety of operations may be performed, such as hoisting or lowering the load, sluing or turning the jib about its supporting centre, and raising or lowering the jib, by which the effective radius of the crane can readily be varied. Fig. 56 illustrates an ordinary type of crane operated by steam power. It consists of a cast-iron base plate *a* and radiating horizontal timbers or *sleepers* *b*. A mast *c* mounted on a revolving bed *d* carries the chain barrels and gearing, and is supported by two back stays or *guys* *e*, *e* secured to the horizontal sleepers. The head of the jib is supported by chains *f* passing over a sheave *g* in the mast head. The boiler *h* is of the vertical type. The engine has two cylinders,

and is fitted with reversible link motion, the revolving and travelling motions being controlled by double friction cones. This type of derrick crane is usually mounted on the gantry tower described in Figs. 34 and 35.

In steel-framed buildings, a different type of high-speed derrick has been used with great success. These derricks are light cranes mounted on and stayed to portions of the structural

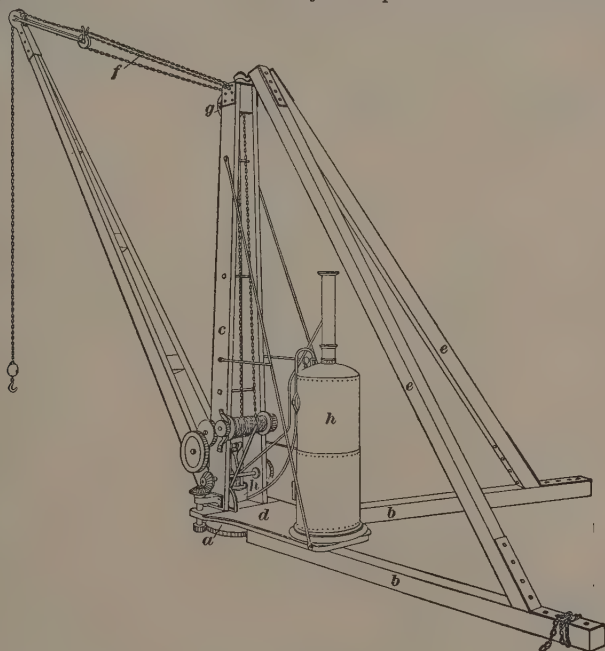


FIG. 56

steelwork, being erected, dismantled, and re-erected whenever and wherever required. The engine is situated frequently on the lowest portion of the building site, ropes being run from the drums of the engine to sheaves at the foot of the crane mast and thence to the jib head. The necessary operations of the crane are then signalled to the engine driver by a series of strokes on a bell.

HOISTS, OR LIFTS

74. Builders' Hoists.—Machines for simply raising or lowering weights, as distinguished from cranes which raise and lower weights and move the same in a horizontal or lateral direction, are called **hoists**, and may be actuated by hand or power. Such hoists range from a simple hand winch to the double-platform high-speed elevator. In both types, an economy of power is effected by installing an ascending and a descending hoist side by side, by which means the descending empty barrow or platform is utilized to hoist the ascending load.

FOUNDATIONS AND FOOTINGS

FOOTINGS

DEFINITIONS

1. The object of foundations and footings is to reduce the settlement of a structure to the minimum, and to ensure that any settlement which may occur shall be uniform. The term *foundation* is commonly applied to the natural material, or the portion of the earth's surface on which the structure rests, and also to those parts of a structure which are in contact with, or contiguous to, this natural surface, which is sometimes called the *foundation soil*, *foundation bed*, or *subfoundation*. Distinguishing terms should be employed to prevent confusion, and hereafter the term *subfoundation* will be used, and the following definitions will be adopted :

The **subfoundation** of a structure is that part of the natural surface of the earth on which the structure is to rest. The **foundation** of a structure is not only that part of the structure which is directly in contact with the subfoundation, but is also so much of the structure as may be needed properly to connect the structure with the subfoundation. Hence, the **foundation wall** of a structure is a wall used to connect the structure with the subfoundation, the necessity for foundation walls arising commonly from some peculiarity in the nature of the subfoundation or in the situation of the proposed structure. In order to distribute the pressure from the superstructure over a greater area of the foundations, the base of a wall is extended by means of a series of steps, technically known as **footings**.

DESIGN AND CONSTRUCTION

2. In many districts the minimum projection of footings is controlled by local regulations. These generally stipulate that the lowest course shall be twice the width of the wall, and that the footing courses shall be regularly diminished by offsets of $2\frac{1}{4}$ inches. Thus, in a brick wall $13\frac{1}{2}$ inches thick, the lowest course of footings must be 2 feet 3 inches wide and project on each side of the wall $6\frac{3}{4}$ inches, which is equal to three offsets of

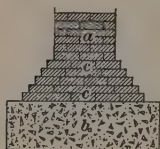


FIG. 1

$2\frac{1}{4}$ inches. This regulation does not, of course, necessarily constitute an accurate method of determining the width of footings for all cases, because to have any useful effect the footings must be of sufficient strength to resist the stresses to which, in each case, they may be subjected. Fig. 1 shows the section of a brick wall *a*, $22\frac{1}{2}$ inches thick, on an ordinary concrete foundation *b*, and having five courses of footings *c, c*. Each course has a clear projection of $2\frac{1}{4}$ inches, and the concrete is 6 inches wider on each side than the lowest course of footings, making the foundations 4 feet 9 inches wide. In

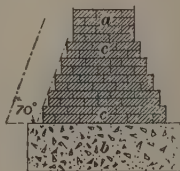


FIG. 2

Fig. 2 is shown a similar wall *a* on a concrete foundation *b*, but with each course of footings *c, c* two brick courses deep. As the angle shown by dotted lines is greater by this method, the strength of the construction is increased, but footings of this type require greater depth and will add somewhat to the cost of the building.

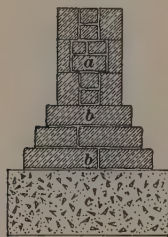


FIG. 3

3. **Footings to Stone Walls.**—An example of a stone wall *a*, 24 inches in thickness, with three courses of footings *b* is shown in Fig. 3. Each course is 8 inches thick and has a clear projection of 4 inches. The concrete *c* is 6 inches wider on each side than the lowest course of footings, and is 24 inches thick.

FOUNDATIONS

MATERIALS

4. **Types of Foundations.**—Foundations may be divided into two types: *deep* and *distributed*. To the former belong piling, caissons, foundation walls, etc.; and to the latter, planking, rafts, grillage, and many forms of reinforced-concrete construction. Combinations of these two systems are also used. The main factors governing the design of foundations are: the nature, and consequently the bearing power, of the subfoundation; the situation and uses of the proposed structure; the weights of the various parts and contents; and the regulations framed by local public authorities. The requirement of first importance is that the foundation shall be of sufficient strength permanently to support the proposed structure; on the other hand, no more should be spent on the construction than is dictated by intelligent skill and prudence. The materials in general use for foundations are: timber, plain and reinforced concrete, steel, cast iron, stone, and brick.

TIMBER

5. Timber has been used largely for foundations, because it is readily procured, convenient, and easily put in place. It is, however, comparatively short-lived, is easily destroyed by fire, and is liable to decay, especially when alternately wet and dry. It is, however, reliable when constantly saturated with water, if of good and suitable quality, but in this case should be thoroughly protected in submerged work against attack by wood-boring worms. The timbers most commonly used for foundation work are oak, elm, teak, beech, blue gum, green heart, pitch pine, Oregon pine, and Baltic fir. All timber should be given some

preservative treatment; creosoting is the most efficient. This consists in forcing into the pores of the wood a quantity of the dead oil of coal tar called creosote oil. Other processes are vulcanizing, kyanizing, burnettizing, and the creo-resinate and zinc-tannin processes.

CONCRETE

6. Plain Concrete.—In many districts it is obligatory to use concrete for foundations under all walls, except in the case of certain subfoundations. The foundations must have a minimum projection of 4 inches beyond the lowest course of footings, and a minimum depth of 9 inches. These requirements, however, must not be regarded as sufficient for all cases. **Concrete** is a mixture of cementing material, called the *matrix*, combined, by means of water, with small fragments of some hard material, called the *aggregate*, into a solid mass. The aggregate should be the hardest available, such as broken granite, limestone, hard varieties of sandstone, broken hard-burnt brick, slag, gravel, etc. For general purposes it should pass through a 2-inch ring, but for special purposes through a $1\frac{1}{2}$ -inch or even a $\frac{3}{4}$ -inch ring. It is not used alone, but is mixed with clean sharp sand for the purpose of filling all voids in the mixture and economizing the cement. When broken brick is used, care should be taken that the material is free from old mortar or plaster. Gravel, being rounded and smooth, does not adhere to the matrix so well as a sharper material. For foundation work in wet situations the matrix should be of *hydraulic cement*, that is, of cement which possesses the property of setting and hardening under water, such as Portland cement. For ordinary work, the proportions of the various ingredients, by volume, should be: 1 part of Portland cement, 3 parts of sand, and 6 parts of aggregate. For important work, where great strength is required, a proportion of 1 part of Portland cement, 2 parts of sand, and 4 parts of approved aggregate is advisable. No concrete should be made unless it is to be used at once, because the cement, forming its most essential part, sets or hardens quickly, and is valueless if it sets before being put in place.

7. **Concrete Foundations Under Water.**—It is sometimes necessary to lay concrete foundations under water, so it is necessary to devise means to prevent the cement from being washed out of the mixture and thus greatly impairing its strength. This is usually accomplished by filling the concrete into loosely woven bags which are lowered into position and carefully deposited, unopened, in regular order in the foundation, which is thus built up very much as blocks of stone in masonry. Sufficient mortar exudes through the material of the bags to cement them firmly together when in place. Specially designed boxes or buckets have been used. These have, at their bottom, an automatically controlled gate, which discharges the contents, when the bucket is sunk to the spot at which the concrete is to be deposited; and a wood or metal tube, called a *chute*, long enough to reach from the top of the water to the foundation, and capable of being moved along as the work progresses, has been largely employed. An additional 10 per cent. of concrete should be used in such cases, to compensate for that which may be washed out of the mixture.

8. **Reinforced Concrete.**—The terms *ferro-concrete*, *concrete steel*, *armoured concrete*, and *reinforced concrete* are applied indifferently to concrete in which is embedded steel in the form of rods, bars, shapes, or netting. The metal used with the concrete is called the **reinforcement**. The term **reinforced concrete** does not apply to those combinations of steel and concrete in which the steel is designed to support all the load; in reinforced concrete, the proportions and design are so arranged that the stresses are properly distributed between the two materials. The best mixture for reinforced-concrete work consists of 1 part of Portland cement, 2 parts of clean sharp river sand, and 4 parts of broken stone, in pieces not exceeding $\frac{3}{4}$ inch in any direction. Concrete for reinforced work should be impermeable, or the steel reinforcement may be attacked by rust, as it is not treated by any anti-rust process, as the adhesion of the concrete to it would be impaired. In designing constructions of reinforced concrete the tensile resistance of the concrete, alone, should be neglected, as a single crack will destroy its tensile power.

STEEL AND IRON

9. At one time foundations were seldom constructed wholly or even mainly of steel or iron, but special conditions have recently led to such constructions in a number of instances. Where great strength must be developed within limited space, or great weight must be transferred from one point to another, steel is the most adequate material. Iron and steel are also used in comparatively small quantities, such as for bolts and cramps in timber and masonry work ; for piles, etc. ; and steel rods are important elements in the construction of reinforced concrete. In foundations of rolled steel, if suitably designed, a maximum stress of $7\frac{1}{2}$ tons per square inch is permissible, where the loads are wholly static or quiescent. The chief objection to the use of wrought iron and steel in foundation work is their liability to oxidize or corrode in the presence of air and moisture. Even where the metal is permanently submerged in water, the oxygen of the air contained in the water may be sufficient to cause serious oxidization, unless the metal is protected in some way. The important fact seems well established, however, though the reason for it is not fully understood, that iron or steel embedded in concrete, whether dry or wet, does not oxidize, and will remain for very long periods of time free from rust or deterioration. Combinations of steel and concrete are specially applicable to certain foundation work, and, when constructed with proper care, such work may confidently be expected to prove durable and satisfactory. All steel and wrought iron used in foundations should be encased in concrete, wherever possible. Cast iron is comparatively free from oxidization, even when exposed to moist air.

STONE AND BRICK

10. *Stone.*—The several varieties of stone are described in *Building Stones*. Those used in foundations are chiefly hard varieties of granite, slate, limestone, and sandstone. They are generally of large size, and laid as squared stone and rubble masonry. The stone should be laid on its natural or quarry

bed, especially if laminated in structure. Stone footings should be laid with large flat stones not less than 8 inches thick. If more than one course is to be laid, the vertical joints should not come over one another, but the stones should be laid to break joint. All stones should be well bedded in cement mortar, and all the joints and spaces between the stones should be completely filled.

11. Brick.—Only sound hard-burnt varieties of brick should be used for foundations, and they should be laid in cement mortar. In footings, all outside bricks should be laid as headers, to distribute the weight of the superstructure over a greater width of the subfoundation or foundation.

PILES AND CAISSONS

CLASSIFICATION OF PILES

12. Piles are columns or beams of wood, metal, or other material driven into the earth to support a structure. They are used when suitable material for subfoundation exists at such a distance below the surface that it is impracticable to reach and utilize it by the ordinary methods of excavation; these are known as *bearing piles*. Piles are also used close together to surround a poor subfoundation and prevent lateral movement under the increased pressure; these are known as *protection piles*. **Bearing piles** may be divided into two types, depending on their application: these are **column piles**, which are used to support vertical forces or loads by being driven down until they reach a hard stratum, and **friction piles**, in which the friction of the earth against their sides is utilized to bear the pressure from the superstructure. **Protection piles** may also be distinguished by two applications: that of consolidating the soil by being driven at close intervals, thus compressing the subfoundation into a compact mass; and as a retaining medium when used to enclose a portion of the subfoundation which is liable to lateral escape, or to protect it from the action of water. Piles for the latter purpose are called **sheet piles** and consist of flat piles driven successively edge to

edge as closely as possible. They are often tongued and grooved, or birdsmouthed, on the edge, for the purpose of forming a water-tight joint.

BEARING PILES

13. Timber bearing piles are generally from 9 inches to 12 inches square and from 5 feet to 20 feet in length, but in certain cases these dimensions have been greatly exceeded. They are round, square, or rectangular in section. The lower end is pointed, as shown in Fig. 4 (a), care being taken that the points are truly on the axis line. The spacing of piles varies with the nature of the soil and the design of the superstructure, but is seldom less than 3 feet between centres. The tops are spanned by timbers, large stones, or concrete, and on the platform thus constructed is built the base of the superstructure. Round piles are sometimes the trunks of large straight trees, driven with their small ends down. The butt end is cut off square, to receive the hammer truly. The small end is usually pointed to an angle of about 30° and is often shod with iron. If the pile is to be used in very soft soil, it is often better to leave the small end blunt, because, when the pile has but little lateral support, any small obstruction is liable to deflect the pointed end, while a blunt pile will drive such small obstacles before it.

14. Shoeing Piles.—When piles are driven through soft material to rock or hard gravel, the force of the blows of the

hammer has a tendency to split the piles, after rock or hard gravel has been reached, thus greatly impairing their bearing capacity; to prevent this, piles are often protected at the end with wrought- or cast-iron shoes. Fig. 4 illustrates three methods of shoeing the ends of

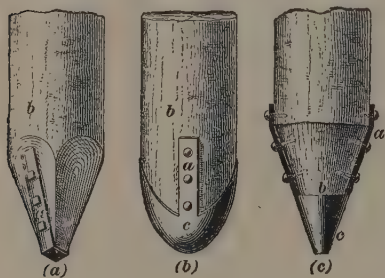


FIG. 4

piles. In (a) is shown a $2'' \times \frac{1}{2}''$ wrought-iron strap *a* bolted

through the pile *b*, forming a shoe, which is the same on both sides of the pile. In (*b*) is shown a cast-iron conical shoe fitted over the end of the pile *b*; the head of the shoe *c* protects the end of the pile, and the straps *a*, one on each side, hold the shoe in place. One of the best forms of cast-iron shoes is shown in (*c*). In this case the pile has a blunt end from 4 to 6 inches in diameter, shown at *b*. The shoe has a solid conical point *c*, the top of its base being of about the same diameter as the end of the pile; the straps *a* then extend up on the sides of the pile, and are bolted or spiked to it, as shown. The straps and bolts hold the shoe in place while it is pushed through the soil. Where square piles are used, a cast- or wrought-iron shoe is fixed on, as shown in Fig. 5.



FIG. 5

15. Ringing Piles.—The tops of each pile should be cut, or chamfered, for a few inches from the end, as shown in Fig. 6, so that a wrought-iron ring *a*, about 1 inch in thickness and 3 inches wide, will fit tightly over the end of the pile when struck with a hammer or ram. Sometimes a ring from 1 to 1½ inches less in diameter than the pile is simply placed on the top of the pile, and driven into it with light blows. This, however, is not so desirable as the former method, as the ring is liable to split long pieces from the sides of the piles, and, being applied generally when the pile is more



FIG. 6

or less battered on the end, is likely to be carelessly fitted and not placed concentrically with the head of the pile. The rings are used in pile driving in order to lessen the tendency of the pile to split and to prevent the splintering of the end fibres on the end of the piles, due to repeated blows of the ram.

16. Protection of Piles.—When piles are exposed to seawater, they are liable to be attacked by various wood-boring worms, which will penetrate the piles and destroy ordinary timber in from 3 to 5 years. Creosoting or charring are therefore generally resorted to, though in some cases it is effectual merely to drive round piles, without first stripping the bark.

METAL PILES

17. **Iron and Steel Piles.**—Iron piles are usually hollow cast tubes or columns; steel tubes consist of single rolled shapes or of two or more such shapes riveted together to form a single member. A cast-iron pile is shown in Fig. 7. This type, owing



FIG. 7

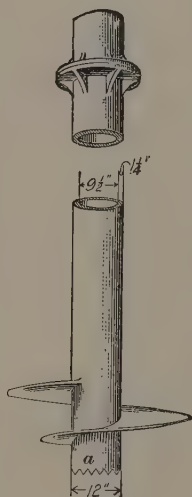


FIG. 8

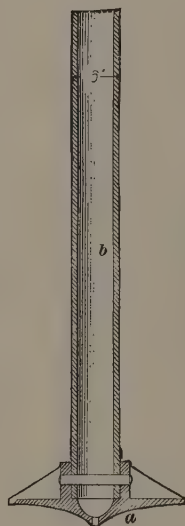


FIG. 9

to its original cost and the difficulty experienced in driving it, is not extensively used.

18. **Screw piles**, shown in Fig. 8, are forced into the ground by means of a cast-iron screw on their lower end. The pile may be of wood, steel, or cast iron, but it is usually made of hollow metal and provided with teeth *a* on its lower edge to assist in

cutting. The screw is usually about 4 feet in diameter, and has a pitch of about 7 inches. Screw piles are driven by a capstan and levers fixed at the top; as they descend, extra lengths are added. Such piles may be used in most soils, as they will sink through sand or gravel and even small boulders. They may be sunk near a building where the shock from a pile driver on an ordinary pile would be dangerous. Hollow cast-iron piles, when used as screw piles, are frequently filled with concrete after they are in place, to strengthen them and to increase their bearing surface.

19. **Pneumatic piles** are cylinders, or caissons, of steel or cast iron which are sunk by excavating from the inside. The pressure of air maintained within them prevents an influx of water. When the piles have been sunk to the proper depth and the interior excavated, they are filled with concrete.

20. A **disc pile**, Fig. 9, usually has a cast-iron disc *a* on a steel shaft *b*. This wide circular flange greatly increases the bearing area of the pile. This pile is sunk by washing away the earth beneath it with a stream of water.

SAND PILES

21. **Sand piles**, Fig. 10, are used where the ground is soft, but not fluid. They are not effectual in very loose or wet soils as the material is liable to work into the surrounding earth. Sand piles are made as follows: An ordinary pile is driven 5 or 6 feet into the ground and is then withdrawn; this operation leaves a hole with the soil packed tightly round it. The hole is then filled with sand, which is put in a few inches at a time and then tamped until the hole is filled. The holes may be made with an auger instead of a wooden pile, and this method is used when the shock of pile driving might endanger adjoining buildings, but otherwise a wooden pile is preferable, as it compresses the soil at the sides.

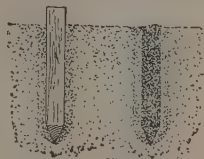


FIG. 10

The bearing power of a sand pile depends as much on the friction against the sides, due to the arching of the grains against one another, as on the direct load carried to the bottom of the hole.

CONCRETE PILES

22. Concrete piles may consist of plain concrete, but are more usually of concrete reinforced with steel rods. They possess the great advantage that they do not decay like wood nor corrode like steel piles. A thin collapsible steel tube or shell, shaped like the intended pile and called a *pilot pile*, is driven into the ground in much the same manner as an ordinary wooden pile. It is then partly collapsed and withdrawn and the hole filled with concrete. Where the ground is too soft to maintain a hole when the pilot pile is withdrawn, a thin casing of metal, or other material, is fitted over the pilot pile and driven with it; then, when the pilot pile is withdrawn, the casing preserves the walls of the hole until it is filled with concrete.

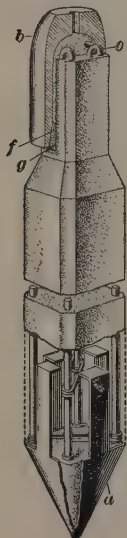


FIG. 11

REINFORCED-CONCRETE BEARING PILES

23. The usual form of reinforced-concrete pile is shown in Fig. 11; it is driven by the water-jet process, or by careful driving with a pile driver. The pile has an iron shoe *a* and is reinforced by four steel rods, which are laced together with wire at regular intervals. When the pile is driven, a steel cap *b* filled with sand *c* acts as a cushion to distribute the force of the blow evenly over the top of the pile. The sand is kept in the cap by means of clay *f*, which in turn is held in position by hemp packing *g*. Where reinforced-concrete piles are used in connection with a raft, or slab, of similar material deposited over the surface of the subfoundation, the top concrete of the piles is broken away and the reinforcement is properly incorporated into the distributed foundation.

DRIVING PILES

24. Pile Driving.—Fig. 12 illustrates the most usual type of drop-hammer pile driver. It consists of a timber frame *a* supporting a pulley over which is carried a rope or wire cable.



FIG. 12

The ram, or weight, *b*, generally a block of iron weighing from 5 to 45 hundredweights, is attached to the rope by a hook, or an automatic clutch, *c*; the other end of the rope is

wound round the drum of a crab winch worked by manual or steam power. In operation, the ram is raised and the pile is hoisted into position and stayed to the frame of the pile driver, after which it is driven into the soil by repeated blows of the ram, which is raised by the winch, and dropped, in rapid succession. Fig. 13 (*a*) shows a detail of the hook. When the cord *a* is pulled, the hook *b* is made to release the ring of the ram. In (*b*) is shown an automatic clutch and release, which dispenses

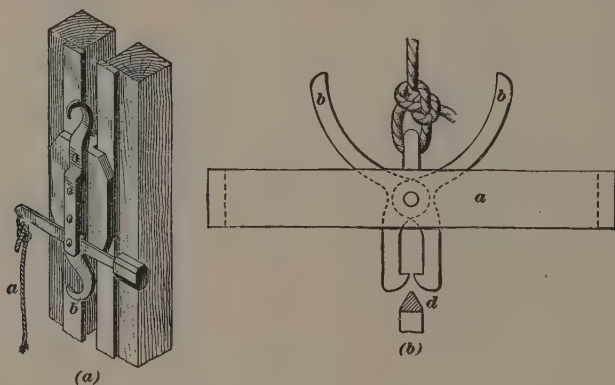


FIG. 13

with the cord *a* shown in (*a*). The short cross-beam *a*, Fig. 13 (*b*), slides in guides on the timber frame and carries a pair of nippers *b b*; the cable, also, is attached to it. When the cross-beam *a* is raised to the top of the frame, the curved arms *b, b* of the nippers are closed by sloped tripping-blocks, so placed that the rounded ends of *b, b* rub against them and are forced inwards, thus opening the nippers and releasing the eye *d* attached to the weight.

25. Steam-Hammer Pile Drivers.—In steam-hammer pile drivers the hammer is directly operated by steam. The general principle is shown in Fig. 14, where *a* is the steam cylinder, *b* the piston rod, and *c c* the hammer; *d, d* are guides, the upper ends of which are attached to the steam cylinder and the lower ends to the anvil block or plate *e*. These rods pass through

holes in both sides of the hammer, thus guiding its motion. The block *e* has a conical hole through its centre, as shown by dotted lines, through which passes the projection *f* on the hammer.

Steam, conveyed to the cylinder by a flexible pipe or hose, is admitted below the piston, forcing it and the attached hammer upwards. At the end of the upward stroke the steam is automatically cut off, the exhaust port is opened, and the hammer and piston fall by their own weight to their original position; steam is again automatically admitted, and the operation repeated. The whole mechanism is placed between guides on the frame, and is suspended by the line that, in the drop-hammer machine, carries the hammer. When the pile is in place, the whole apparatus is lowered on to its top, which is usually dressed approximately to fit the conical hole in the block *e*. Steam is then turned on and the hammer begins to work, striking rapid blows and following the pile as it descends. Experience shows that a heavy hammer and continuous quick blows, even up to thirty per minute, are best in pile driving. For this reason a steam pile driver is to be preferred.

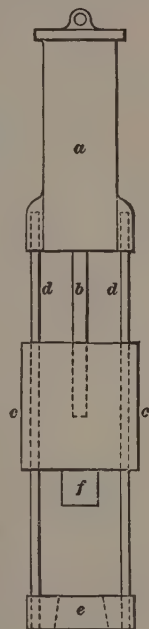


FIG. 14

26. Sinking Piles by Water Jet.—Where piles are to be used in sand or comparatively loose soil, they may be sunk into position by the use of a jet of water. In this method the lower end of the pile is fitted with a nozzle discharging downwards, generally connected by means of a fixed pipe running through the centre of the pile to a hose or pipe which in turn is attached to a force pump. When the pile is placed in position, the pump is started and a strong jet of water is discharged against the sand or other material immediately below the end of the pile. The result is to soften and cut out the material, allowing the pile to settle by its own weight, or to be forced into place either by additional weight placed on its top, or by comparatively

light blows of the driver. When the operation is completed and the current of water discontinued, the surrounding material settles into close contact with the pile, giving it the necessary stability and bearing power. Under favourable conditions, piles may be driven by this method more rapidly and cheaply than with the ordinary pile driver, especially in dry sand, which offers great resistance to piles forced into it.

CALCULATIONS FOR PILE FOUNDATIONS

27. The efficient bearing capacity of piles in different soils is very indefinite. A pile may go down through stiff clay or gravel for some distance, sinking very little at each blow of the ram, and may then penetrate a soft stratum in which it will sink nearly as much at the last blow as at the first. In this case, the friction on the sides of the pile keeps it in place. But this friction is so great that even in marshy ground the ultimate bearing capacity of a pile 30 feet long has been found to be from 6 to 9 tons ; in alluvial soil, or moderately soft clay, the bearing capacity is from 9 to 13 tons ; and in stiff clay, or compact sand, from 13 to 14 tons. Knowing the weight of the ram, the height it falls, and the penetration of the pile at the last blow of the ram, the following rule may be used for determining the ultimate load on a pile, in hundredweights :

Rule.—Multiply the weight of the ram, in hundredweights, by the height it falls, in inches, and divide the product by eight times the distance, in inches, through which the pile is driven at the last blow of the hammer.

Expressed as a formula :

$$L = \frac{Wh}{8d}$$

in which L = safe load, in hundredweights ;

W = weight of ram, in hundredweights ;

h = height through which ram falls at last blow,
in inches ;

d = distance through which the pile is driven at
the last blow, in inches.

EXAMPLE.—The weight of a pile-driver ram is 12 hundredweights; height of fall, 15 feet, or 180 inches; penetration of pile at last blow, 2 inches. What is the safe bearing capacity of the pile?

SOLUTION.— $L = \frac{12 \times 180}{8 \times 2} = 135 \text{ cwt. or } 6.75 \text{ tons, the safe load}$
which the pile will carry. Ans.

28. In all calculations of the bearing power of piles, a large factor of safety is necessary, owing to the many uncertainties connected with the subject. The weight on each pile should, in general, be limited to $2\frac{1}{2}$ hundredweights per square inch of sectional area, or 18 tons on a $12'' \times 12''$ pile. If the supporting material is clay or wet ground, this load should be reduced to 12 tons, but in gravel the load may be correspondingly increased.

TERMS USED IN PILE DRIVING

29. The technical terms used in pile driving are descriptive of the form or position of the piles, or the manner of their driving. A **close pile** is one set close to another pile when the one already driven shows signs of weakness. A **false pile**, or **follower**, is an additional length added to a pile for deeper driving. Fig. 15 shows the manner of connecting piles; *a* is the first pile, *b* is the follower, *c* is a dowel to preserve the alinement of the two piles, *d, d* are wrought-iron straps, made usually of $20'' \times 2'' \times \frac{1}{2}''$ iron, to bind the two piles together. A **wale** is a horizontal string piece to bind the piles together. **Pile hoops** are the bands placed round the tops to prevent splitting.

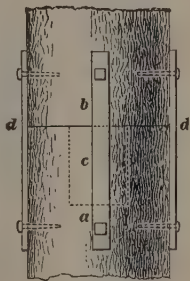


FIG. 15

PROTECTION PILES

30. Consolidating piles are similar in all respects to bearing piles, and require, therefore, no separate description. Sheet piles are flat piles cut obliquely at the foot for the purpose of drawing

them together in driving; they are chamfered to a cutting edge, and are generally shod with iron, as shown in Fig. 16.

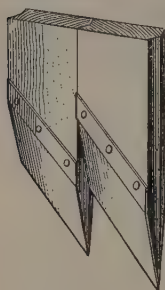


FIG. 16

The driving of sheet piles to enclose a portion of the subfoundation, as shown in Fig. 17, is accomplished by first driving guide piles *a*, similar to ordinary squared bearing piles, at the corners of the portion of subfoundation to be maintained and at intervals of from 5 to 10 feet between. These piles are driven to within 3 feet of the surface of the subfoundation. Stout waling pieces *b*, about 9 inches by 6 inches, are notched and bolted in two pairs to the portion left standing, one pair on each side of the guide piles. The sheet piling *c* is then inserted vertically between the top and bottom waling pieces, and is driven into the subfoundation. Fig. 18 shows the application

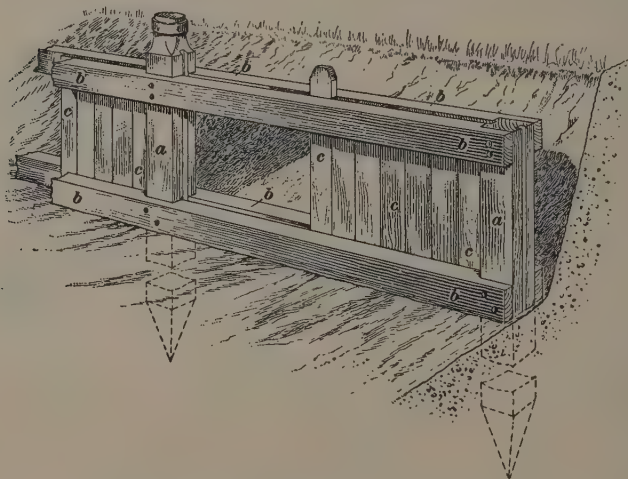


FIG. 17

of sheet piling to confine quicksand under a concrete foundation. In this case, sheet piling *a*, *a* is placed 4 feet apart; the concrete *b*, in quicksand *c*, is 2 feet thick and extends the full width of the piling, supporting the 22 $\frac{1}{2}$ -inch brick wall *d*.

31. Reinforced-Concrete Sheet Piles.—Fig. 19 shows reinforced-concrete sheet piles, which are similar in construction to the reinforced-concrete bearing piles described. The foot of the pile, however, is shaped obliquely and the close joint is formed by means of a cement tongue. A projection *a* near the base of each pile slides in the groove of the pile last driven. Above this is placed a closely fitting iron pipe, which serves as a guide in driving the

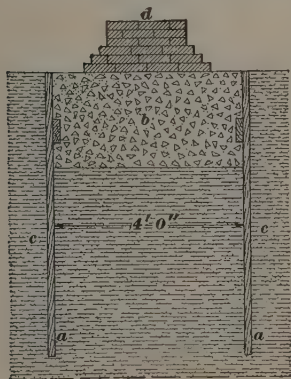


FIG. 18

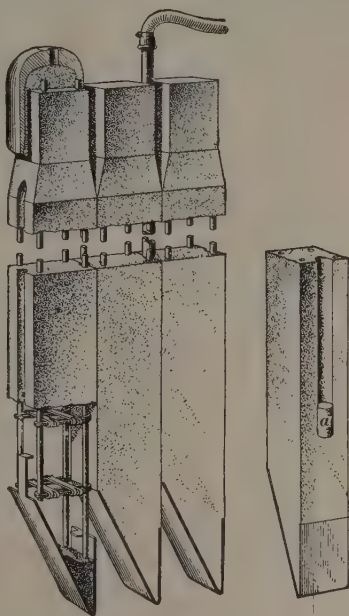


FIG. 19

pile. Any sand that might jam the grooves, which are in both sides of each pile, may be forced out by water from a pump with which the pipe is connected. After the pile is driven, this pipe is withdrawn and a water-tight joint made by filling the grooves with cement.

32. Steel sheet piles are manufactured, and have been largely used, principally in the construction of coffer dams, the consideration of which is outside the scope of this Course, for, if an architect should be called upon to design any construction of this nature, he should at once consult an eminent engineer thoroughly acquainted with such work.

CRIBS, OR CAISSONS

33. A caisson is a chamber of iron or wood used in the construction of deep foundations. Formerly, they were used exclusively for the foundations of bridge piers, but the advent of steel construction and the consequent erection of very tall buildings have caused them to be used for building foundations. There are two methods of reaching the required foundation. In the first, the desired depth is reached by excavating the material from the interior of a large timber or iron box, or cylinder, which is then forced to sink by placing a sufficient weight or load in it until the required depth is reached. Structures of this kind are called *open caissons*. The second method is used when the foundations are to be carried to a great depth in very soft material or in water. The lower part of the caisson is formed into an air chamber resting on the soil at the bottom. Air is pumped into this at a pressure corresponding to that at the depth below the surface, and the excavation is carried on by men working at the bottom, as in a large diving-bell. This structure is called a *pneumatic caisson*. After the caisson is sunk to the proper level, either to rock or compact gravel, the inside space is filled with concrete. If the open caisson is used, the piers are built on this concrete, but if the pneumatic method is used, the piers are built on top of the caisson itself. The work of sinking the caissons is, as a rule, outside the province of the architect, and is usually entrusted to some firm of engineering contractors who make a speciality of work of this description, and who leave everything in readiness for the construction of the foundation walls or piers.

FOUNDATION WALLS

STONE RUBBLE FOUNDATION WALLS

34. Stone foundation walls below ground, when concealed from outside view, are usually constructed of rough rubble, as shown in Fig. 20. This represents an elevation (*a*) and

section (b) of a 20-inch rubble stone wall *a, a*, 10 feet high, with footing stones *b* 8 inches thick and 2 feet 8 inches wide. A

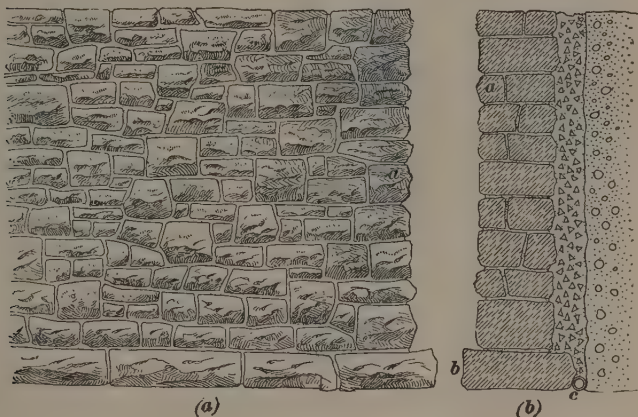


FIG. 20

drain *c* is used to keep water from the surface of the wall. All stone walls, 24 inches or less in thickness, should have at least one header *a*, Fig. 21, extending through the wall every 3 feet of height and every 4 feet of length. In walls over 24 inches thick there should be at least one header that runs into the wall at least 2 feet for every 6 superficial feet of wall space. The headers should not be less than 18 inches wide and 8 inches thick,



FIG. 21

and should be good flat stones. They serve to bind the stones together in the walls and keep the foundation from splitting apart crosswise when weight is placed on it. All wall angles should be well tied by means of long stones *a, a*, Fig. 22, laid alternately in the wall. In this way the weight on the corners of the wall is equally distributed, and the wall can be kept plumb and true.

35. Foundation stones should always be laid on their natural, or quarry, beds. The

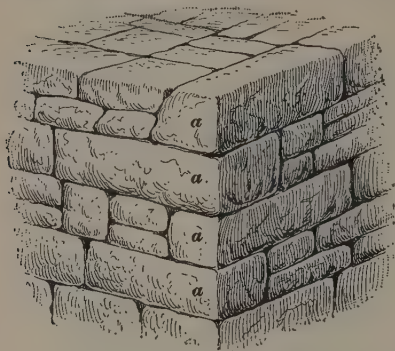


FIG. 22

tendency to splitting, or cleavage, in a stone is with the grain or bed; so that when the stone is laid on its natural bed, the weight of the material placed above it comes against the grain or lamination. No stone should have more face than bed, and one side, at least, of each stone should be reasonably

flat. Every stone laid should be well bedded in mortar.

36. The usual practice among masons in rough walling is, after setting the larger stones, to fill the interstices with spalls, or chips, of stone, or even pebbles, more or less carefully fitted, and put in dry; then to dash in mortar, trusting that it will work its way into the crevices. It does so to some extent, but the method is not a good one. A good, conscientious workman will place no stone, even the smallest chip, except in a bed of mortar prepared to receive it, rubbing it well in, and settling it with blows of the trowel and hammer, so that all the stones have a layer of mortar between them. A common practice is to *grout* the walls with mortar made so thin that it will flow into the spaces between the stone.

37. For good work, it is necessary that the outside of the wall, even when concealed by the excavated bank, should be carried up with a good face, and the joints should be well filled with either cement, or lime-and-cement mortar. If this is properly done, any moisture which runs out from the bank or descends from above, so as to flow down over the outer face of the wall, will drip off, instead of running into the joints. The space between the outside of the foundation and the bank of the excavation should be filled with gravel or sand, preferably

the former, well packed. Water will then soak through this sand or gravel to the drain below ; this method makes the cellar drier and warmer, and keeps much of the moisture away from the foundation walls.

FOUNDATION WALLS PARTLY ON ROCK

38. A very faulty construction, sometimes met with, is that in which the foundation is built partly on a ledge of rock and partly on the footings. This is shown in Fig. 23, where *a* is the footing course ; *b*, the rock projecting into the foundation wall ; *c*, the thin wall in front of the rock to bring the foundation to the thickness required by the plans ; and *d*, the wall of the building resting partly on the thin wall *c* and partly on the ledge of rock *b*.

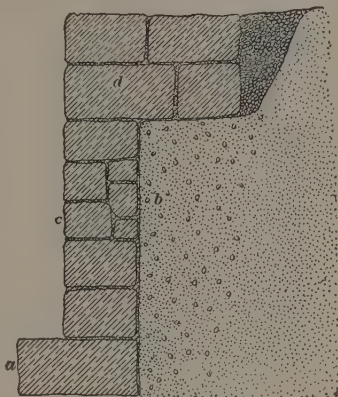


FIG. 23

In a wall so built, the water will find its way through the imperceptible seams of the ledge of rock *b*, or over its top into the body of the masonry, keeping it constantly damp. Besides, under the heavy weight of the upper wall, the thin lining built up against the ledge, but in no way bonded to it, may separate from it and fall away, leaving the superincumbent masonry most insecurely supported upon the rock alone. There is, also, the certainty that the foundation wall, built partly on unyielding rock and partly on softer soil, will settle unequally, and crack, perhaps injuring the masonry above beyond repair, and at least opening an inlet for moisture.

The proper method of construction is to cut away the ledge so as to leave ample space for the whole thickness of the foundation walls down to the footings, with sufficient space between the wall and the ledge of rock for packing gravel. This will intercept the water and carry it away from the wall.

THICKNESS OF WALLS

39. A very good rule for determining the thickness of foundation walls, where the matter is not controlled by the regulations of any local authority, is, that they shall be at least 9 inches thicker than the wall next above them, for a depth of 12 feet below the ground level. They should be increased $4\frac{1}{2}$ inches in thickness below that point, for every additional 10 feet or less in depth. Thus, if the ground-floor walls are $13\frac{1}{2}$ inches thick, the stone foundation walls would have to be $22\frac{1}{2}$ inches thick for 12 feet in depth, and 27 inches thick below that point for 10 feet or less. Rubble-stone foundation walls are seldom made less than 18 inches in thickness. A wall 18 inches thick is not always necessary to carry the superimposed weight, but smaller walls are relatively more expensive to build and consequently are seldom constructed. The thickness of foundation walls in all towns, cities, and in almost all the Rural Council districts is controlled by the building by-laws. Where there are no existing laws Table I will serve as a guide.

TABLE I
THICKNESS OF FOUNDATION WALLS

Height of Building Stories	Dwellings, Hotels, Etc.		Warehouses	
	Brick Inches	Stone Inches	Brick Inches	Stone Inches
Two	$13\frac{1}{2}$ to 18	22	18	22
Three	18	22	$22\frac{1}{2}$	27
Four	$22\frac{1}{2}$	27	27	32
Five	27	32	27	32
Six	27	32	32	36

OPENINGS IN BASEMENT AND CELLAR WALLS

40. When there are window or door openings in a basement wall, the stones under the opening should be laid in steps, as shown at *a*, *b*, *c*, Fig. 24. This distributes the weight under

the windows. When there is to be a heavy weight on the foundation wall, the window sill *d* should be made just the length of the opening and should not project into the wall. This allows the sill to move if there is pressure against it, and thus reduces the liability to fracture. This style of sill is called a slip sill. Precautions should be taken, where slip sills are used, to prevent water from percolating into the wall at all vertical joints.

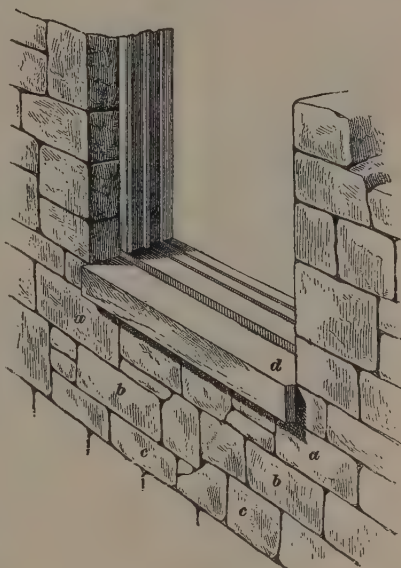


FIG. 24

BRICK FOUNDATION WALLS

41. Brick foundation walls do not differ materially from ordinary brick walling. The bricks should be of a hard sound variety, laid in cement mortar and carefully bonded. In the example of subfoundation of unequal character shown in Fig. 25, view (*a*) shows the rock *b* and gravel *a* before levelling or excavation. In (*b*), portions of the rock have been removed to a certain level *b* and the softer soil *a* excavated and levelled as at *c, c*. A bed of concrete *d* about 3 feet thick has then been deposited to the level place in the rock, and on this the brick or stone foundation wall *e* has been built.

42. It is not necessary, on solid rock, to have the foundation bed cut level over its entire surface, nor even cut into a series of horizontal surfaces resembling steps, as is frequently done in softer soils; but the surface of the rock should be roughened so

that the possibility of slipping of the foundation will be prevented. After this is done, concrete may be put in to bring the foundation to its proper level. Where the proposed structure

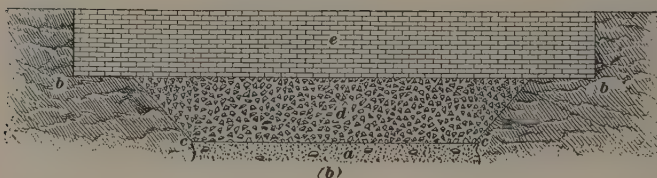
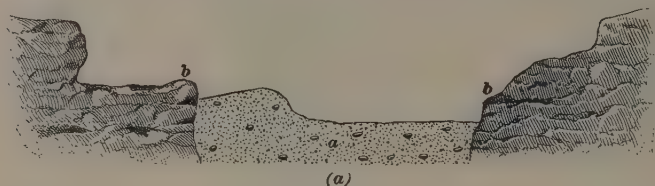


FIG. 25

is three or four stories in height, stone or brick may be used in place of concrete, but a concrete base is usually preferable.

FOUNDATIONS ON SLOPING GROUND

43. Foundations built on slopes, especially of clay, are always likely to slide. This may be avoided by cutting horizontal

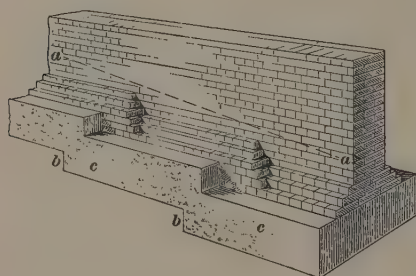


FIG. 26

steps in the slope, as shown in Fig. 26, where the original slope of the ground shown in relation to the wall by the dotted line *aa* is stepped off as shown at *b, b*, in order that the foundations *c, c* may have a horizontal bearing. These foundations may

be either of stone or concrete, but when the former material is used, great care must

be exercised to secure a perfect bond at the stepping places, and the foundations should be laid in as long sections as possible.

44. Arches in Foundation Walls.—For the purpose of concentrating the distributed weights of a structure on isolated

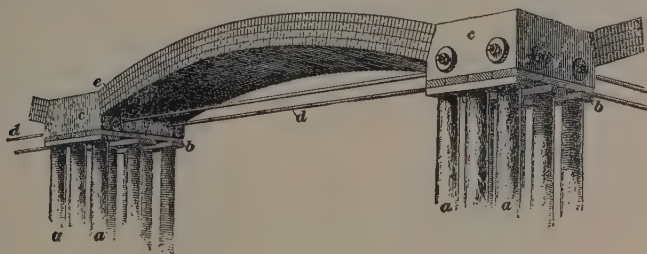


FIG. 27

portions of the subfoundation, arches of brick or stone are used, if the depth of the proposed foundation is not strictly to be limited. Fig. 27 shows a brick arch utilized to concentrate the weight of the superstructure on clusters of timber piles *a*. On

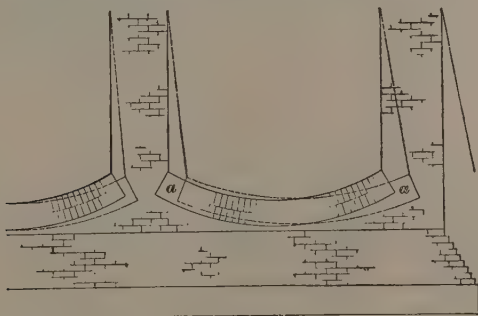


FIG. 28

the piling, 12" $\times$ 3" planks *b, b* are laid transversely and longitudinally, and spiked to the top of the piles. On these are laid stone skewbacks *c, c* which are tied together with 1 $\frac{3}{4}$ -inch iron rods *d, d* secured with screws, washers, and nuts. The brick

segmental arch *e* is built up of four brick rings, which spring from the stone skewback.

45. Inverted arches are used in foundations for the converse purpose of distributing the concentrated weights imposed by piers, continuously on the subfoundation. Care should be exercised in the use of inverted arches, for unless properly designed they are apt to exercise thrusts on the piers which are less heavily loaded. It is necessary also to provide extra abutment for the end arches of the system, otherwise the weight might throw out the pier, as shown by the dotted lines at *a, a*, Fig. 28. This difficulty may be overcome by the use of iron

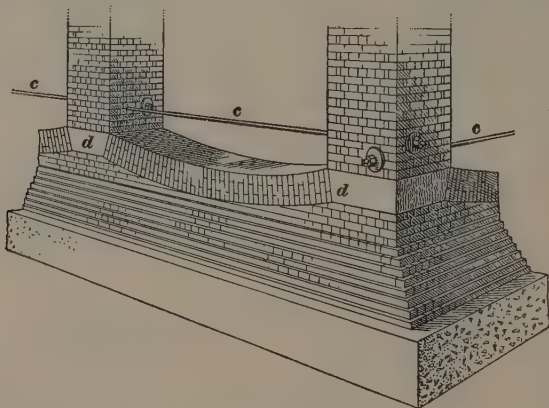


FIG. 29

rods *c*, with washers and nuts, as shown in Fig. 29, which hold the bases of the piers in place and prevent the skewbacks from slipping.

46. The best form of inverted arch is the three-centred, or elliptical; next, the pointed; third, the semi-circular; and lastly, the segmental arch. A method of setting out the lines for centring an elliptical arch is shown in Fig. 30.

On the springing line *ab* of the arch, which is divided into six equal parts, are drawn three circles having a radius equal to one of these parts as *ac*. Then a line *ef* is drawn

perpendicular to ab through the centre c of the middle circle and intersecting the circumference at f ; ce is the height of the arch. The point f will then be the centre from which the curve of the arch from g to h is to be struck; the points g and h being

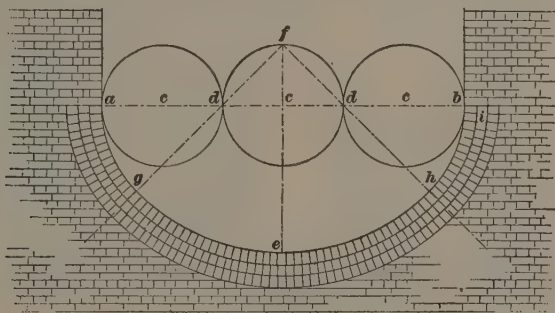


FIG. 30

those where lines drawn from f through d intersect the arch line. The points d are the intersection of the circles with the springing line. The curves ag and hb are drawn with d as centres. The arch i consists of three rows or rings of bricks in this case, but may be constructed of any number of rings necessary for the strength required to distribute the concentrated weights over the subfoundation.

CONCRETE FOUNDATION WALLS

47. Concrete foundation walls are built either in large blocks, which are cast in moulds and laid like blocks of stone in masonry, or in monolithic courses from 6 to 18 inches deep. Where the concrete wall is below the surface of the subfoundation, timbering or forms may be necessary, unless the sides of the trench are self-sustaining. Concrete is deposited in the trench in layers, usually from 6 to 12 inches deep, which are thoroughly rammed until the water in the mixture flushes. If one layer sets before the next is applied, the surface of the first is swept, raked, or picked to form a key, and is sprinkled with water before fresh concrete is deposited.

DISTRIBUTED FOUNDATIONS

PLANKING

48. The method of utilizing planking under walls for foundations is shown in Fig. 31. Here $12'' \times 3''$ planks *a* are cut into

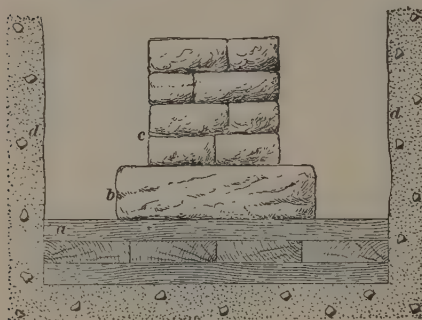


FIG. 31

short lengths and laid in three or more layers on the subfoundation at the bottom of the trench *dd*, the direction of the planking in alternate layers being reversed. The large stone footing course *b* is laid under the foundation wall *c*. Fig. 32 shows a foundation composed of planking and larger timbers, with the addition of concrete. Upon 2-inch planks *b* laid across a 12-inch bed of concrete *a* placed in a trench are laid the $8'' \times 8''$ timbers that support the large footing stones *d* which support the foundation wall *e*. The $8'' \times 8''$ spaces between the timbers are filled with concrete, and the spaces *f* between the wall and side of the trench are filled with sand, which is carefully rammed down. A foundation of this type has been used for the erection, upon marshy ground, with water-soaked sand at a depth of about 5 feet, of a factory building, 80 feet by 50 feet

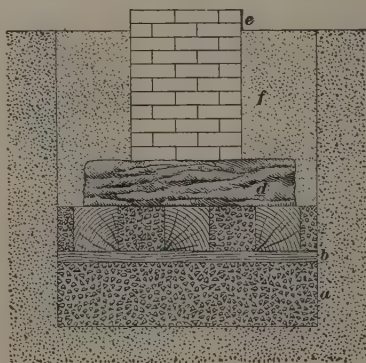


FIG. 32

and 40 feet high, equipped with boiler, engine, shafting, and machinery, and employing a large number of operatives. Fig. 33 (a) and (b) shows in plan and elevation an isolated

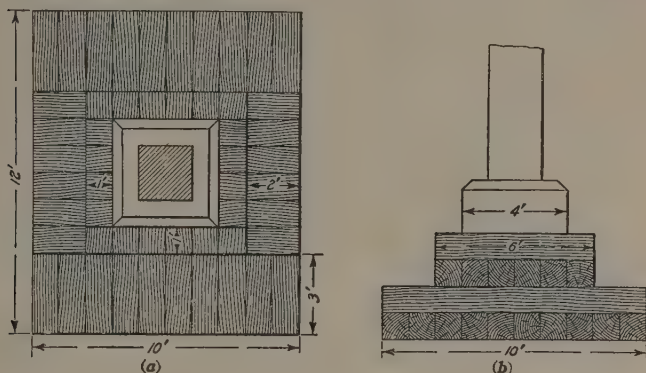


FIG. 33

foundation for a pillar, composed of $12'' \times 12''$ timbers laid side by side in four rows, the two lower rows being increased in length to distribute the weight over the subfoundation. This form is permissible only in a permanently wet stratum.

RAFTS

49. Rafts are foundations covering the whole, or the greater part, of the site of a building, and are formed of a continuous layer of cement concrete, upon which the structure is erected. A foundation of this type must carry safely all the varying loads of the several parts of a structure, and may therefore be uneconomical if certain portions of the building are very heavy.

GRILLAGE FOUNDATIONS

50. Timber Grillages.—Grillage foundations consist of two or more layers of horizontal beams placed in tiers crossing each other. Timber grillages, Fig. 34, consist of square barks *a, a* called *stringers*, and cross-pieces *b, b* notched or framed together. Where used over timber piling they are secured to the pile heads by drift bolts. If the pile heads and timber grillage are liable

to alternate wetness and dryness, the ground should be excavated

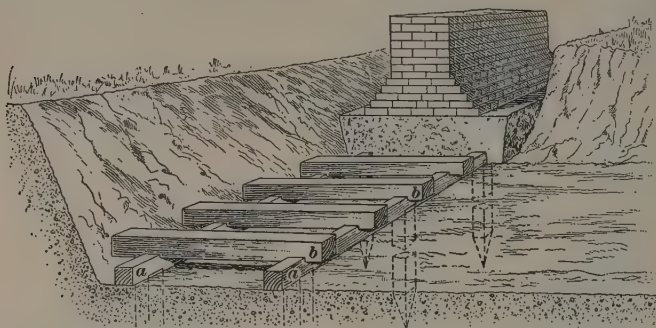


FIG. 34

and a sufficient depth of concrete deposited and consolidated between the piles and between the members comprising the grillage.

51. Iron and Steel Grillages.—Originally, steel rails crossed in alternate layers were used, for they were easily obtained, shallow in depth, and offered a considerable resistance to transverse stress. But, since rolled structural shapes are easily obtained, they are more economical, as their weight is less in proportion to their strength. They are also more readily connected together into a system by means of bolts and separators. A foundation constructed of steel beams embedded in concrete and carrying a wall, though not strictly to be called a grillage foundation, is shown in Fig. 35. Steel beams *a* 10 inches deep, embedded in cement concrete *b*, are placed transversely to the direction of the wall *c*, the weight of which they distribute over the subfoundation. In order to thoroughly protect the steel,

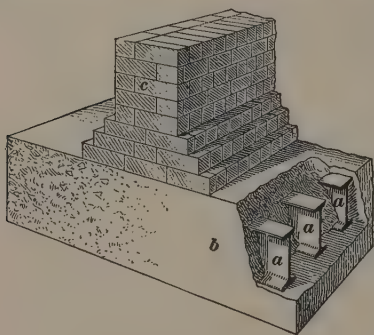


FIG. 35

tion constructed of steel beams embedded in concrete and carrying a wall, though not strictly to be called a grillage foundation, is shown in Fig. 35. Steel beams *a* 10 inches deep, embedded in cement concrete *b*, are placed transversely to the direction of the wall *c*, the weight of which they distribute over the subfoundation. In order to thoroughly protect the steel,

the concrete foundation should be made 3 inches wider at each end than the length of the beams.

52. Steel grillage foundations are used chiefly under piers, or stanchions, carrying great concentrated loads. In Fig. 36 is shown the effect of transverse loading on a plain concrete slab, which illustrates the necessity for some form of strengthening other than the plain material where great depth is inconvenient or not economical. Fig. 37 illustrates a grillage foundation suitable for a column, pier, or stanchion. At *a* and *b* in the plan (*a*) are shown the beams, and at *c, c* the foundation. The

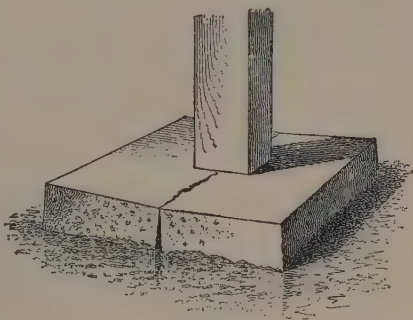
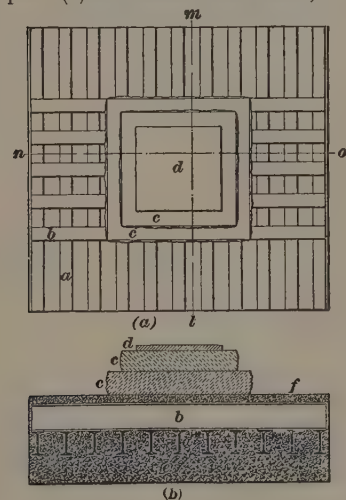


FIG. 36



upper tier of beams *b* is laid transversely on the lower tier *a*, and the stone footings *c, c* carry the 3-inch iron plate *d* under the column. In (*b*) is shown a section of the foundations taken on line *no*, on the plan (*a*), showing the lower tier of beams in section and the upper tier in elevation.

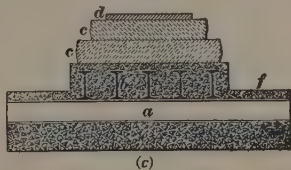
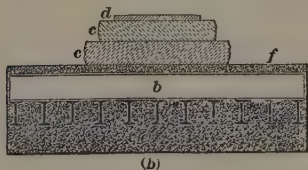


FIG. 37

The 12-inch course of concrete is shown at *e*; the lower tier of beams at *a*; the upper tier of beams at *b*; the concrete covering the beams at *f*; the base stones of the pier at *c, c*; and the iron plate under the iron column at *d*. In (*c*) is shown a section taken on line *l m* on the plan (*a*), where the concrete base, on which the lower tier of beams rests, is shown at *e*, the lower tier of 10-inch beams at *a*, and the concrete over the upper tier of beams at *f*; *b* is the upper tier of 10-inch beams shown in section, and *c, c* the stone footing-courses under the iron plate *d*.

Fig. 38 shows a similar foundation in which steel rails *a* are used instead of the beams; *b* is the base stone supporting the

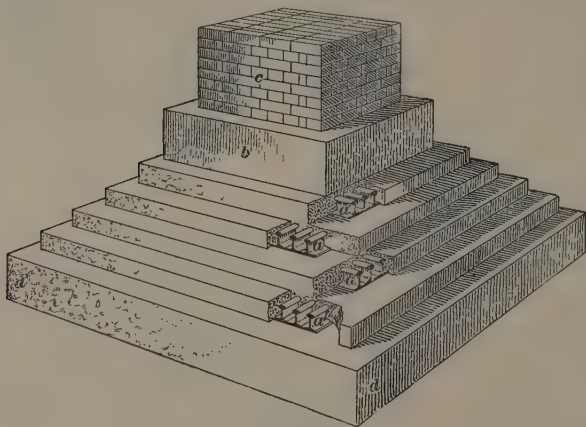


FIG. 38

brick pier *c*; and *d* is the concrete base. In this case each layer of rails diminishes in length and number until the area of the top layer does not greatly exceed the size of the pier base, the foundations and footings thus assuming a pyramidal form.

53. In the construction of grillage foundations, the nature and bearing power of the subsoil should be thoroughly investigated at the actual points at which the isolated foundations will occur. The weight of the foundation itself must be added to any calculation of the load upon the superstructure. All steelwork should be cleaned with wire brushes, to remove all rust and

mill scale, and thoroughly painted or washed over with cement grout. It must be carefully bedded and encased in concrete, for which purpose a space of not less than 3 inches should be left between the flanges of adjoining beams. Concrete, having a smaller aggregate, should be used between the flanges and carefully rammed. The beams should be bolted together through the webs, and separated by cast-iron or rolled-steel distance pieces. The position of all holding-down bolts, for securing the baseplate of the stanchion, or column, should be ascertained before the foundation is laid.

REINFORCED-CONCRETE FOUNDATIONS

54. The most usual types of reinforcement for concrete are steel bars or netting. Ordinary round bars have been very largely used, but within recent years many patent bars, with mechanical bond of corrugated, twisted, or other surfaces, have

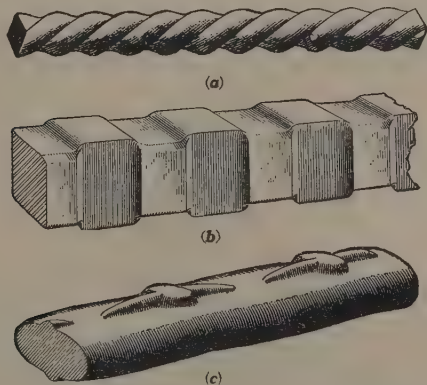


FIG. 39

been put on the market. The makers of these bars state that there is a tendency for the smooth bars to slip through the concrete, and claim that twisted or corrugated bars increase the grip of the concrete on the steel. Fig. 39 illustrates three types : (a) represents a bar used in the Ransome system of concrete reinforcement, and is called the *Ransome twisted bar* ; (b) is the

indented bar; and (c) is the *Thacher bar*. Whether it is better to use a few patented bars for a certain piece of work or a greater number of ordinary round bars is simply a question of expense. The patented bars cost more, but are more efficient. Their

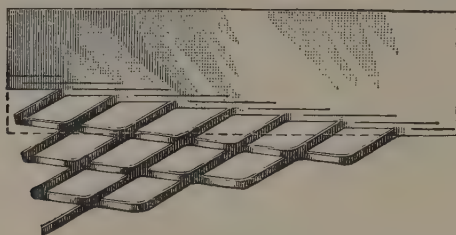


FIG. 40

greater cost is due to their being patented and to the fact that more labour is required in their manufacture. Another kind of reinforcement which is sometimes used for foundation work is *expanded metal*. This is made from sheet steel, which is slit and then stretched out, or expanded. Fig. 40 shows a sheet cut and partly expanded. The steel is coated to prevent corrosion.

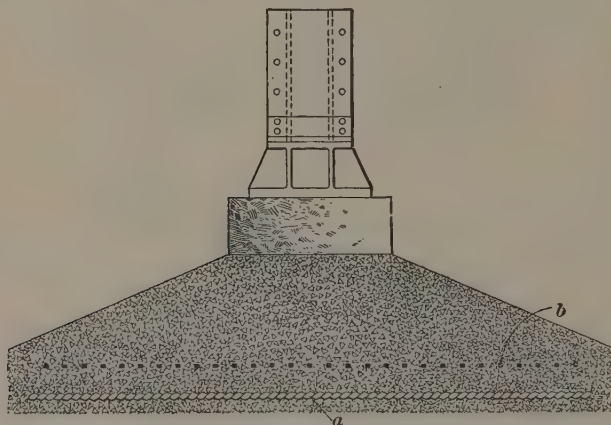


FIG. 41

55. Fig. 41 shows a section of a concrete foundation reinforced with steel bars of the Ransome type. The bars are laid

in rows at right angles to each other, as shown at *a* and *b*. In building these foundations, a layer of concrete from 3 to 6 inches thick should first be laid and the first tier of longitudinal bars placed and tamped. Another layer of concrete 4 inches in thickness is spread and the transverse bars are then laid; in this manner, as many courses of concrete and steel as are necessary may be laid and tamped. Fig. 42 illustrates a spread foundation reinforced with expanded metal, which is laid in the same manner as the bars in Fig. 41. The stress on the foundation is greater in the centre than toward the edges; an extra piece of expanded metal *a* is therefore placed in the middle of the foundation.

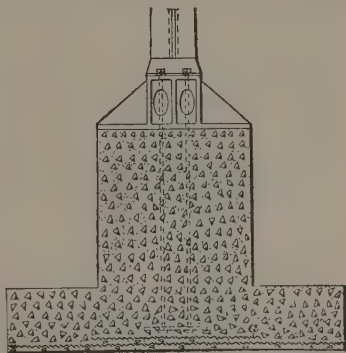
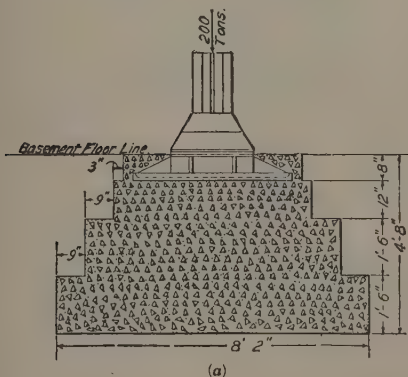
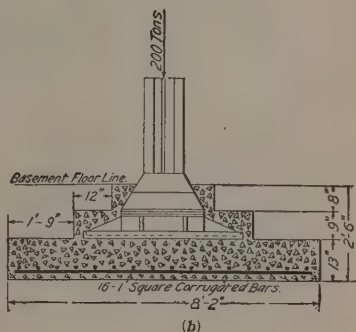


FIG. 42



(a)



(b)

FIG. 43

56. Economy of Reinforced Concrete.—Fig. 43 illustrates the saving of material and labour over plain concrete foundations by the use of steel reinforcement. In (a) is shown a concrete

foundation without steel reinforcement, while in (b) is shown a foundation designed for the same strength but reinforced with the indented bar. The cost of the foundation shown in (a) is :

Excavation, 11.53 cubic yards at 3s. . . .	£1	14	7
Concrete, $7\frac{1}{2}$ cubic yards at 20s. . . .	7	10	0
Total cost of solid-concrete foundation .	£9	4	7

The cost of the foundation shown in (b) is :

Excavation, 6.18 cubic yards at 3s. . . .	18	6	
Concrete, 3.18 cubic yards at 20s. . . .	3	3	7
Extra length of column, 53 pounds at $1\frac{1}{2}$ d. .	6	7	
Corrugated bars, 720 pounds at 1d. . . .	3	0	0
Total cost of reinforced-concrete foundation	£7	8	8

These foundations have been proportioned to sustain a column load of 200 tons, or about 3 tons per superficial foot on the ground.

DESIGN OF FOUNDATIONS

PROPORTIONING FOUNDATIONS

57. It is very important that the foundations, whether continuous, as in a foundation wall, or isolated, as in piers, should be designed in proportion to the weight which they will be required to carry and to the bearing capacity of the soil. The weight is divided into several classes or loads, among which are the **dead load**, or the weight of the materials of which the building is made, and the **superimposed load**, or the weight of furniture, occupants, goods in storage, machinery, etc. The *wind load* and *snow load* are of less importance. The pressure on the soil from each superficial foot of the foundations should be the same, where the soil is uniform, and at no place must the safe bearing power of the soil be exceeded. To secure the most satisfactory results, therefore, the foundations must be proportioned to properly distribute the weight which is to be carried over a sufficient area of ground to secure uniform settlement in each case. If these conditions are properly considered, there will be few

cracks in the building, as such cracks are caused usually by unequal settlement. A uniform settlement even of an inch or more is, in most buildings, unimportant.

58. Fig. 44 (a) shows a tower with continuous foundation. The masonry of the walls does not bear on the foundations under the door, but only at the piers at both sides of the door, as indicated by the two arrows pointing downwards. The foundation at these two places must therefore carry a greater load than the foundation in the middle and will therefore settle more,

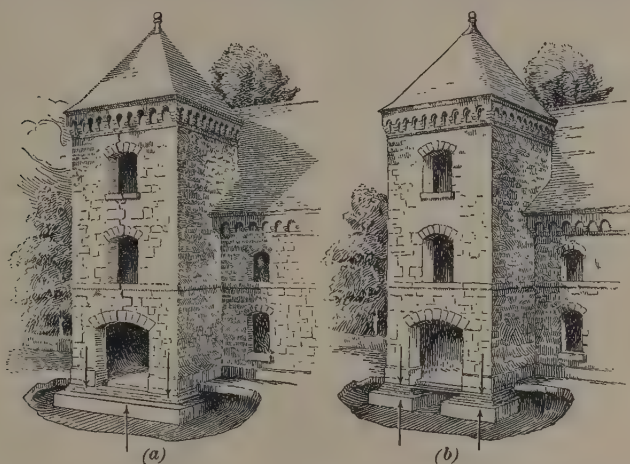


FIG. 44

causing the wall above to crack, as shown. To prevent this, the foundation should be isolated, as shown in Fig. 44 (b). In this case, if the foundations are properly proportioned, the walls will settle evenly at all points and will not crack. Of course, for buildings of light weight and with small doors, this precaution need not be taken.

59. In order to determine the area of isolated foundations in buildings with continuous outside foundations, the weights to be carried by each internal pier, or stanchion, and the weight of, and loads carried by, all the walls, should be ascertained and

entered in a book for reference. The ground should be examined, and the load per superficial foot which it is deemed advisable for the footings to carry should be determined. See table in *Preliminary Building Operations* for bearing power of different soils. The load on the various foundations divided by this unit load will give the proper area of each in square feet.

60. Where the exterior walls of a building are the support for the floors and are of brickwork or masonry, it would be theoretically correct, in designing the footings under the cast-iron or steel columns in the interior of the building, to have them exert 10 per cent. more pressure, per unit, on the soil than is exerted by the footings under the walls. This greater settlement under the columns, or stanchions, will compensate for the settlement which will take place in the body of the walls owing to the shrinkage of the mortar joints. This condition, however, does not exist to so great an extent when cement mortar is used, and can then be neglected.

61. One object of proportioning foundations is that the building shall settle evenly without cracking the walls. In order to obtain this result, the foundations must be proportioned according to the *average load* which they will carry and not to the *greatest load*, although, of course, they must be sufficiently large to support safely any weight which is likely to come on them. That is, the superimposed load which would actually exist in the building should be considered, and not the maximum superimposed load that might be, but probably never would be, applied to all the floors at one time. If the maximum superimposed load is used, the foundations under piers, which support a very large floor area compared with those under the walls, will be so large that the settlement will be very little compared with that of the foundations under walls, and, consequently, cracks will occur. If the average load which the building will carry is used, the settlement will be uniform; but in proportioning the foundations for this average load, a smaller bearing value of the soil should be assumed so that the footings will be large enough to support the maximum load if it should ever be applied. It is therefore necessary to observe

the rule that foundations shall be proportioned to the actual average loads they will have to carry in the completed and occupied building, and not to the theoretical or occasional loads. Care must be taken, however, when proportioning foundations to the average load, that the maximum load per superficial foot on footings does not exceed the safe bearing value of the soil.

62. For factories, stores, etc., 50 per cent. of the superimposed load which the floors are to carry should be added to the dead load carried on the foundations. For office buildings, hotels, dwelling houses, etc., the weight of the people occupying them need not enter into the calculations for proportions of foundations, and only from 25 to 30 pounds per square foot of floor need be allowed for the weight of furniture, books, safes, etc. Statistics prove that the average permanent loads do not exceed these limits, just stated. For theatres, halls, etc., a larger allowance should be made for the weight of people, but even a densely packed crowd of men will not weigh more than 120 pounds per foot superficial of floor area. As the result of experiment, it is customary in practice to provide for the following loads on floors, which loads include also the weight of the floor itself:

CLASS OF BUILDING	HUNDREDWEIGHTS PER SQUARE FOOT
Ordinary dwelling houses	$1\frac{1}{4}$
Public buildings, lecture rooms, etc.	$1\frac{1}{2}$
Warehouses, factories, etc.	$2\frac{1}{2}$ to 4

The loads in the last case depend on the nature of the goods to be stored or the kind of manufacture to be carried on.

CALCULATING REQUISITE AREA OF FOUNDATIONS

63. Fig. 45 represents a six-story warehouse that is to be built on a thick bed of ordinary dry clay and have a basement or cellar. The plan is shown in (a), the longitudinal section in (b), and the transverse section in (c). The building is 50 feet wide inside the walls at the lowest story and a double longitudinal

row of stanchions *a*, *a* support the rolled-steel girders *b*. The load on 1 linear foot of the side walls will be $167\frac{1}{2}$ cubic feet

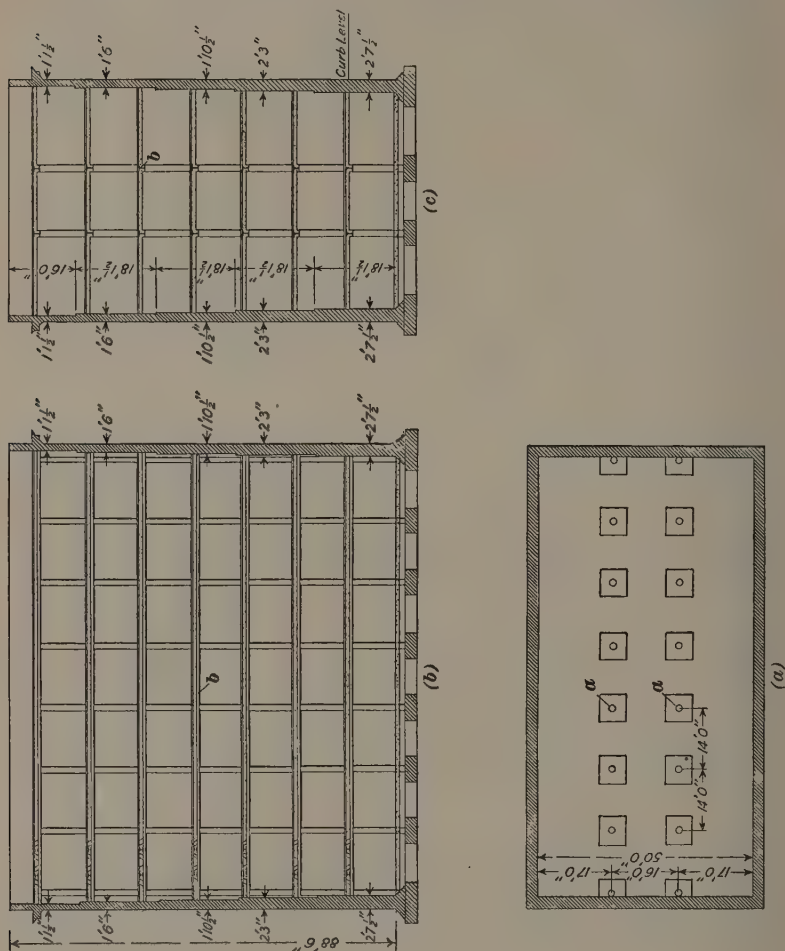


FIG. 45

of brickwork weighing 112 pounds per cubic foot, or 18,760 pounds. One foot of wall must support $8\frac{1}{2}$ feet superficial of

each floor and the same amount of roof area. This area can be calculated from the plan (a).

64. The floors are assumed to be constructed of steel joists filled in with hollow terra-cotta blocks, with cement filling on top. This floor will weigh, altogether, 75 pounds per foot superficial. The roof is also of fire-resisting construction, but has lighter steel joists, and weighs 60 pounds per foot superficial. Thus, the dead load from the six floors and the roof will amount to $8.5 \times (6 \times 75 + 60) = 4,335$ pounds. The first, second, and third floors are supposed to carry 168 pounds per foot superficial, and the fourth, fifth, and sixth floors 112 pounds per foot superficial, the weight of snow on the roof being taken as 12 pounds per foot superficial. The total superimposed load on the footings amounts to $8.5 \times (3 \times 168 + 3 \times 112 + 12) = 7,242$ pounds per foot superficial. If the wall, floor, and half of the superimposed load are added, the weight per linear foot of wall that the foundations are required to carry will amount to $18,760 + 4,335 + \frac{7,242}{2} = 26,716$ pounds. The soil can safely

support a load of 8,000 pounds per square foot, according to the table of bearing power of different soils in *Preliminary Building Operations*. In this calculation, only one-half the superimposed load has been considered. This was done so that the building would settle evenly without cracking; therefore, a smaller safe load for the soil must be used, or 5,000 pounds per square foot. Dividing 26,716 by 5,000, gives approximately 5 feet 6 inches as the required width of the foundations.

65. To obtain the load on the foundations under the columns, the weight of the floors and the roof, together with the superimposed load, is calculated, the weight of the columns themselves being so little, in proportion to the other loads, that it need not be considered in the present example. Suppose that the columns are spaced 14 feet apart longitudinally and 16 feet transversely, then each column will support $14 \times (8 + 8\frac{1}{2}) = 231$ square feet of floor; thus the dead load on the foundations under the columns amounts to $231 \times (6 \times 75 + 60) = 117,810$ pounds, and the superimposed load is $231 \times (3 \times 168 + 3 \times 112 + 12)$

= 196,812 pounds. Therefore, the foundation must be designed to carry $117,810 + \frac{196,812}{2} = 216,216$ pounds on each pier.

According to Art. 60, the safe load per square foot must be 10 per cent. greater than 5,000 pounds, the safe load per square foot for the exterior walls. Therefore, the safe load for the column bearing will be 5,500 pounds. Dividing 216,216 by 5,500 gives 39.31 square feet, or about $39\frac{1}{2}$ square feet. The columns in the end walls, since they carry one-half the load each of an interior column, have each one-half the area of footing.

66. The dimensions of the front and back wall foundations are easily obtained; since these walls carry no load except their own weight, they are termed **curtain walls**. They contain $167\frac{1}{2}$ cubic feet of brickwork for each foot of length. Taking the weight of brickwork as 112 pounds per cubic foot, the weight of the walls per linear foot is $167\frac{1}{2} \times 112 = 18,760$ pounds. Dividing this weight by the safe allowable bearing of the soil, or 5,000 pounds, gives 3.75 feet, or 3 feet 9 inches for the width of the foundations.

67. Besides being correctly proportioned, the foundations must be given their proper position under the wall or pier. The wall or column in house building usually bears evenly on the foundation, that is, the centre of pressure is at, or almost at, the centre of the wall, as shown by the arrow *a*, Fig. 46 (*a*). If it is not so situated, it is usually due to heavy wind or cantilever loading, and the problem is more especially within the province of an engineer. The line bisecting the wall should cut the base of the foundation in its middle third; that is, assume that the base of the foundation *f e*, Fig. 46 (*a*), is divided into three equal parts, *f c*, *c d*, and *d e*. The line *a* must cut the base between *c* and *d*. If it does not, the foundation is liable to turn, as shown in (*b*). The centre line of the bottom of the foundations is shown at *b g*. It is not necessary that the lines *a h* and *b g* coincide, but simply that *a h* cuts between *c* and *d*. In fact, the foundation is usually built so that the line *b g* will lie outside of *a h*. This has a tendency to make the foundations tip the

wall slightly inwards, so that it will press against the joists and cross-walls, and will be stayed and buttressed by them, which

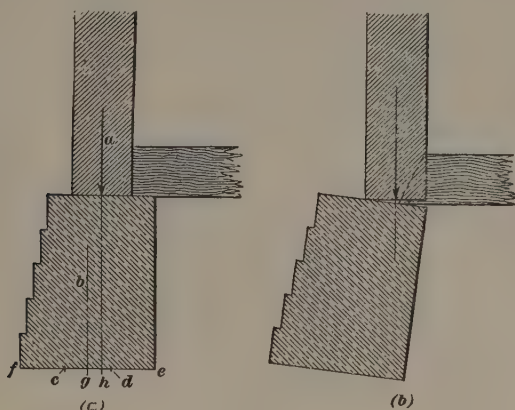


FIG. 46

is better than allowing the wall to tip outwards and bulge and thereby cause the ends of the joists to slip off their bearings.

PROTECTION OF BASEMENT WALLS

DAMPNESS IN CELLAR WALLS

68. Damp cellar and basement walls are due either to water soaking through from the outside or to wet soil underneath, from which water rises in the walls by **capillary attraction**, that is, the water is soaked up by the stone or brick in the wall as it would be by a sponge. This action is increased by vertical joints in the materials of which the wall is made. The effect of



FIG. 47

capillary attraction is shown in Fig. 47, which illustrates a corner in a stone wall. This effect is more noticeable in corners than in a straight wall, and more in brick than in stone.

DAMP-PROOF COURSES

69. To prevent the absorption of water into the building by capillary attraction, it is necessary to interpose a layer of impervious material between the wet earth and the wall to be protected, or between those portions of the wall which are built into and directly in contact with the earth and those portions which it is essential to protect against dampness. The impervious layer is called a **damp-proof course**, and is commonly of slate, asphalt, stoneware, sheet lead, or similar materials in various forms.

70. Slate Damp-Proof Course.—Two courses of sound thick slates, bedded in cement and laid to break joint, form an inexpensive damp-proof course and one readily obtainable in most districts.

71. Asphalt Damp-Proof Course.—The asphalt damp-proof course is perhaps the most efficient and most general for good work. Natural asphalt should be laid while hot in two coats of $\frac{3}{8}$ inch each, and is equally suitable for horizontal or vertical waterproofing. Since inexperienced workmen find it difficult to lay natural or rock asphalt, sheet asphalt may be obtained in widths corresponding to the thicknesses of the walls. These rolls are simply opened on the wall at the requisite level, special welts or lapped joints being made where necessary. The most general market forms are: asphalted felt, fibrous asphalt, reinforced asphalt, and asphalt lead, the latter forms being sheets of asphalt surrounding a central layer of woven mesh or thin sheet lead. The use of felt, in such a situation is to be deprecated, as it cannot be regarded as a permanent building material. A **sheet-lead damp-proof course** is efficient, but expensive.

72. Vitrified Stoneware Damp-Proof Course.—The vitrified stoneware damp-proof course is made in the form of grooved

blocks, Fig. 48, from 2 to 3 inches thick, and pierced transversely with a series of holes, which are useful in providing ventilation to the space below floors. This material is expensive, and necessitates the ordering and use of special blocks at the angles, and special widths to suit the walls, involving also the question of transit, and in certain situations might be considered unsightly.

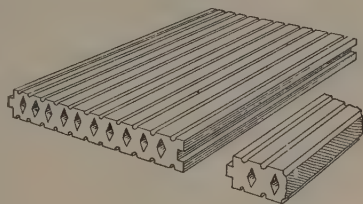


FIG. 48

73. Ground Layer.—The surface earth and vegetable mould should be removed from that portion of the surface of a site

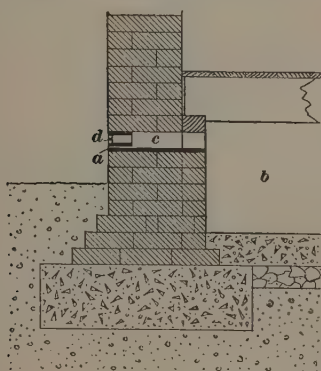


FIG. 49

which is actually to be covered by the proposed building, and a layer of cement concrete 6 inches thick, called the **ground layer**, should be spread over the whole area thus exposed. This will prevent the penetration of ground air and ground moisture into the building. When the floor of the lowest story is above the level of the ground, as shown in Fig. 49, the damp-proof course *a* should be placed not less than 6 inches above the level of the adjoining earth, or paved surface, and should be taken through the full thickness of the wall.

If the floors are of wood construction, the whole of the woodwork in contact with the wall of the building should be placed above the level of the damp-proof course. The space *b* thus formed between the ground layer and the floor-joists should always be ventilated, to obviate the danger of dry rot by which the woodwork might be attacked. This is accomplished by

means of channels *c* formed through the walls at intervals of about 5 feet, the outer mouth of such channels being protected by a grating *d*.

When the floor of the lowest story is below the level of the adjoining ground, or paved surface, one of the examples shown in Figs. 50 and 51 will be suitable.

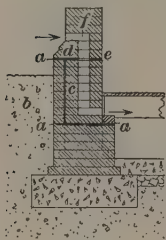


FIG. 50

74. In Fig. 50, two horizontal damp-proof courses are shown at *a, a*. If these alone were used, the portion of wall between the two damp-courses would be without protection from the wet earth *b* adjoining and from surface moisture. Vertical damp-proofing *c* is therefore introduced, to protect the basement room or cellar from dampness. Where economy is an important consideration, the portion *d, e* of the upper damp-proof course might be omitted, care being taken to ensure good joints in the waterproof material wherever a change of direction occurs. Because the outer portion of brickwork between *a, a* cannot be toothed or bonded into the brickwork adjoining, without unduly complicating the damp-proofing, this portion of brickwork is sometimes built as an addition to the general thickness of the wall, and is formed into a plinth as shown. The alternative arrangement is indicated on the figure by dotted lines. Ventilation to the space under floor boarding is secured by means of the traversed, or bent, channel *f*. In Fig. 51, the vertical damp-proofing material is replaced by the cavity *a*.

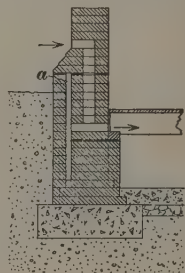


FIG. 51

DAMP-PROOFING CELLAR WALLS

75. Some clay soils, though sufficiently solid to support the walls of dwelling houses, retain moisture in wet seasons which is not carried away into the earth, but rises in the cellar, keeping it nearly always damp. Fig. 52 shows a method of

forming a damp-proof cellar floor and walls which has proved successful. The floor is first prepared by laying 3 or 4 inches of broken brick or stone *a* over the ground, which is then covered with a 3-inch layer of Portland-cement concrete *b*. When the cement concrete is thoroughly dry, it is covered with a $\frac{1}{2}$ -inch layer of natural asphalt *c*, which is carried through the walls at *d*, then up the outside of the wall at *e*, and through the wall again

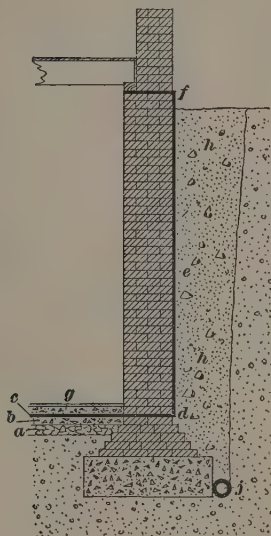


FIG. 52

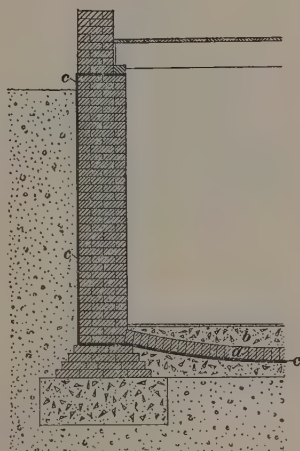


FIG. 53

at *f* to form a damp-proof course above the ground line. Another layer of concrete *g* is laid on the asphalt if there is a tendency of the water to accumulate with any pressure under the floor, as it might otherwise burst the floor upwards. The space *h, h* should be filled with rubble and an agricultural drain pipe be laid at *j*, and provided with an outlet; this will help greatly to keep the wall dry.

76. In the case of basements and cellars below the permanent level of the ground water, or below the flood level, the upward

pressure on the floor may be considerable, and an inverted arch *a*, Fig. 53, may then be necessary. The space *b* above the arch *a* is filled with concrete up to the level of the floor. The damp course *c* is continuous, forming a kind of water-tight tank. When the ground is only moderately damp, and there is no additional pressure of moisture after rains, a good floor may be prepared as shown in Fig. 54. The ground is covered with a layer of broken brick or stone *a*, on which is placed a layer of concrete *b* composed of 6 parts of sand and 1 part each of cement and lime. On this are laid 3" × 3" creosoted redwood nailing fillets *c*, the spaces between which are filled in with concrete *d*. On this is laid the wooden flooring *e*.

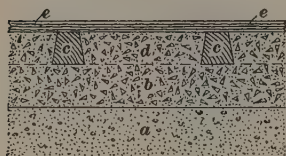


FIG. 54

77. A durable composition for a cellar floor is made of 6 parts of asphalt, 1 part of coal tar, and 3 parts of sand. It should be used while hot. Cement and asphalt in equal parts are also used and form durable cellar floors. The ingredi-

ents should be placed over a fire in a large pan or boiler and thoroughly mixed, after which the composition should be spread over the surface.

SUBSOIL DRAINS

78. A simple method of excluding moisture from cellars is shown in Fig. 55. The excavation *a* is made wider than the building, so that there will be about 1 foot of space left between the bank and the foundation wall. A tile drain *b* should be placed at the bottom of this trench and connected with a drain discharging either into a surface-water drain or a properly trapped connection to an inspection chamber of the ordinary drainage system. The trench *a* should then be filled with loose stone, coarse gravel, and sand. If the top, for about 2 or 3 feet from the building, is then covered with stone flagging or cement *c*, it will assist greatly in keeping the walls dry. If the soil round the building is drained in this way, and the wall is coated with asphalt *d*, *d*, a perfectly dry

wall can be obtained. The asphalt should be carried to the bottom of the footings, as at *e*, and through the wall to the centre of the cellar floor, as seen at *f*.

79. A danger which may be encountered during excavation is the presence of a clay bank between limestone, sandstone, or sand beds. Being water-tight, the clay beds do not permit rain-water to penetrate them, so it flows on their surface and may cause great damage to the foundations, especially those built against a clay bank or stratum. The rain which penetrates the gravel or loam or percolates through crevices of the rock will be arrested by this clay and, forming cavities, will produce a concealed current.

If the cellar wall extends below this current, the water will penetrate it and fill the cellar. To avoid this, it will be necessary to turn aside this stream of water by means of a drain, as shown in Fig. 56 (*a*). Here *ab* is the gravel, sand, or loam bed which can be penetrated by water; *cd*, the clay bed which collects water after every rain storm. This water will be stopped by the foundation of the cellar *e*, and will soon penetrate it. The transverse drain *f* has openings formed in the top to receive the water, thus leaving the wall *e* dry. In (*b*) is shown part of the intercepting drain; *a, a* are openings for the water to enter.

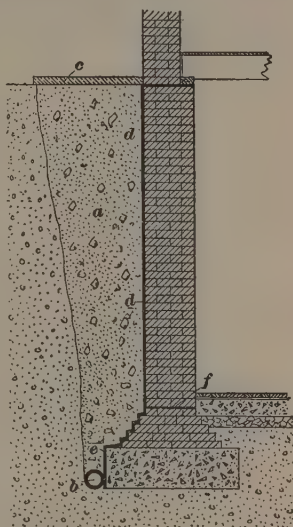


FIG. 55

80. If foundations are laid in open clay, another difficulty to be guarded against is the slipping of the entire bed. Clay beds having a slope like that shown in Fig. 57, where *a, a* is a rock bed under the clay bed *b* and *c* is gravel or loam, are apt to slip after a long rainy season. The rain water accumulating above will

pass at *e* under this bank, and will form from *e* to *c* a thick soapy bed, so that the clay bed may slip by its own weight, especially

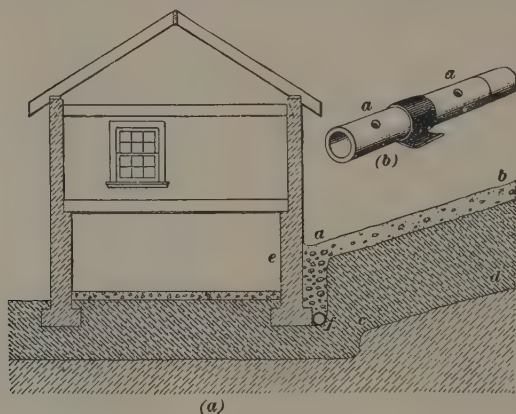


FIG. 56

if that weight is increased by that of the building *d*. This danger may be averted by building a drain *f* at the upper end of

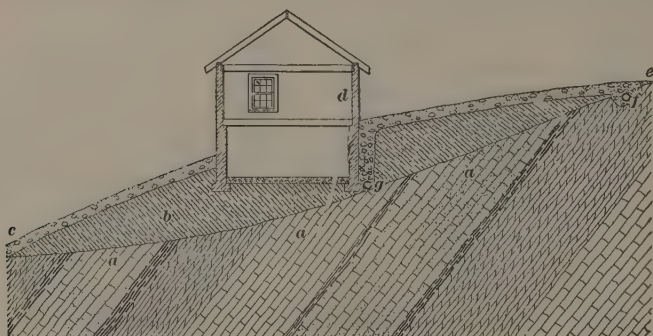


FIG. 57

the clay bed *ce*, to intercept the water running between the clay and the ledge of rock. It would be well also to extend the

wall *d* down to the rock, thus presenting a solid face to the pressure of the clay, and preventing absolutely the upheaval of the cellar floor and bulging of the wall *d*. Another drain must be placed at *g*, to intercept the water flowing through the upper strata of soil and keep the foundation walls dry.

AREAS FOR KEEPING CELLARS DRY

81. Fig. 58 (*a*) shows one form of area used for keeping dry the walls of basements and cellars. The area wall *a* is arched and buttressed to meet the wall of the building as shown in the plan (*b*), thus forming a wall strong enough to resist the pressure

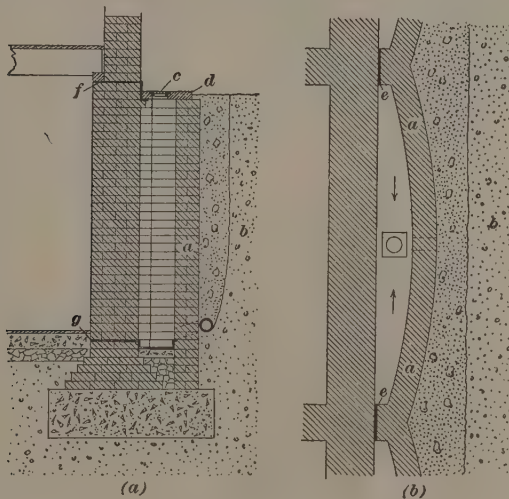


FIG. 58

from the bank of earth *b*. The dry area enclosed by the wall of the building and the segmental area wall is drained by a pipe discharging over a trapped gulley and connected to the general drainage system of the building. The area is ventilated by means of a small grating *c* set in a hole formed in the cover stone *d*.

Where the buttress of the area wall adjoins that of the building, a

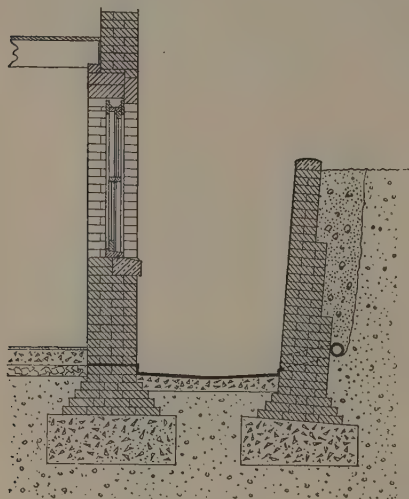


FIG. 59

vertical damp-proof course *e* is interposed, to prevent the communication of moisture at that place. Horizontal damp-proof courses are shown at *f* and *g*. Fig. 59 shows an area of more common form. This style, although not necessarily as deep as shown, is often used to admit light to the cellar windows. An area wall can sometimes be used to advantage when a building is situated in a sloping clay bed.

AREAS, VAULTS, AND RETAINING WALLS

AREAS

1. **Area Walls.**—It often becomes necessary to excavate trenches, called **areas**, outside the foundation walls of a building in order to give light and access to the basement, and, in some cases, to keep the outside of the cellar wall dry. A wall is built in order to keep the bank of earth from falling in. Because stone walls in damp situations offer greater resistance to sliding on the bed than do brick walls, it follows that area walls are often built of stone.

2. An area wall should be built strong enough to withstand the pressure of the earth at the back of it. For this reason, when putting in such a wall, it is best not to disturb the earth behind it any more than is absolutely necessary. If possible, the space that has been excavated behind it should not be filled until the mortar has set, so that the brickwork or masonry can be carefully pointed on the back; then the wall is not so liable to be pushed out of line.

3. In building area walls, cement mortar should always be employed; lime mortar rots by continued moisture. When built of stone, the beds between the stones are generally made horizontal, but, when the face of the wall is battered, the stones are sometimes laid with the beds perpendicular to this face, which makes a stronger wall, but in such a case the joints must be well pointed, otherwise rain will penetrate through to the back of the wall, moistening the back filling, and subjecting it, as well as the walling, to the action of frost. On the whole, the safest way is to make the bed joints horizontal.

4. It is customary to build area walls with both faces plumb, if not too high. A 24-inch stone wall may be safely built to a height of 5 feet without any danger of being tipped over by the

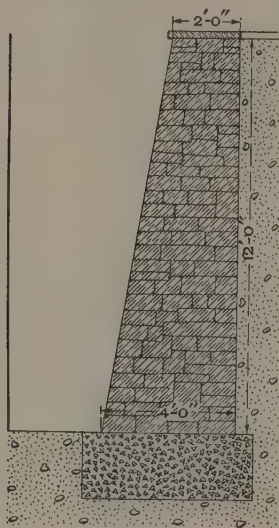


FIG. 1

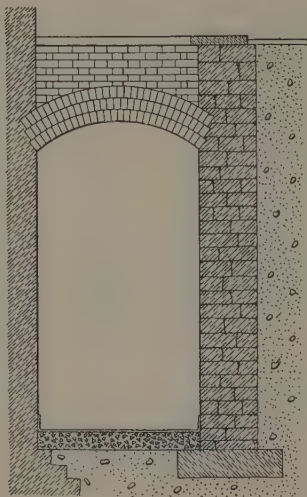


FIG. 2

pressure of the earth behind it. In building an area wall higher than 5 feet, however, it is customary to put a batter of about

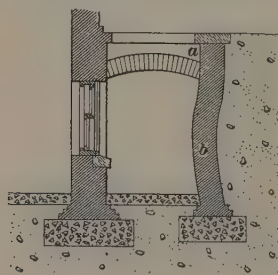


FIG. 3

2 inches to the foot on the outside, as shown in Fig. 1. When an area wall is longer than, say, 10 feet, it may be braced by arches that spring from the main wall of the building at intervals, as shown in Fig. 2. This bracing is not necessary unless the wall is so thin that there is a liability of its overturning. A wall that is not thick enough will probably bulge as seen from Fig. 3, where an area wall is shown supported at the top by arches abutting as at *a*. The bulging usually occurs at about one-third the

height from the footings, as at *b*. Sometimes bulging is caused by filling in the earth behind the wall before the mortar or cement has had time to set properly.

5. Coping for Area Walls.—It is well to cap an area wall with a stone or tile slab, known as the **coping**, which is as wide as, or a trifle wider than, the wall itself. This will ensure that most of the water will be thrown off instead of soaking down into the wall. In Fig. 4 is shown a stone coping *a* which is wider than the wall *e* on which it rests. It is sloped or weathered on top from *b* to *c* to allow the water to run off easily; the groove or throat *d* causes any wet that has run down from the weathered top of the coping to drop to the ground instead of running down the face of the wall. The earth behind the wall is shown at *f*.

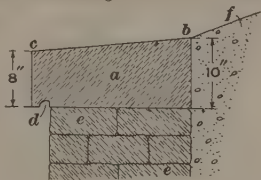


FIG. 4

Stone coping is usually made of granite, freestone, or some stone that will not readily absorb moisture. Copings are sometimes of brick, specially made for the purpose, or of ordinary brick laid on edge and set and pointed in cement mortar. In cheap work, the coping on a wall is sometimes dispensed with, the top of the wall being coated with about 1 inch of Portland cement mortar instead.

6. Area Pavement.—Whether or not an area should be paved depends entirely on circumstances. If there is any danger of water rising up through the ground into the area, it should be both paved and drained. Rain-water is drained from a paved area by means of a gutter and trapped gully, which should be connected to a sewer or a surface-water system of drainage, as circumstances permit.

7. Window and Entrance Areas.—Window areas should be large enough to admit plenty of light into the cellar. They are often made rectangular with side walls, which assist in bracing the area wall, but it is generally better to build them semicircular, as shown in Fig. 5. At *a* is shown the foundation

wall of the building; at *b*, the window and window opening through the wall; at *c*, the area; and at *d*, the semicircular area wall, in this case a 9-inch brick wall. When the area

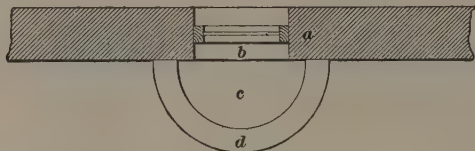


FIG. 5

is 3 or 4 feet deep and the same in length and width, the brick wall should be at least $13\frac{1}{2}$ inches thick, and if a stone wall is built, it should be 18 inches thick.

8. **Drainage of Areas.**—Areas should be drained in order to dispose of rain-water and melted snow. To do this effectually, the bottom of the area should be made of cement concrete made of cement, gravel, and sand, floated to a fine finish with neat

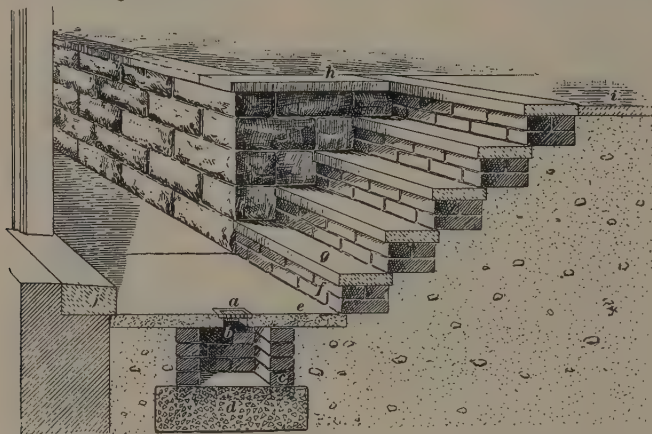


FIG. 6

cement; or of stone flagging; or even of brick laid in cement. Whatever is used, it should be carried up to about 6 inches below the window sill.

If the area is a large one, a small cesspool, about 12 inches square, with $4\frac{1}{2}$ -inch brick walls built in cement, should be built

and connected by means of a trapped gully and drain pipe to the disconnecting chamber leading to the main drain. All open well holes and lighting areas should have similar traps and drain pipes.

9. An arrangement for draining an outside area is shown in Fig. 6. At *a* is shown a removable cast-iron grating, which fits over the cesspool and prevents rubbish, dead leaves, etc., from being washed into the trap and clogging the drain pipe; at *b*, the drain pipe leaves the cesspool to connect with the main drain or sewer. At *c* is the $4\frac{1}{2}$ -inch brick cesspool wall built on the concrete foundation *d*; the stone, brick, or cement area pavement is shown at *e*; while at *f* are shown the brick risers of the area steps. At *g* are shown the stone treads; at *h*, the stone coping of the area walls; *i* is the pavement; and *j*, the stone sill of the door opening into the area.

10. **Area Steps.**—Area steps should be built either of stone or of a combination of brick and stone, and should be laid in cement mortar, no lime being used. Concrete steps are likely to become slippery if too much cement is used in making them, and are liable to wear away if too little cement is employed. However, concrete steps may be successfully made with a reeded face to prevent slipping, as shown in Fig. 7. Being *monolithic*, that is, literally, in one piece of stone, they are more liable to crack than steps made of separate parts. To prevent this uncertain cracking, concrete steps are often built with *planes of least resistance*; that is, certain sections are made weaker than others by laying in the mass sheets of tar paper or other non-porous material. Then, if the steps do crack, they will part at these sheets of paper, which are so placed that the crack will appear at the angle between the riser and the tread, a place where it will hardly be noticed.

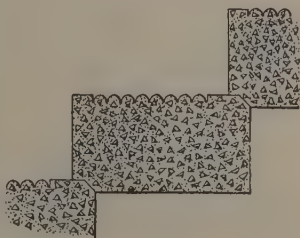


FIG. 7

If the ground is soft or if the area is deep, the steps should have a wall on each side of them. The treads of the steps should be set in this wall at each end, in order not to have a bearing in the middle. If the risers do not rest on the treads, they should also be set in the wall. These side walls should go down into the ground below the frost line, so as not to heave up and displace the steps. If the ground is composed of compact gravel and there is no liability of water or frost affecting it, the steps may be cut directly out of the soil. The risers in this case are generally made of two courses of brick, and the treads are made of 2-inch flags, as shown in Fig. 6.

11. On the whole, stone is the most satisfactory for area steps, especially when the ground is soft or the area deep. The steps may be solid stone or 2 or 3-inch flags; the former are the stronger, but are also the more expensive. Each block of the solid steps should be laid so as to lap from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches over the step below it, as shown in Fig. 8 (a). Two ways of making steps of flag are shown in Fig. 8 (b) and (c). The method shown at (b)

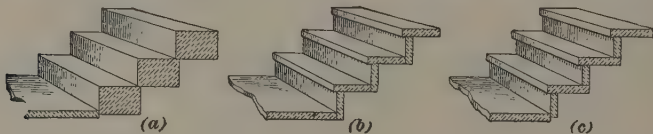


FIG. 8

has the advantage that the risers are not so likely to be pushed out of place by the movement of the earth behind them, due to rain or frost, while the method shown at (c) possesses the advantages of having no vertical joints in which water may penetrate and of having the load partly transferred from one step to another. When flag steps are more than 5 feet wide, they should be supported in the middle by a stone or concrete dwarf wall.

12. Wooden plank steps supported on plank string pieces are used in many localities; also iron strings with plank treads. The ground should be excavated, and the area walled up under them. When the plank treads decay or are worn out, it is

a matter of small expense to replace them. The steps should finish on a platform made of cement, flagging, or brick.

In order to shed water properly, steps of stone, wood, or any material should always slope from the rear to the front, the pitch being from $\frac{1}{8}$ to $\frac{3}{16}$ inch, according to the width of the treads.

VAULTS

13. Vaults Under Pavements.—Where land is expensive, it is necessary, when building, to use all the available space. For this reason, the cellar is generally extended under the pavement and a retaining wall is built to sustain the adjacent soil. This extra space is called a **pavement vault** or **house vault**.

14. Vault Construction.—A method of construction in use where partition walls can be placed in basements under pavements is shown in Fig. 9. The partition walls *a* are spaced about every

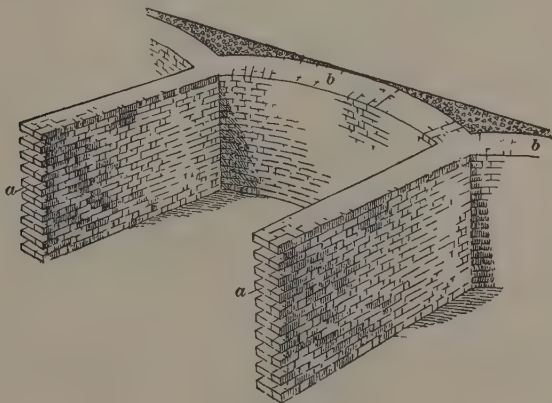


FIG. 9

10 feet, and are usually $13\frac{1}{2}$ inches thick. The outer or street wall is shown at *b*. It is built in the form of an arch, of hard brick laid in cement mortar. These arches are usually about 18 inches thick, and the rise, or height of the arch above the

springing line, is one-sixth of the span. Vaults similar to the one illustrated in Fig. 9 can be roofed over with brick arches springing from the partition walls, and the tops of the arches can be levelled off with earth, broken bricks, sand, or, preferably, with concrete on which the pavement can be laid.

15. When it is desired to have the vault unobstructed by partition walls, the pavement may be supported on iron **I** joists as at *a*, Fig. 10, which are spaced at about 2' 6" centres, with one end of each joist resting on the concrete wall *b*, while the other end rests on the lower flange of the steel joist *c*. The joist *c* is in turn supported at intervals of about 20 feet by steel stanchions *d*, which rest on a stone pad *e* bedded direct on to the concrete foundation *f*. The concrete wall *b* is protected by

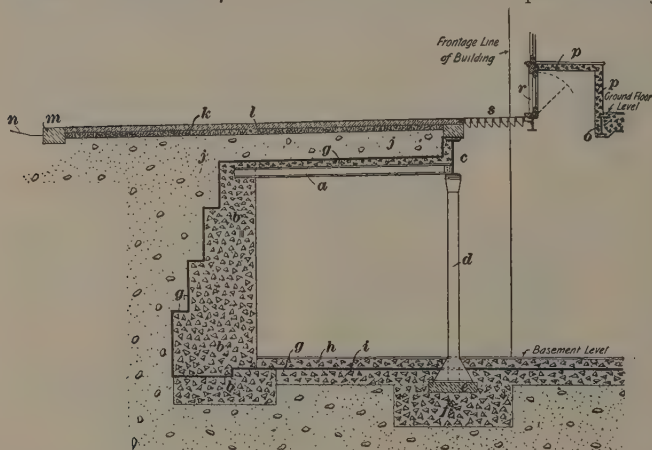


FIG. 10

means of the asphalt or other waterproofing material *g*, which is taken along the top of the concrete covering to the joists *a*, and between two thicknesses of concrete *h* and *i* forming the basement floor, so preventing the entrance of water into the vault. The earth filling *j* over the vault roof is covered with 3 inches of gravel *k*, on top of which are the 3-inch York stone pavement slabs *l*, or other suitable material. At *m* is the kerbstone and at *n* the roadway.

A method of lighting the vault and basement is also shown in Fig. 10. The ground floor is trimmed or stopped off about 3 feet 6 inches from the frontage line of the building by means of the trimming joist *o*, and on this is erected, preferably in concrete, what is called a **bulkhead** *p*. The lower portion of the shop front *r* is formed with glazed sashes made to open, thus admitting light and air into the basement. Additional light is obtained through the pavement lights *s*.

16. Rules for Building Pavement Vaults and Areas.—The following quotations are typical of the rules usually issued by local authorities with regard to the construction of pavement vaults. It will, of course, be understood that the rules will vary in detail with the locality.

House Vaults.—The walls and arches of house vaults are to be constructed of good hard brickwork or stone, set in mortar or cement. The walls are not to be less than $13\frac{1}{2}$ inches thick. For a span not exceeding 10 feet, the arches are not to be less than 9 inches thick; exceeding 10 feet but not exceeding 15 feet, $13\frac{1}{2}$ inches thick; exceeding 15 feet, the thickness must be approved by the surveyor. Every arch is to have a rise of not less than 1 foot to every 5 feet of span.

Springing walls of all arches are to be at right angles to the line of frontage, and the end walls must be parallel to the line of frontage, where straight on plan, but where practicable they are to be curved on plan. The tops of arches must not be less than 15 inches below the surface of the pavement. The top of any roof built over a vault must be at least 15 inches below the surface of the footway paving.

Failing special permission, the external face of the end wall must not project beyond the outer line of the kerb of the footway, and in no case may it project more than 10 feet from the line of building frontage.

No water closet will be allowed beneath the public footpath.

Coal Shoots and Plates.—Coal shoots into house vaults must not be more than 14 inches in internal diameter, and they must be formed of hard brickwork, not less than $\frac{1}{2}$ brick in thickness, set in mortar or cement. All new coal plates are to be circular, and

not more than 14 inches in diameter, and be properly let into an iron frame inserted in the paving stone; they are to be formed of iron deeply chequered or otherwise roughened on the surface, and should not be less than $\frac{3}{4}$ inch thick. If required to give light to the vaults beneath, they are to be formed of iron frames with glass lenses inserted therein, each lens to be not more than $3\frac{1}{2}$ inches in diameter, and the iron in no part of the frame to be less than $\frac{3}{4}$ inch in sectional area. Every coal plate or illuminating plate is to be securely fastened and held in its place by means of chain, shackle, bar, or other permanent fastening.

Areas, Area-Gratings, and Pavement Lights.—No areas beneath the public way, in front of premises, are to project more than 2 feet 6 inches, measuring from the line of building frontage to the external face of the brickwork of the end wall. They are to be formed of hard brickwork or stone, set in mortar or cement, and, excepting where covered with iron gratings, shall be arched over with hard brickwork, not less than 9 inches thick, or with Yorkshire stone landings, not less than 4 inches thick, or any other approved covering. No area grating shall be larger than 7 feet, or project more than 1 foot 6 inches, measuring from the line of frontage. All gratings are to be of iron, and be securely let into hard stone kerbs, and run with lead; the kerb shall be fixed at the level of the footways, be not less than 6 inches wide and 6 inches deep, and be securely pinned into or rest on the brickwork for at least 4 inches at each end. All open iron gratings are to be made with frames or borders; the bars of the gratings are to be fixed at right angles to the line of frontage of the houses; the space or width between the bars is not to exceed $1\frac{1}{4}$ inches; each bar shall be not more than $\frac{3}{4}$ inch wide on the surface, and the iron must not be less than $\frac{3}{4}$ inch in sectional area. Where the area gratings or frames are to be filled in with glass slabs, the spaces between the bars must not be more than 3 inches wide; the bars must be fixed at right angles to the line of frontage, the iron must not be less than $\frac{3}{4}$ inch in sectional area, and the glass slabs must not be less than 1 inch thick. Where the area gratings or frames are to be filled in with glass lenses the thickness of iron between the lenses must not be less than $\frac{3}{4}$ inch in sectional area, and the glass lenses must not be more than $3\frac{1}{2}$ inches in diameter.

RETAINING WALLS

17. When a wall has to keep in place a filled-in backing of earth, rock, or gravel, it is called a **retaining wall**, because it retains the earth in its place, and resists its natural tendency to cave in. Foundation walls also act as retaining walls, with this difference, that they support a superstructure whose weight is usually found sufficient to overcome the thrust of the earth against the wall. A true retaining wall carries itself, and also must have strength enough to resist the pressure of the earth behind it. Area walls are, to a certain extent, retaining walls ; but, as has been already shown, they are usually braced by arches or cross-walls from the foundation wall, and hence do not require the same thickness as a retaining wall proper.

18. The walls of swimming baths are really retaining walls. When the tank is full, the pressure of the water in the tank is practically balanced by the pressure of the earth on the outside ; but when the tank is empty, the pressure of the earth on the outside is unbalanced and the wall is a retaining wall.

19. There are many formulæ for finding the strength of retaining walls, but they all assume certain facts about the earth behind the wall, its tendency to slide, the pressure it exerts, and other conditions that may or may not be true. It can readily be seen that even the changes in the weather will have a great effect on these properties of the earth backing, which may be wet or dry, soft or frozen. Many of the formulæ take no account of these changes. It seems reasonable, therefore, that a retaining wall should be proportioned like retaining walls that have been actually built and found to be efficient, rather than be designed according to a theoretical formula that is known to give results that are not exact. The proportions given hereafter are therefore taken from actual practice.

high, is 5 feet thick. At b , b are shown the footings, each course being made as a rule 12 inches thick, and each projecting at least 6 inches beyond the course above.

The stability as well as the appearance of a retaining wall is increased by battering, or sloping, it outwards. This batter is usually about 1 inch to the foot.

21. Openings d , called *weep holes*, are provided to carry off surface water that would otherwise run down behind the wall and affect its stability. These can be formed by leaving out stones through the thickness of the wall at intervals, or by building in ordinary agricultural drain pipes of about 3 inches bore. Along the bottom row near the lower ground level they should be placed at intervals of about 10 feet, or closer if the ground is very damp behind the retaining wall. At the back of the wall behind the weep holes should be placed layers of clean loose stones about 1 foot in depth to allow water to run through easily. The weep holes can be allowed to drip over the face of the wall, but this results in damage to the foundations by reason of water collecting at the base of the wall. A good method is to form an agricultural drain at the base of the wall as shown at e , covered over with rough gravel or stones f ; the pipes can be connected to a drain or allowed to run into a pond or cesspool.

22. When the earth is terraced or banked up above the wall, as shown by the dotted line $g g$, additional thickness is required. It is generally agreed that if the embankment is one-third the height of the wall, the wall should have a thickness at each step equal to one-half the distance from that point to the top. Therefore, the wall at a_2 would be 3 feet thick; at a_3 , 4 feet 6 inches thick; and at a_4 , 5 feet $7\frac{1}{2}$ inches thick. The stability of a retaining wall is increased by stepping the wall on the back, because the steps bond the wall into material behind it, and the weight of the soil resting on the steps being added to the weight of the wall, it is not so likely to slide out at the bottom.

The footings should be carried well below the base of the wall, to ensure against displacement from frost or settling. It is also well to plaster the back of the wall and the top of the steps with cement, to a depth of 3 or 4 feet from the top of the wall.

The proportions of retaining walls built of brick or concrete follow to a large extent those shown in Fig. 11.

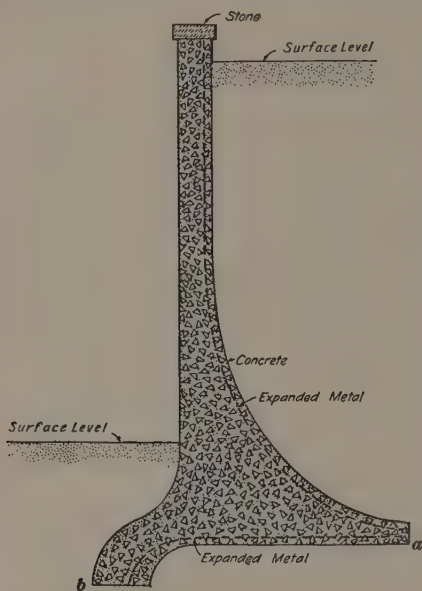


FIG. 12

23. Reinforced-Concrete Retaining Walls.—Fig. 12 shows a retaining wall built of concrete and expanded metal. A wall of this kind is able to withstand considerable pressure from the earth behind it and still be quite stable. To overturn a wall built in this manner, it would be necessary to lift the filling that rests on the part *a*, as the wall has a tendency to revolve about the point *b*. It is, therefore, evident that, although the wall itself may be quite

light, it borrows part of its stability from its backing, and for certain classes of work it is an economical design to use.

24. Anchored Retaining Walls.—In Fig. 13 is shown a retaining wall equipped with land ties. These ties *a* consist of wrought-iron rods 1 inch in diameter, which are anchored through the cast-iron plates *b*. The rods should pass through the plates at about two-thirds of their height from the top, and the anchor plates should be placed far enough from the retaining wall to be in solid ground that is not liable to slip. Cast-iron washers *c* are placed under the nuts of the bolts, as otherwise the nuts might pull through the wall. This method of securing a retaining wall cannot be recommended very highly, as the rods are liable to rust through and allow the wall to overturn.

25. **Relieving Arches.**—When great strength is required for retaining walls, relieving arches may be used. Fig. 14 (a) shows a section through a retaining wall that is strengthened by two relieving arches; Fig. 14 (b) is an elevation of the arches. The wall may be reinforced by either one or two arches. The filling *m* does not press against the wall, but rests on the arches as shown, except above the top arch. This is clearly shown in the sectional view, Fig. 14 (a), where the filling *m* is seen to slope in a forward direction from the under

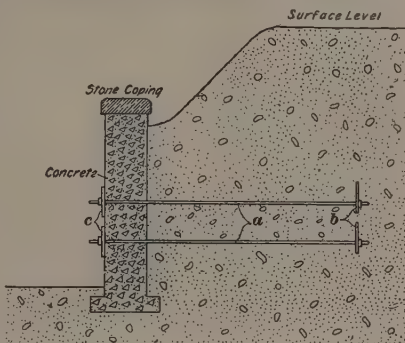


FIG. 13

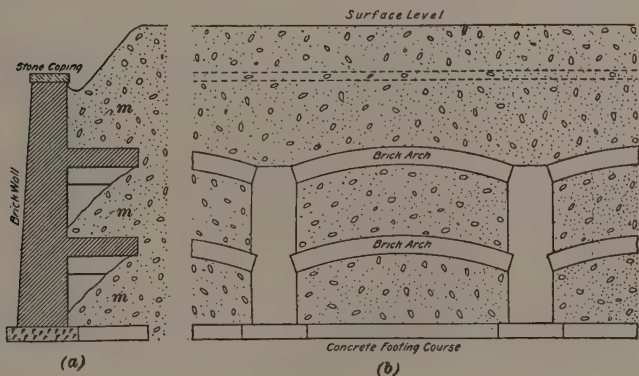


FIG. 14

side of each arch toward the inner side of the brick wall. This style of wall is very strong, although expensive to construct.

PAVING AND PAVEMENTS

26. Pavings may be made of flagstones, concrete, or brick. A flag is a thin slab of stone, which is generally used in pavement work. Concrete pavings are usually finished on top with cement and crushed granite. The bricks used for paving work should be hard and of the variety known as paving bricks.

27. Stone Paving.—If stone of a texture that readily splits into flags can be obtained, it will probably make a better and cheaper paving than one made of concrete. A flag paving can be taken up and relaid better than one of concrete, is easier to repair, and is also more durable.

28. Stone for paving should be 2 or 3 inches thick when used in areas or similar places, and the flags should be cut rectangular.

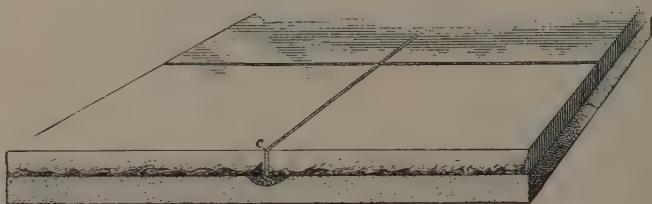


FIG. 15

They should be laid on a sand bed 2 or 3 inches in thickness. The edges of the stones usually rest on a small bed of concrete, and one-to-one cement mortar is put into the joints, as shown at *c*, Fig. 15. This makes a joint that will prevent water from soaking down between the flags; the subsequent freezing of water under the flags would upheave them. Even in countries where there is no frost, this concrete and cement should not be omitted, or vegetable growths may take place in the joints and ultimately disturb the flags.

29. Stone Pavements.—A stone pavement is laid in a similar manner to any other paving, but owing to the fact that it is subjected to more traffic, it should be built more substantially. The flags used for pavements are therefore generally 3 or 4 inches thick. It is always best, if possible, to have the stones run all the way from the kerb to the building line, but for wide pavements

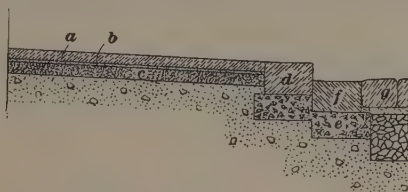


FIG. 16

this is impracticable. The section through a flagged pavement is shown in Fig. 16, where *a* indicates the flags bedded on a layer of sand *b*, on the top of a 3-inch thickness of gravel *c*. The granite kerb is shown at *d*. The channel or gutter *f* is laid on a bed of concrete *e*; at *g* is shown the roadway.

30. Cement - Concrete Pavements.—The method of laying cement-concrete pavements is as follows: The ground should be levelled from 18 to 20 inches below the finished level of the pavement, and should be well settled by ramming, care being taken that the excavation is drained to one point. A foundation of large stones, preferably packed by hand, and about 12 inches thick, should then be laid, and rolled with a heavy roller. About 2 inches of finer material, hard stone or brick, should be placed over this and again well rolled. The concrete should then be prepared in the proportion of one part of Portland cement and three parts of broken hard material to pass a $\frac{3}{4}$ -inch gauge, with a sufficient quantity of water to make a stiff mortar. It should be thoroughly mixed and worked while being laid, its thickness being 3 inches. The top, or finishing, coat should be laid immediately, and only as much concrete should be laid as can be covered with cement on the same day, for if the concrete gets dry on top, the finishing coat will not adhere to it. The top coat should be prepared by mixing one part of the best Portland

cement with two parts of clean, sharp, crushed granite or flint rock.

A $\frac{1}{2}$ -inch space should be left between the kerb and the pavement and between the building line and the pavement, to allow for expansion and contraction. This space should be filled with cinders or ashes. The pavement itself should be set out into blocks 6 feet square or less. These blocks should be separated from one another by thin wood strips, which should extend all the way through the concrete. When the various blocks have set and hardened, the wood strips can be removed and the spaces filled in with cement and pointed to a neat finish on the surface.

31. Hair cracks are often caused by the mortar in the top coat being too rich in cement. If the pavement is trowelled too much, it has a tendency to make the cement float to the top, which is as liable to cause hair cracks as the use of too much cement. If the top coat is put on too wet, it has the same effect.

32. In towns, concrete pavements should be finished with a rough surface. Such a surface is not so slippery in winter as a smooth finish ; also, it is easier to construct and does not show any hair cracks. In laying such a surface, the top coat is levelled

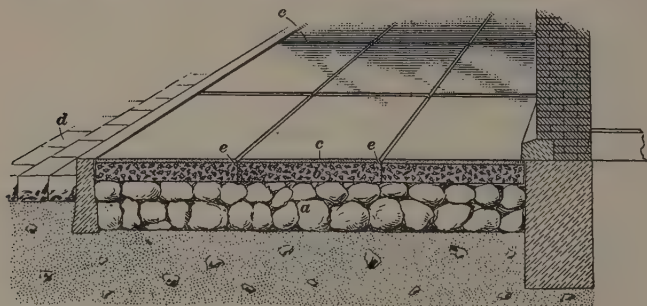


FIG. 17

with a straightedge running on battens, one set on each side of the pavement. The battens are arranged so that the pavement at the kerb will be lower than at the building wall. This gradient is controlled by local regulations, and is usually 1 inch in 1 yard.

The pavement is then left until it has almost set, before it is trowelled. It should be trowelled as little as possible and with a wooden trowel instead of a steel one. After trowelling, it should be covered with straw and kept moist for at least a week. The less the pavement is smoothed with the straightedge or trowel, and the more it is rammed instead, the better it will be.

Fig. 17 shows a section of a concrete pavement, *a* being hard stone core ; *b*, the first coat of concrete ; *c*, the finishing coat ; *d*, the roadway ; and *e*, the joints with wood strips in them.

33. Brick Pavements.—In constructing brick pavements, good hard paving bricks, sound and square, should be used. These bricks should be laid flat, herring-bone fashion, on a bed of sand that is from 4 to 6 inches thick. After the bricks are laid and levelled, the entire surface should be covered with sand, which should be swept over the bricks until the joints are thoroughly filled. If an extra thickness of wearing surface is desired, the bricks may be set on edge, and covered with sand as described.

LIMES, CEMENTS, AND MORTARS

LIMES

INTRODUCTION

1. Any substance which is plastic at one degree of moisture or heat and tenacious at another is, in a broad sense, a **cement**. However, the word cement, as now used in building construction, means more particularly a certain one of the many varieties made from a mixture of limestone and clay. This substance has the peculiar property of *setting* or getting hard under water. For this reason it is often called *hydraulic cement*. Of the many kinds of cementing materials those principally used for building purposes are limes and hydraulic cements, plaster of Paris, and Keene's cement.

Limes and cements, in building construction, have three principal uses: first, as binding materials to hold other bodies together; second, as materials with which to coat parts of structures; and third, as building materials in themselves. The first use is illustrated by the mortar in the joints in stone and brick work; the second, by the plastering material used to coat walls; and the third, by concrete foundations, walls, piers, etc.

LIMESTONE AND THE BURNING OF LIME

2. **Occurrence and Forms of Limestone.**—Limestone may be defined as rock composed of calcium carbonate, or carbonate of lime. It includes such substances as marble, chalk, etc. It

is one of the most widely distributed rocks in nature, and it is found in almost all countries in stratified beds of considerable extent. It occurs in all geological formations from the earliest to the present, and some periods and epochs were distinctly limestone-forming times, at least in some sections, while at other points they may have been sandstone or clay-forming periods.

3. Impurities in Limestone.—The principal impurities in limestone are silica, iron, alumina, magnesia, and sulphur. Silica occurs in the free state as sand or combined as silicate of alumina, while alumina occurs only in the latter form. Iron may occur as the carbonate, oxide, or sulphide, and magnesia is generally found as carbonate. Sulphur may occur as sulphide of iron or as sulphate of lime or magnesia. In the burning of lime, these impurities are of very little importance when in small amounts, while if the proper ones occur in larger amounts, the process passes from the manufacture of lime to the manufacture of hydraulic limes or hydraulic cements.

4. Burning of Limestone.—Lime is a white or light-coloured earth-like calcium oxide produced by heating a limestone. The stone is placed in a kiln, where it is heated to that temperature at which it will give off part of its constituents in the form of carbon dioxide, or carbonic-acid gas. The heat must be applied gradually and for a definite time only; but the time varies considerably, for it depends on local conditions. When taken from the kiln, the product is known as **quicklime**. It is hard and rock-like and has a great affinity for water, which it absorbs with a marked evolution of heat when it is exposed to moist air. The heat is caused by the chemical union of lime with water, which forms calcium hydrate, or **slaked lime**. The slaking is accompanied by an increase in volume as well as weight, and may be accomplished by exposure to moist air or the addition of water. If lime is underburnt, it will not slake evenly, and there will be some hard pieces that will not disintegrate when water is added. If this lime is used, especially for plastering walls, the unslaked particles will disintegrate after being incorporated in the wall, and will swell or *blow*, as it is called, throwing out bits of plaster

and thus injuring the surface. Overburnt lime also slakes slowly, and for that reason should not be employed. Though the lime-burning industry has suffered considerably from the introduction of natural and artificial cements, it is still one of great importance, and improvements in the process of manufacture or in the methods of handling are constantly being made.

5. Importance of Slaking Lime Before Using.—All lime, whether used in sand mortar for brickwork, masonry, or plaster, or used almost pure for finishing work in plastering, must be perfectly slaked before it is used. Very frequently too little attention is paid to this, and defective work results. If lime is only partially slaked, it consists of a mixture of quicklime and calcium hydrate, which, when mixed with sand and put in work, even with excess of water will retain some quicklime. The water will evaporate rapidly and the mass may become sound and hard, but the quicklime will still have a strong affinity for moisture, which it will obtain from the atmosphere. As slaking is accompanied by increase in volume, the mass will swell or expand and peel or crack. It is therefore recommended that lime should be thoroughly slaked for at least a week before it is used; and in some cases specifications require even a longer period.

CLASSIFICATION OF LIMES

6. Commercial lime may be divided into three general classes, namely, *rich*, *fat*, or *pure lime*, *poor lime*, and *hydraulic lime*. These varieties differ from one another in the amount of clay, sand, or iron they contain.

7. Fat Lime.—Fat lime is almost pure oxide of calcium; that is, it consists chemically of calcium and oxygen. It contains only about 5 per cent. of impurities and has a specific gravity of 2.3 and a great affinity for water, of which it absorbs about one-quarter of its weight. This absorption is accompanied by a great rise in temperature. Finally, the lime crumbles to a powder, which occupies from two and one-half to three and one-half times the bulk of the original lime, the exact amount

depending on its initial purity. When slaked, the lime is unctuous and soft to the touch, and from this peculiarity derives the name of *rich* or *fat*.

8. Poor Lime.—Poor lime consists of from 60 to 90 per cent. of pure lime, the remainder being impurities, such as sand or other foreign matter. These impurities have no chemical action on the lime, but simply act as adulterants. Poor lime slakes more slowly and evolves less heat than fat lime. The resulting paste is also thinner and not so smooth, strongly resembling fat slaked lime mixed with sand. Poor lime is not so good for building purposes as fat lime, and should only be used when no better can be obtained.

9. Hydraulic Limes.—Hydraulic limes are those that contain enough quicklime to slake when water is added, and enough clay or sand to form a chemical combination when wet, thus giving them the property of setting under water. Hydraulic limes are made from limestone containing from 5 to 30 per cent. of clay or sand. They are often considered as divided into three classes, namely, *feebly hydraulic*, *ordinarily hydraulic*, and *eminently hydraulic*, in proportion to the amount of clay and sand present. The slaking qualities of this lime vary; at the one extreme is a feebly hydraulic lime which slakes in a few minutes after water is added, and at the other is the strongly hydraulic lime which slakes only after many hours with practically no evolution of heat and without cracking or powdering. The time of setting under water varies from setting as hard as soap in 20 months with the feebly hydraulic, to becoming as hard as stone in 3 or 4 days with the strongly hydraulic. If carbonate of magnesia is present in the lime, it reduces the energy of the slaking but increases the rapidity of the setting and the ultimate strength when set. As the amount of clay is increased, hydraulic lime gradually merges into hydraulic cement, so that it is difficult to determine whether a material about on the border line is a lime or a cement. All hydraulic limes and cements should be put in place on the work before they begin to set, because if they are disturbed during the process of hardening, they will never attain their full strength.

10. Hydrated Lime.—Hydrated lime is a lime slaked by a patent process by means of which the exact amount of water is added thoroughly to slake the lime and to leave a perfectly dry powder, at the same time avoiding the absorption of carbon dioxide with its deteriorating effect on the lime. This operation is performed in a shallow pan furnished with ploughs, which turn the lime over so that all parts are thoroughly wetted. Immediately after hydration the lime is passed through sieves of fine wire cloth, which screen out any particles of underburnt or overburnt lime or foreign matter. Hydrated lime can be kept in stock for an indefinite period without deteriorating. Being quite cool it does not swell and burst the bags containing it, and is said to be as easily handled and stored as Portland cement. By the addition of a proportion of hydrated lime to Portland cement, first thoroughly mixed in a dry state, cement concrete blocks can be rendered waterproof, and cement mortar can be made to work much better on the trowel and to produce watertight results.

VARIETIES OF LIMES

11. Lias Lime.—Lias lime is manufactured from a limestone found in what is called the lower lias formation, and varies to a certain extent with the locality from which the limestone is obtained. The original stone is of a bluish colour, therefore the finished product is often termed *blue lias lime*. The analysis of blue lias lime is somewhat similar to Portland cement, but there is a certain percentage of coal ash left after the burning of the limestone in the kiln. It contains about 63 to 65 per cent. of lime, 18 to 20 per cent. of silica and from 10 to 12 per cent. of alumina and ferric oxide. Some blue lias limes are strongly hydraulic, while others are only moderately so; they can be obtained either in lumps or finely ground. Great care should be taken in slaking lump lime; manufacturers recommend that it should be broken into small pieces and then evenly sprinkled with water from a watering-can with a fine rose. The whole should then be covered quickly with sand and left about 24 hours before being used.

12. **Tensile Strength of Blue Lias Lime.**—In Table I is shown the results of a series of tests made on a blue lias lime obtained from Warwickshire, England. Sample 1-inch cubes, or bri-

TABLE I
TENSILE STRENGTH OF BLUE
LIAS LIME AND SAND
BRIQUETTE

Age of Sample When Tested	Tensile Strength, in Pounds Per Square Inch	
	Three Parts of Sand, One Part of Lime	Six Parts of Sand, One Part of Lime
2 weeks	103	45
4 weeks	123	58
8 weeks	128	89
12 weeks	203	104
26 weeks	234	124
1 year	228	162
2 years	272	154
3 years	286	168

quettes, were made up, in one case, of 3 parts of sand and 1 part of lime, and in the other case of 6 parts of sand and 1 of lime, and kept for different periods. When tested, it was seen that the longer the briquette was kept, or, in other words, the longer the lime was allowed to set, the greater resistance to tensile stress it had.

13. Greystone Lime.
Greystone lime is obtained from beds found in the South of England. It is

not very strong, but is often used when mixed with sand for mortar in brick and stone walls. It is feebly hydraulic and slakes very freely.

CEMENTS

CLASSIFICATION AND GENERAL PROPERTIES

14. The cements used by builders and engineers are divided into two main classes; namely, *natural cements*, and *artificial cements*. These cements differ from the limes by not slaking after calcination. They can be formed into paste with water, without any sensible increase in volume and with little, if any, disengagement of heat, except in certain instances among those varieties that contain the maximum amount of lime. As they do not shrink in hardening, they can be used with or without sand, but for the sake of economy sand is used.

The *activity*, or *time of setting*, of cements is variable ; some set under water at 65° F. in 3 or 4 minutes ; others require as many hours. The point at which the setting is considered to begin is when the stiffening of the mass first becomes perceptible, and the end of the setting is when cohesion extends through the mass sufficiently to offer such resistance to any change of form as to cause rupture before any appreciable deformation takes place. The binding and economic value of a cement is affected by the degree of fineness to which it is ground. Coarse particles have no setting power, and act as an adulterant.

15. Natural cements are made by calcining natural argillaceous or silicious limestones at a heat just below fusion and grinding the product to powder. On account of the difference in the rocks from which they are made, natural cements are extremely variable, for, as a rule, the raw material undergoes no preliminary preparation before burning, but is taken from the quarry or mine direct to the kilns. In several mills there is a sorting process of material into coarse and fine, but these are exceptional. After burning and grinding, the cement is ready for use.

16. Artificial cements are made by making an accurate and uniform chemical mixture of the raw materials, in order to obtain which, the mixture is finely pulverized before burning. The percentages of the various ingredients are confined to rather narrow limits, so that if the burning is conducted properly these cements are not subject to such variation as in natural cements. Artificial cements are higher in lime, specific gravity, and tensile strength than natural cements.

VARIETIES OF NATURAL CEMENTS

17. For all practical purposes natural cements may be treated as one class. Their setting time may range from 5 minutes to as many hours, but the former time may be increased by the addition of a little sulphate of lime (raw gypsum or plaster of Paris). The cements on hardening attain a tensile strength

greater than hydraulic lime under most conditions, but less than Portland; but there are cases on record where hydraulic lime and sand mixtures have developed greater tensile strength on long-time tests than natural cements. The chemical composition varies greatly in different brands, silica, iron, and alumina having wide limits, lime even greater, sulphuric acid still more marked, while the variation in magnesia is exceptionally great. Table II shows the wide limits in the percentages of various constituents.

TABLE II
ANALYSIS OF NATURAL CEMENTS

Analysis	English Roman Per Cent.	English Medina Per Cent.	Foreign (Grenoble) Per Cent.	Rosendale Per Cent.
Silica	18.48	18.20	19.88	27.30
Insoluble residue . .	8.74	5.40	1.94	
Alumina	17.42	15.70	12.70	7.14
Lime	43.29	45.96	54.65	35.98
Magnesia	2.74	8.80	4.45	18.00
Sulphuric anhydride .	1.75	1.58	3.32	
Volatile matter . . .	5.40	4.02	2.34	
Alkalies	2.18	.34	.72	6.80

All of these cements, however, show decided hydraulic properties, have stood the test of time, and are well suited to work to which they have been applied. In regard to magnesia, which is always less than 4 per cent. in an artificial cement, it should be stated that in a natural cement it possesses hydraulic properties and combines with silica and alumina to form silicates and aluminates, just as lime does, and these compounds, when crystallized, are stated to be equal in hardness to the lime compounds. The reason for this action in natural cements (when in artificial cements it is said to possess no hydraulic properties and to exercise an injurious effect) may be ascribed to the burning. In natural cements calcined at a low temperature,

magnesia hydrates readily, but if the hard and highly burnt pieces are picked out and ground, it will be found to hydrate very slowly. In making natural cements high in magnesia, the hard pieces are picked out and discarded. The process of making artificial cement differs radically, since all of the material is calcined at a high temperature and brought to incipient fusion. The principal natural cements are *Roman*, *Medina*, *Rosendale*, and *Pozzuolana*. Other natural cements are made from stones found in Boulogne, Yorkshire (England), Scotland, Belgium, and Germany; but all are being superseded for building work by Portland cement.

18. Roman Cement.—Roman cement is made from stones found in the London clay, which are simply dug out and burnt in a kiln. The resulting matter, which is finely ground, is not nearly as strong as Portland cement, by which it is being almost entirely superseded. In recent years Roman cement was used a great deal for brick or stone work in damp situations or under water, such as for piers, breakwaters, harbour walls, etc., as its property of setting very rapidly eminently suited it for such positions. It should be mixed only in small quantities, as it will set in about $\frac{1}{4}$ hour after mixing.

19. Medina Cement.—Medina cement is made in Hampshire and the Isle of Wight, England, and is very similar to Roman cement. It sets in about the same time, but its ultimate strength is not so great.

20. Rosendale Cement.—Rosendale cement is an American natural cement, made in Rosendale, New York, whence it derives its name, and in other places. It contains from 20 to 40 per cent. of clay, the remainder being mostly carbonate of lime, with a little carbonate of magnesia.

21. Pozzuolana Cement.—Pozzuolana is a name given to certain clayey earths containing about 80 or 90 per cent. of clay, with only a small portion of lime and small quantities of magnesia, soda, potash, etc. Natural pozzuolana is an earth of volcanic origin naturally burnt and found in Italy, near Mount Vesuvius, and in other volcanic centres of Southern

Europe. It never reaches the same degree of hardness as Portland cement, is used sometimes in damp situations, but will not endure long in dry, exposed positions, neither will it stand much wear.

Another cement of this class is manufactured from an earth called *trass*, which is also of volcanic origin, and is found in Germany and Holland.

VARIETIES OF ARTIFICIAL CEMENTS

ENGLISH PORTLAND CEMENT

22. Portland cement was discovered by Joseph Aspdin, in 1824, and was given its name from the resemblance of its finished surface, when set, to the building stone of that name. It is made in nearly every country, as the demand for it is ever increasing. Until recently, the mixture was made according to a rather lax rule-of-thumb method; but now more care is taken and better results secured. As the quality and strength of Portland cement depend on the mixture of the raw materials, the composition of each constituent is carefully determined and the amount necessary to produce the desired result calculated. These calculations are then checked at frequent intervals, so that the cement is of uniform quality. These calculations also determine whether the cement shall contain 61 per cent. or 63 per cent. of lime, for this depends on the relative amounts of silica and alumina present.

23. *Mixing.*—The mixing of the raw materials is done in what is called a *wash mill*, the clay being fed in, by weight, on one side and the chalk, by measure, on the other. In this mill, water is added and thoroughly mixed with the clay and chalk to form a creamy liquid called *slurry*. The slurry is washed through grids by gravitation from one mill to another and passes through seven or eight before it enters the final or finishing mill, which is constructed similar to the others except that it has fine sieving all round it. Here the slurry is agitated, washed through the sieves, and pumped into tanks, where it is chemically

analyzed to ascertain whether or not the correct amount of carbonate of lime is present. If the mixture is not correct, the necessary additions are made when the slurry is in these tanks, but if it is found to be correctly constituted it is ready for burning.

24. Burning.—From the tanks, the slurry, still in its liquid state, is pumped into one end of a **rotary kiln**, where it is burnt. These kilns consist of a slightly inclined cylinder, which varies from 45 to 70 feet in length. The outer casing is made of steel, but has a firebrick lining that varies from 9 inches at one end to about 3 inches at the other. As the kiln revolves, the material inserted at the upper end slowly works its way toward the front end, which, while open, is protected by a hood or shield of firebrick. The fuel used is coal. During its progress through the kiln, the material is first calcined and then brought to incipient fusion, and is delivered at the mouth of the kiln in the form of small kernels about the size of a walnut, called *clinker*. When received, it is at a white heat, so it is allowed to drop into another rotary kiln called a *cooler*, from which it is discharged into trucks that convey it to the mill where preliminary grinding takes place. During this preliminary grinding the setting time of the finished cement is determined by the addition of gypsum. After the preliminary grinding is completed, a final grinding takes place in a different kind of mill, reducing the cement to the required fineness of the finished material. The cement is then taken and packed into casks or sacks and is ready for market.

MISCELLANEOUS CEMENTS

25. There are a number of materials used in architecture that can be classed as cements. Many of these, as plaster of Paris and Keene's cement, are used almost exclusively in plastering work, and will therefore be treated in another Section.

26. Silica, or Sand, Cement.—Silica, or sand, cement is made by grinding together silica, or sand, and Portland cement. The

sand acts as an adulterant, but on account of the extra-fine grinding of the cement, which enables it to cover more surface per unit of weight, this silica, or sand, cement is sometimes as strong as Portland cement. For the same richness of mixture, the mortar made from this kind of cement is more dense than that made from ordinary Portland cement.

A cement in some respects very similar to silica cement is **slag cement**, a mixture of granulated blast-furnace slag and slaked lime. It is produced at a price considerably less than Portland cement, from which, when analyzed, it is found to vary considerably. It deteriorates very quickly and is not to be recommended for enduring work.

27. Bituminous Cement.—Bituminous cement is made from bitumen and sand or stone dust; the bitumen is sometimes adulterated with coal tar. This cement has two important physical properties, being both waterproof and elastic. Bituminous cement is largely used for street pavements, for waterproofing cellars, and for covering the extrados of arches, the tops of walls, and similar places. It is also used for the foundations of small machines placed in the top stories of tall buildings when it is desired to eliminate all vibration. The cost of bituminous cement, however, has somewhat limited its use.

28. Cements for Fastening Ironwork.—Fastening iron rods into stone may be accomplished by pouring hot lead or hot sulphur round the rods after they are placed in the holes drilled in the stone to receive them. Neat Portland cement is sometimes used for this purpose, and also a mixture of equal parts of sulphur and pitch.

29. Cements for Fire-Resisting Purposes.—A clay known as **fireclay** is extensively used to withstand high temperatures; it does not possess a high tensile strength. The interior walls of brick-lined furnaces are often laid in fireclay mortar, while in many instances the bricks are omitted and the interior is plastered to the required thickness with the clay. This clay does not set in the sense of a chemical reaction taking place, but simply dries out. Another cement that will resist great

extremes of heat is made from asbestos powder moistened with silicate of soda. **Ganister**, made from ground quartz and fireclay, is also often used for fireproof purposes. **Parget**, made of lime, hair, and cow dung, was used formerly for house flue lining, but is not now much employed.

TESTING CEMENT

30. Character of Tests.—The quality or constructive value of a cement is ascertained by submitting a sample of the cement to a series of tests. The properties usually examined are *specific gravity, activity, soundness, fineness, and tensile strength*. Chemical analysis is sometimes made to detect adulterations and to ascertain if harmful constituents are present in inadmissible quantities. As the tests cannot be made at the site of the work, it is usual to sample each lot of cement as it is delivered, and send the samples to a testing laboratory.

31. Sampling.—The cement is sampled by taking a small quantity (1 to 2 pounds) from the centre of the package. The number of packages sampled in any given lot depends on the character of the work, and varies from every package to 1 in 5 or 1 in 10. When the cement is brought in barrels, the sample is obtained by boring with an auger either into the head or into the centre of the barrel, drawing out a sample, and then closing the hole with a piece of tin tacked firmly over it. For drawing out the sample, a brass tube sufficiently long to reach the bottom of the barrel is used. The tube is thrust into the barrel, turned round, and pulled out; the core of cement is then knocked out into the sample can, which is usually a tin box with a tightly fitting cover. When the cement is in bags, the sample is taken from the surface to the centre. Each sample should be labelled, stating the number of the sample, the number of bags or barrels it represents, the brand of the cement, the purpose for which it is to be used, and the date of delivery and that of sampling. The sample should be sent at once to the testing office, and none of the cement should be used until the report of the tests is received. The testing

of cement ordinarily consumes 30 days ; therefore, the supply must be gauged so that a sufficient quantity will be kept on hand to allow the tests to be made without delay to the work of construction.

METHODS OF TESTING

32. Test for Specific Gravity.—The specific-gravity test is used to determine whether the cement is thoroughly calcined and whether it is adulterated. It is usually made by a specially qualified chemist. Although the architect very rarely makes this test, he often has it made for him, as by its result he knows the state of the cement, for adulteration and underburning reduce its specific gravity, and overburning increases it.

33. Test for Fineness.—The fineness is determined by measuring the percentage that will not pass through sieves having a certain number of meshes per linear or per square inch. The British standard specification for fineness of cement stipulates that if 4 ounces of cement is sifted continuously for 15 minutes the residue on a sieve containing 5,776 meshes per square inch shall not exceed 3 per cent. ; the residue on a sieve containing 32,400 meshes per square inch shall not exceed 18 per cent.

34. Test for Activity.—The activity, or time of setting, is ascertained by submitting pats of cement paste to the penetrating effect of weighted needles. To make the test, prepare a plastic mortar by mixing the cement with from 25 to 30 per cent. of its weight of clean water having a temperature between 65° and 70° F. ; then make a cake or pat 2 or 3 inches in diameter and $\frac{1}{2}$ inch thick, immerse it in water having a temperature of 65° F., note the time required for the cement to set hard enough to bear respectively a needle made of steel wire $\frac{1}{16}$ inch in diameter and loaded to weigh $\frac{1}{4}$ pound, and a needle $\frac{1}{8}$ inch in diameter loaded to weigh 1 pound. In use, the points of the needles rest on the cement, and are held between the fingers in a vertical position. The moment when the coarse needle fails to sink into the cement is called the time of initial setting ; when the fine needle no longer penetrates, the moment of final setting.

35. Setting of Cement.—A very important consideration is the time required for cement to set. This depends on a great many conditions, such as the amount of water added to the dry cement, the temperature of the atmosphere, and various other things. If the minimum amount of water is added, English Portland cement will have an initial set in about 30 minutes and a hard set in from 2 to $2\frac{1}{2}$ hours. Natural cement, under the same conditions, will develop an initial set in from 2 to 3 minutes and a hard set in from 4 to 6 minutes. The British standard specification, issued by the Engineering Standards Committee, stipulates that there shall be three distinct gradations of setting time, *quick*, *medium*, and *slow*. In the first, the final setting shall take not less than 10 minutes or more than 30 minutes. In the second the final setting shall take not less than $\frac{1}{2}$ hour or more than 2 hours, and in the third it shall take not less than 2 hours or more than 7 hours.

Cement has a tendency to get stronger and stronger for some time after it has set ; and cements that set very rapidly do not develop so much additional strength after the first month of set as do the slower-setting cements. Cement should be put in place before it begins to set. Some architects and builders have an idea that if they let the cement get an initial set and then add more water and remix it, or *retemper* it, as they say, a greater final strength will be developed than if used the proper way. This notion has been proved incorrect, and the practice should never be followed.

36. Various substances may be mixed with cement to retard the setting, that is, to make it set slowly ; among these is *sulphate of calcium* (plaster of Paris). A small addition of this material will delay considerably the setting of cement. When cement is mixed with sand, it sets slower than does the neat (pure) cement. The British standard specification limits the addition of plaster of Paris to not more than 2 per cent. by weight. Whether or not cement will set when frozen is a disputed question. However, it remains undisputed that cement sets very slowly when cold, and therefore, if placed when the temperature is below freezing, it will again get soft in a few days, provided a thaw

comes. Some engineers claim that natural cement will not set at all when frozen.

37. An effective method for regulating the setting of cement has been patented by the Associated Portland Cement Manufacturers. It consists of subjecting the cement in the final stage of grinding to a moist atmosphere by injecting into the tube mill in which the cement is ground a quantity of steam, which keeps the atmosphere saturated with moisture, so that, as the mill revolves, every particle of the cement is subjected to a continuous process of superficial hydration. This process is based on the well-known fact that a quick-setting cement, if allowed to become aerated by exposing to the atmosphere, becomes slow setting, and is also improved in quality by the neutralizing of any particles of free lime present.

38. Tests for Soundness.—The tests for soundness have for their object the determination of those properties that tend to impair the strength and durability of cement. The tests for soundness are carried out by making pats of pure cement paste which for a **normal** test are immersed in water for 28 days. For the **accelerated** test the pats are placed in water of a temperature of about 70° F., which is gradually raised to the boiling point. This temperature is maintained for 6 or 8 hours, when the pat is taken out and allowed to cool. If as the result of both tests the pats show no signs of cracks or distortion, the cement is said to be thoroughly sound.

39. Test for Strength.—The strength of cement is determined by submitting a specimen, called a **briquette**, of known cross-section to a tensile stress. The reason for adopting tensile tests is that comparatively light stresses produce rupture, and that, since mortar is less strong in tension than in compression, it fails in most cases by tensile stress, even though the brickwork or masonry is under compression. The briquettes are tested at two periods: the first, 7 days, and the second, 28 days after moulding. In each case the briquettes remain for the first 24 hours in moist air, and the rest of the time in fresh water at 70° F. A cement giving an extremely high strength at the 7-day period should be

regarded with suspicion, and the tests for soundness should be made with great care.

40. The method employed to ascertain the tensile strength is to make two tests, one with neat or pure cement and one with a mixture of cement and sand.

1. A certain quantity of pure cement is placed on a glass plate and mixed with just sufficient water to make it plastic. The paste is then pressed firmly into a brass mould, the shapes of which vary considerably. After the mould is filled, it is covered with a damp cloth, or is placed in a moist closet and allowed to stand for 24 hours. At the end of this period, the block or briquette of cement is removed from the mould and immersed in water, where it remains until the time for testing arrives.

2. One part of cement and 2 or more parts of sand are carefully measured by weighing, and then thoroughly and intimately mixed dry on a plate of glass or other non-absorbing surface. The mixture is then drawn together in the form of a mound, and a depression made in its centre into which just sufficient water to produce the desired consistency is poured. The material on the outer edge is turned into the depression, and the whole mass is thoroughly mixed for about 1 minute. The mortar so made is immediately placed in the mould and compacted with a moderate pressure of the fingers; the filled mould is then covered with a moist cloth or is placed in a moist closet and allowed to stand for 24 hours, after which it is removed from the mould and immersed in water, where it remains until tested.

In order to compare different brands of cement, it is essential that the sand used for the mortar should be of exactly uniform composition and quality as regards size, sharpness, surface of grains, and degree of dampness. On important works, the tests are usually made with the actual sand that is to be used. The briquettes are removed from the water at the end of 7 and 28 days, respectively, and subjected to a tensile stress until rupture takes place. For this purpose, several machines are used.

41. **Official Standard.**—The British standard specification stipulates that cement mixed neat with just sufficient fresh water to make a smooth paste shall be filled into moulds, without

TABLE III
TESTS OF CEMENTS

Kind of Cement	Residue Left on Sieve Per Cent.				Amount of Water Used in Gauging Per Cent.	Time Taken in Setting		Tensile Strength Pounds Per Square Inch Average of Two Tests Made					Amount of Water Used in Gauging Per Cent.	
	Number of Meshes Per Inch							Neat		Gauged with Sand 1 and 3				
	50	76	100	180		Initial	Final	3 Days	7 Days	28 Days	7 Days	28 Days	Neat	With Sand
English Roman .		19.0	20.5	29.5	35	6 min.	10 min.	240	265		80		30.0	11.0
English Medina .		6.2	8.2	18.2	40	3 min.	4½ min.	290	310		85		35.0	12.0
French Natural (Grenoble)	3.0	8.4	13.8	23.4	24	2½ min.	4 min.	345	387	320	145	185	22.0	9.0
English Portland	none	7.5	3.0	14.5	22	30 min.	2 hr. 15 min.	585	705	875	270	365	17.00	7.5

mechanical ramming, to make the briquettes of a standard shape; these briquettes shall be allowed to set sufficiently to permit of removal from the mould without damage to the briquette and shall be kept in a damp atmosphere for 24 hours, when they shall be placed in fresh water, and allowed to remain there until required for testing. The water shall be renewed every 7 days and the temperature thereof maintained at between 58° and 64° F. On the seventh and twenty-eighth days, respectively, steady tensile stress shall be applied. The minimum stress before breaking shall be 400 pounds per square inch of section after 7 days and 500 pounds after 28 days. Similar briquettes formed of 1 part of cement mixed with 3 parts of dry sand, by weight, should bear a minimum tensile stress of 120 pounds per square inch after 7 days and 225 pounds per square inch after 28 days before fracturing.

42. Results of Actual Tests.—From actual tests carried out, the results shown in Table III were obtained, showing the difference between English Roman, Medina, French natural, and English Portland cement. In all cases the temperature of the room in which the tests were made was between 58° and 62° F. The table shows the time taken in setting, the residue left on sieves of varying mesh, the tensile strength of samples examined, and the percentage of water used in gauging the various samples before testing.

43. Rough Tests.—As it is frequently necessary to use cement before the laboratory tests can be completed, as well as to ascertain if the unsampled barrels are running the same as the sampled ones, it is essential that rapid or rough tests should be made at the work. These may be performed as follows: The setting and hardening are determined by noting the time that elapses until pats of the cement and the mortar resist penetration under a light pressure of the thumbnail. The soundness is ascertained by placing a pat of the cement in water and raising the temperature to the boiling point; if the pat shows no signs of checking or cracking, the cement may be considered sound.

Another method often adopted on small works is to plunge one's hands into the centre of a sack of cement. If the cement feels hot, it should not be used until it has been exposed to the air, but if no appreciable heat is felt it may be used.

Builders and architects often test cement by mixing some to the consistency of a paste and filling it into a glass bottle while still wet. If, when it has dried, it shrinks and moves as a lump in the bottle the cement is stale and should not be used. If it expands on setting and cracks the bottle, it should likewise not be used, as it is too fresh. But if, on the other hand, it just fills the bottle when set, without cracking it, the cement is just in a fit condition for use.

MORTARS

LIME AND CEMENT MORTARS

44. Definition.—Mortar is used in building operations to bind together the various materials in brick or stone construction, and to fill up the voids between the individual bricks or stones in a wall and so help to render the joints weather-tight. For structural purposes, it is composed of lime or cement, called the *matrix*, and sand, called the *aggregate*, mixed to the proper consistency with water. The proportions of the ingredients depend on the character of the work in which the mortar is to be used. The quality of the mortar depends on the quality of its constituents, the proportions in which they are used, and the method by which they are mixed.

INGREDIENTS

45. Sand.—The sand used for mortar must be clean, that is, free from dirt and organic matter and an excess of clay (5 per cent. of clay is considered detrimental). If the sand contains much loam, when it is rubbed between the fingers they will become stained; but if they do not become soiled, the sand is probably clean. If squeezed in the hand, the particles of sand

should fall apart when the fingers are opened. Sea sand does not make a good sand for mortar. In the first place, the grains are round from being washed over one another by the action of the sea ; this does not allow the cement to get as good a grip on the sand as if the grains were sharp and angular. Sea sand contains salts that are *hygroscopic*, that is, attract moisture ; this property causes the mortar, even after becoming set, to be always damp and discoloured. If sea sand must be used, it should always be washed free from its impurities and mineral salts before mixing in the mortar. Sand is prepared for use by (a) screening to remove the pebbles and coarser grains, the fineness of the meshes of the screen depending on the kind of work in which the sand is to be used ; (b) washing to remove salt, clay, and other foreign matter ; (c) drying, if necessary. When dry sand is required, it is obtained by evaporating the moisture either in a machine, called a **sand dryer**, or in large, shallow iron pans supported on stones, with a wood fire placed underneath.

46. The fineness of sand will depend on the character of the work for which the sand is to be used : if in rubble masonry and concrete, it should be a mixture of coarse and fine grains, the coarse predominating ; if in brickwork or ashlar masonry, a fine sand should be used. The best sand is that in which the grains are of different sizes ; the more uneven the sizes, the smaller will be the amount of voids, and hence the less the amount of cement required. In the absence of suitable sand, stone dust, pulverized slag, and clean broken bricks may be substituted without apparently affecting the strength or durability of the mortar. The British standard specification for sand used in testing cements is that it shall pass through a sieve of 20 by 20 meshes per square inch and be retained on a sieve of 30 by 30 meshes per square inch, the wires of the sieve being $\cdot 0164$ and $\cdot 0108$ inch in diameter respectively.

47. **Water.**—The water should be fresh and clean, and free from mud and vegetable matter. Salt water should not be used if it can be avoided, as it retards the setting. The quantity of water should be just sufficient to produce a plastic consistency

suitable for the work for which the mortar is intended. The consistency of mortar for brickwork and masonry should be such that it will stand in a pile and not be fluid enough to flow.

48. Mixing the Ingredients.—Sand is added to lime or cement to increase the bulk and thus cheapen the material. It also has the advantage, when mixed with lime, of reducing the shrinkage that occurs while setting. Combined with cement, sand makes a mortar that sets more slowly than would the neat cement. When sand and lime are mixed to make mortar, there should be enough lime in the mixture to cover completely each grain of sand. More lime than this causes the mortar to shrink excessively on drying; less lime weakens the mortar. The correct quantity of lime or cement to be added to sand to get the desired results, according to different authorities, varies from 1 part of lime to $2\frac{1}{2}$ parts of sand, to 1 part of lime to 4 parts of sand.

In mixing mortar, it is usual to designate the amount of each ingredient by a ratio, such as 1 to 1, 1 to 2, 1 to 3, etc., which signifies that the mortar is composed of 1 part of lime or cement to 1, 2 or 3 parts of sand, respectively. The first number of the ratio always indicates the amount of lime or cement, which for convenience is taken as unity. The proportion of the ingredients varies according to the work for which the mortar is used. The unit for measuring these ingredients is usually the commercial barrel of cement. The cement is sometimes sold in bags, but as a certain number of bags, usually three for natural and four for Portland cement, represents a barrel, the barrel still forms the most convenient unit. In specifying the barrel as the unit, it must be clearly stated whether it is the volume of packed or loose cement that is intended, because packed cement when emptied from the barrel increases in volume to such an extent that a nominal 1-to-3 mortar is changed to an actual 1-to-4.

LIME MORTAR

49. For mixing lime mortar, the lime and sand are first measured in order that the proportions specified may be obtained. The sand is then formed into a bed in the mortar box and the

lime is distributed over this as evenly as possible. The lime should then be slaked by pouring on water, after which it should be covered with a layer of sand or, preferably, a tarpaulin, to retain the vapour given off while the lime is slaking. Care should be taken to add just the proper amount of water to slake the lime completely to a paste. If too much water is added, the lime becomes too wet and will never attain its proper strength. If too little water is used at first, and more is added during the process of slaking, the lime will have a tendency to chill, and thereby injure its properties. It is considered better practice to make lime mortar in large quantities and leave it standing in piles for use as may be needed than to make it in small batches.

50. Using Lime Mortar in Freezing Weather.—Cold weather has two effects on lime mortar: it retards the setting and expands the mortar. The retarding of the setting is not necessarily detrimental, providing the weather remains cold for a long period; but mortar should not be laid during freezing weather if it can possibly be avoided.

51. Advantage of Lime Mortar.—Lime mortar has one property that cement mortar can never have: it has a certain amount of elasticity or give in it. This is an important point in favour of lime mortar, for this elasticity prevents many cracks in the structure which might otherwise occur.

CEMENT MORTAR

52. Proportion of Materials.—The same thing may be said in regard to the proportion of materials for cement mortar as was said about lime mortar; the cement should be in the correct proportion to just fill the voids between the grains of sand. If more cement is added, the cost of the mortar is increased; and if less cement is used, the strength of the mortar is decreased. A fact to be remembered, however, is that in either lime or cement mortar, the shrinkage caused in setting is not in the sand, but in the interstices between the grains of sand. Therefore, the more sand that a mortar has, the less it will shrink when setting.

53. Mixing.—The ingredients of mortar are usually mixed by hand. Sometimes, on extensive works where large quantities can be used within a short time, mechanical mixers are employed with the advantage of lessening the labour and ensuring good work. In mixing by hand, a platform or box should be employed. The sand and cement should be spread in layers, with a layer of sand equal to about one-half that to be used in the batch, spread evenly as a bed; the dry cement should be spread over this layer, and the remainder of the sand spread over the cement. The layers of sand and cement should then be turned and mixed with shovels, until a thorough incorporation is effected. The dry mixture should then be spread out, a bowl-like depression formed in the centre, and all the water intended to be used poured into it. The dry material from the outside of the basin should be thrown in until the water is taken up, and then worked with shovel and hoe until it assumes a plastic condition. Or, the dry mixture may be shovelled to one end of the box, and the water poured in at the other end. The mixture is then drawn into the water with a hoe, a small quantity at a time, and mixed with the water until a plastic mass is formed.

54. Frost has the bad effect of retarding the setting of cement mortar. If the weather remains cold long enough for the water in the mortar to evaporate, there may not be enough moisture left for the chemical reaction of setting when the mortar gets warmer. On the other hand, if wet mortar begins to set and then freezes, the ice may break the set and ruin the work. The effect of a continuous spell of cold weather on cement mortar is not as bad as alternate freezing and thawing. Portland cement is less affected than natural cement by a freezing temperature.

If it is absolutely necessary to continue building operations with mortar in freezing weather, one or more of the following precautions should be observed, according to the temperature and the importance of the work. Use a quick-setting cement. Make the mortar richer in cement than is customary. Heat the building, stone, cement, sand, or water, or all of these, as circumstances permit. Make the mortar as dry as possible.

Add salt to the water to lower its freezing point. Cover up the finished work with clean straw, or a tarpaulin, but not with manure, as is sometimes done, because the acids in the manure are liable to injure the cement. Of course it is very much better to suspend operations during freezing weather than to proceed and take any of these precautions.

STRENGTH OF MORTAR

55. The strength that mortar should possess is of three kinds; namely, *compressive*, *cohesive*, and *adhesive*. The degree to which it should possess any one of these depends on the position in which it is employed. In ashlar masonry, resistance to compression is all that is required; in uncoursed rubble masonry and in brick work, it must possess adhesiveness, or the capacity of adhering to the surface of the stones or brick in order to prevent their displacement. In brick work and masonry of all classes that may have to develop transverse stresses, it must possess cohesiveness or tensile strength.

The tensile and the compressive strength of a given mortar depend on the adhesive strength of the cementing medium and on the character of the aggregate. Coarse and fine sand in the proportion of about 4 parts of coarse grains and 1 part of very fine grains usually produce the strongest mortar. Screenings from broken stone usually produce stronger mortars than sand, because of their greater density. Mixtures of sand and screenings often produce stronger mortar than either material alone. With the same aggregate, the strongest and most impermeable mortar is that containing the largest percentage of cement in a given volume of the mortar. With the same percentage of cement in a given volume of mortar, the strongest, and usually the most impermeable, mortar is that which has the greatest density, that is, which in a unit volume has the largest percentage of solid materials.

56. The tests that have been made to determine the various strengths of mortar show varying results, owing to the differences in the sands employed and the methods followed in conducting

the tests. The resistance to compression is ascertained by submitting cubes of mortar to compressive stress in suitably arranged machines. The crushing strength of mortar is usually taken as equal to eight or ten times the tensile strength.

57. The resistance of mortar to tensile stress is measured by breaking briquettes composed of cement and sand mixed in the desired proportions and tested in the manner similar to that described for cement. The tensile strength of mortars composed of various cements is shown in Table III. These values may be expected to apply to good materials; as a rule, they are much greater than the values required by specifications. Thus, for mortar composed of 1 part of Portland cement and 3 parts of sand, tensile strengths of 100 and 150 pounds per square inch, at the end of 7 and 28 days, respectively, are required by some specifications; and for neat cement, 450 and 550, respectively, for the two ages just mentioned.

In comparing the tensile strengths of mortars, it is generally considered that a mortar composed of 1 part of Portland cement and 4 parts of sand is equal to a mortar composed of 1 part of natural cement and 2 parts of sand. In testing the tensile strength of Portland-cement mortar composed of 1 part of cement and 1 part of sand, it frequently happens that this mixture shows a greater strength than a paste made of cement. This result is due to the thoroughness with which the mortar is mixed and compacted in the moulds.

58. The adhesive strength of a mortar depends not only on the materials of which it is composed, but also on the character and porosity of the materials to which it adheres. The experiments so far made show that the adhesion of mortar to brick surfaces is from 25 to 85 pounds per square inch, and to iron bolts from 200 to 300 pounds per square inch.

MORTAR COLOURS AND STAINS

59. Artificial colouring, in mortar, has been known for hundreds of years, but its general use dates from a comparatively recent period. Coloured mortar is used to obtain one of two

entirely different ends : one is to get the effect of a mass of colour by concealing the joints, the other is to use a contrasting colour to emphasize the joints. Common bricks lose much of their rough effect when mortar of the same colour as the brick is used, and the chipped or uneven edges do not show so plainly as when the bricks are laid in white mortar.

Most of the mortar colourings consist of mineral pigments, sold either as a dry powder or in the form of a pulp or paste. Pulp colours seem to mix better with the mortar than dry colours, and are therefore preferable for the better class of work. Mortar colours should never be mixed with lime until the lime has been slaked at least 24 hours. The colour, however, should always be mixed very thoroughly with the lime putty before any sand is added ; if the work is of a very fine quality, the coloured mortar putty should be strained through a coarse sieve. When cement mortar is to be made, the colour is mixed dry with the cement and sand. The coloured mortar looks different in the bed than when dry. The final colour can be seen by taking a small quantity of the mortar from the bed and permitting it to dry thoroughly.

WATERPROOFING MORTAR

60. Ordinary lime mortar when set absorbs about 50 per cent. of its weight of water. Cement mortar absorbs about 12 per cent. of its weight. If a joint in a wall is saturated with water and then allowed to freeze, the moisture in the mortar joint, as it congeals, expands and has a tendency to crack off the pointing. In order to overcome this difficulty, it is sometimes deemed advisable to make the mortar waterproof. This is accomplished by adding 1 per cent. of powdered alum to the sand and 1 per cent. of soft soap to the water. These two substances, when combined, form an insoluble compound. With lime mortar, however, the waterproofing just mentioned retards the drying out of the interior portions of the work, on account of making those parts air-proof. Nevertheless, waterproofing makes the mortar somewhat stronger both in the initial and in the final set.

GROUTING

61. **Grout** is thin lime or cement mortar used, instead of stiff mortar, to pour in places where the latter would be difficult to apply. It is also used to fill up completely any spaces not properly filled up during the laying of the bricks. Grout should be used sparingly, as the excess of water seems to lessen the strength of the cement considerably. It should only be allowed to be applied at every third or fourth course of brickwork, and then only sparingly. The empty joints it fills are due only to bad workmanship.

CONCRETE CONSTRUCTION

COMPOSITION AND USE OF CONCRETE

INTRODUCTION

1. The value of concrete as a substitute for stone and brick has been known for many centuries. Concrete found extensive employment in the construction of ancient Roman temples, palaces and public baths, being used for domes and arches, for the interiors of brick-faced walls, and for the foundation work.

Although, for many years past, engineering works of great magnitude have been constructed of concrete, it is only within a quite recent period that this valuable material has come into extensive use for building purposes, except in foundation work, where it has long been used. While it may be concluded that concrete will never entirely take the place of stone and brick, the material must be recognized as one of great value for various constructional purposes, used either alone, as a casing to iron or steel, or in combination with steel as reinforced concrete. Suitable materials for the manufacture of concrete can be obtained cheaply in almost every locality, and this renders the material an important substitute for brick and stone in ordinary structures.

2. **Component Parts.**—The component parts of concrete are (1) the **matrix** or cementing material, which is usually Portland cement and sand, and (2) the **aggregate**, which is either gravel, shingle, ballast, broken stone, furnace clinker, coke-breeze, or broken hard bricks. An aggregate is used in order to make the resulting mass cheaper than pure cement mortar would be.

Some engineers consider that the word *aggregate* includes the sand also. This may be correct, but it is not customary and will not be so used in this Section.

An aggregate must be free from clay or any other substance which might prevent the matrix from coming into direct contact with the gravel or broken stone. Although gravel, ballast, and shingle are frequently used as aggregates, they are smoother than broken stone, and, consequently, the matrix will not adhere so well. The particles of the aggregate should not be too large, or

TABLE I
CRUSHING STRENGTH OF CONCRETE

Parts by Volume			Age of Concrete			
			7 Days	1 Month	3 Months	6 Months
Cement	Sand	Broken Stone	Crushing Strength in Pounds Per Square Inch			
I	I	3	1,600	2,750	3,360	4,300
I	2	4	1,400	2,400	2,900	3,700
I	2½	5	1,300	2,225	2,670	3,400
I	3	6	1,200	2,050	2,440	3,100
I	3½	7	1,100	1,875	2,210	2,800
I	4	8	1,000	1,700	1,980	2,500
I	5	10	800	1,350	1,520	1,900
I	6	12	600	1,000	1,060	1,300

they will not fit into small corners in the work. Broken stone makes a better aggregate than a rounded material such as gravel, but it is often more expensive.

The size of the aggregate depends on the character of the work. In concrete for ordinary foundation work, it is usually specified that all the aggregate shall pass through a sieve of 2½-inch mesh. In concrete for walls and floors a 1½-inch mesh sieve is usually specified, and in the case of reinforced concrete the material of the aggregate is required to be still smaller, and should all pass through a sieve of ¾-inch mesh. It should be

clearly understood that it is customary to fix only a maximum size, so that, unless the material is so fine as actually to constitute sand, there is practically no minimum size. The material of a good aggregate should be of various sizes, so that the particles interlock with one another.

In the case of a crushed aggregate, the small pieces of stone caused by the crushing should not be screened out, as is often done, because they help to fill up the voids between the larger stones and thus decrease the amount of matrix required.

3. Compressive Strength of Concrete.—On account of advances in the use of concrete, particularly when reinforced with steel, the compressive strength that may be developed in the material is becoming an important consideration. Table I gives the crushing strength of concrete made with several proportions of materials.

MIXING AND LAYING CONCRETE

MIXING AND MIXERS

4. Proportions.—Concrete should be made by mixing a good quality of Portland or best natural cement with clean sharp sand and a proper amount of aggregate. The proportions vary for different classes of work, from 1 part of cement, 2 parts of sand, and 3 to 4 parts of aggregate in reinforced-concrete construction to 1 part of cement, 3 parts of sand, and about 6 parts of the aggregate for the less important kinds of foundation work. Natural stone, gravel, broken brick, etc., may be used as an aggregate, but limestone should not be used in other than foundation work, as it has been found to disintegrate under the action of fire. If, in the construction of walling, a very close imitation of a natural stone is desired, the stone to be imitated should be crushed and used as an aggregate. In addition, colour to correspond with that of the stone should be mixed with the cement. The finer the stone is crushed, the nearer will be the resemblance to real stone.

Gauging boxes should always be provided for measuring the materials. A ton of Portland cement contains about 25 cubic feet. The cement is packed in bags for inland use, there being usually ten bags to the ton, each bag containing $2\frac{1}{2}$ cubic feet. The size of a gauging box can, therefore, easily be calculated. As, however, the cement is sometimes delivered in eleven or twelve bags to the ton, it is usually necessary to verify the number of the bags. A gauge box consists of four sides without a bottom. Thus, when the box has been filled, the workmen have merely to lift it up and the whole of the material is left behind.

5. Hand Mixing.—In preparing concrete by hand labour, the materials should be worked on a platform made of boards. This platform should have sides about 10 inches high, battened on the back, and should be laid on the ground near the work in course of construction. The platform is necessary, as it keeps the concrete off the ground and thus prevents loam or clay from mixing with the mass. As either of these substances would adhere to the stone and destroy close contact with the cement, the effect would be to lessen the strength of the concrete.

The measured portions of cement, sand, and broken stone, while dry, should be thoroughly mixed by shovelling them together at least twice, so that there will not be varying proportions of aggregate to matrix in different parts of the concrete. Water is then sprinkled with a rose on this mixture, which is shovelled over again, at least twice, until it has a pasty consistency, after which it is ready to be put in place.

6. Continuous Mixers.—When large quantities of concrete are needed, mixing machines are usually employed. Machine mixers produce much better results than ordinary mixing by hand and are much more economical. There are a large number of various kinds on the market. One type of machine, known as a **continuous mixer**, consists of a long screw about 18 inches in diameter, which is enclosed in a slightly inclined iron casing of a little greater diameter; the screw is operated by a crank which may be turned either by hand or by power. The cement, sand, and stone are introduced through hoppers at the upper

end and the water is added through pipes at intervals along the length, and, as the screw revolves, the materials become thoroughly incorporated. The concrete is forced out at the lower end of the machine, falling into a bucket or other con-

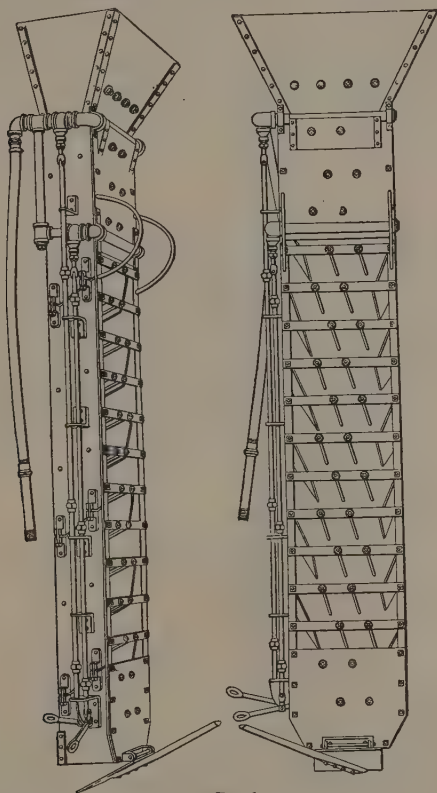


FIG. 1

veyer. After the concrete is made, the time between the mixing and the ramming should be as short as possible, in order to prevent the cement from setting before it is in place.

To hoist the prepared material, a travelling crane is sometimes used, the concrete being placed in buckets, which are

transported to the proper place by the crane. Another plan is to have an elevator built at a central point on the site and carried to the full height of the intended building, runways being made to each floor as the work progresses.

7. **Gravity Mixers.**—Another style of continuous concrete mixer is termed a **portable gravity mixer**. This mixer, as shown in Fig. 1, consists of an inclined trough with projecting steel pins, so arranged that the ingredients of the concrete will be deflected from one side to the other in descending, and thus become thoroughly incorporated. At the upper end of the

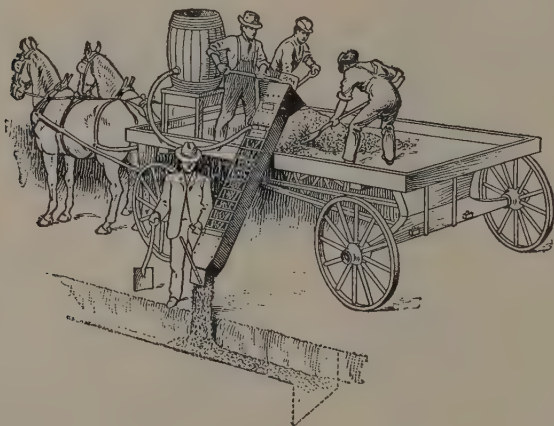


FIG. 2

trough the pins are placed close together, so that any pieces of stone passing the first row will be certain not to clog in the other pins. In order that the concrete may be mixed dry before the water is added, the water connection is placed part-way down the mixer. Water is supplied through a hose as shown in Fig. 2, which illustrates one of the many uses to which this mixer may be put.

8. **Batch Concrete Mixers.**—A batch mixer consists essentially of a large cylindrical or cubical box placed on trunnions. There are many types of these mixers, which are operated in various

ways. In the oldest type the ingredients are placed inside the box, which is securely closed and revolved on the trunnions. In other mixers the box, which is cylindrical, is open at both ends, and the materials are mixed by vanes attached to the inside of the cylindrical walls, the concrete being discharged as the machine revolves on its axis. With some types of mixers this discharge is accomplished by tipping up the cylinder, and in others by inserting a shoot in the end of the machine while the cylinder is in motion.

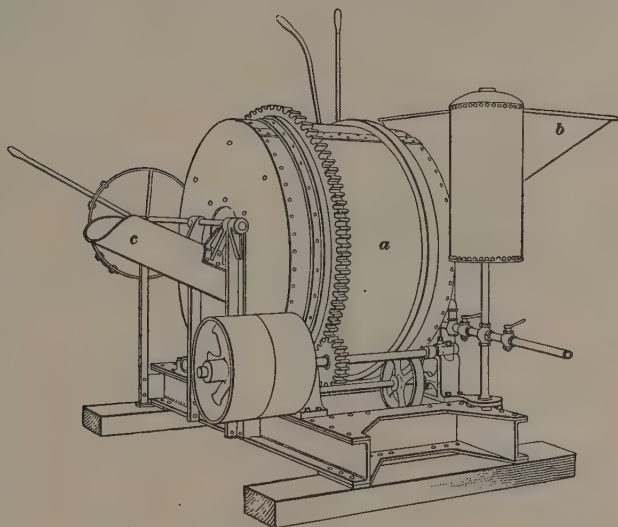


FIG. 3

9. The Ransome batch concrete mixer is illustrated in Fig. 3. The portion of the apparatus which effects the mixing consists of a revolving cylinder or drum *a*, having fixed blades or scoops in the inside. The hopper by which the drum is filled is shown at *b*, and the shoot by which the concrete is discharged is shown at *c*. The shoot remains in the position shown in Fig. 3 until the concrete is properly mixed, then it is lowered, and, the machinery being still kept in motion, the concrete is discharged by means of the shoot.

LAYING CONCRETE

10. Method of Laying.—Concrete should not be thrown to its final position from a height. It should be carefully lowered, or tipped out of barrows, and then spread out and settled, by ramming, which should be continued until the concrete is thoroughly consolidated, and the cement begins to appear on the surface. The surface of each layer should be left rough for the succeeding layer, which is applied in the same way, until the required thickness is obtained. In order to ensure that the concrete shall be a homogeneous mass, it is necessary to deposit a fresh layer before the previous one has become set, otherwise cracks are liable to develop in the finished work at the points where the layers join.

The making and placing of concrete should be very closely inspected, to ensure the specified quality and proportions of cement and other materials, and also the proper manipulation. The cement, too, should be frequently tested during the progress of the work. It is desirable that a clerk of works should be constantly at hand to see that the concrete is properly mixed and laid.

PLAIN CONCRETE CONSTRUCTION

CONCRETE IN SITU

WALL CONSTRUCTION

11. When placing concrete, it is necessary to confine it in forms, on account of its tendency, when wet, to run and spread. Forms for concrete work are usually made of wood, and are removed after the concrete has set. While still wet, concrete exerts considerable pressure against the side of the forms.

12. Form Construction.—When building concrete walls, piers, and partitions, the concrete is deposited in forms, made somewhat as follows: Timber posts, 4 inches by 4 inches or 6 inches by 4 inches, are set up in pairs on opposite sides of the wall which is

about to be constructed, each pair being strongly braced to prevent springing. Against the inner sides of the uprights are laid planks 1 or 2 inches thick, the distance from face to face of the boards being just the thickness of the wall. The planks frequently have bevelled ridges formed on the inner side, to produce the appearance of joints in the wall. This is shown in Fig. 4, in which *a* represents the posts; *b*, the inside face of the boarding; and *c*, the moulded or bevelled strips which make the apparent joints when ashlar is to be imitated. It is thus evident that concrete may be made to resemble any kind of stone masonry by varying the position and shape of the strips.

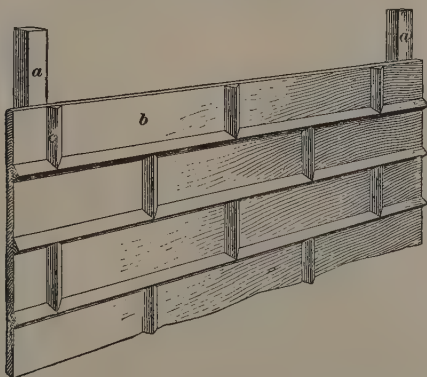


FIG. 4

The inside surfaces of the forms should be planed, smoothed, and then washed over with some substance such as whitewash, soft soap, or oil, in order to prevent the adhesion of the concrete to the timber.

After the lower portion of the concrete wall has set, the braces against the uprights are loosened sufficiently to permit the withdrawal of the boards, which are put in place higher up. A moveable arrangement is often adopted. This consists of slotted standards which may be raised, when necessary, without interfering with the work of laying the concrete.

13. The forms for rough concrete work do not necessarily require the same amount of care as the one just mentioned,

and sometimes the boards are not even dressed. An ingeni-

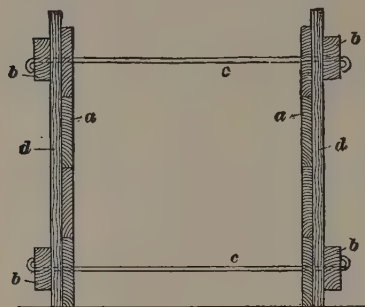


FIG. 5

ous method used to support the forms for large work, and which is cheaper than the one shown in Fig. 4, is illustrated in Fig. 5. Here, *a, a* represent the form proper, which is held in place by uprights *d, d*, spaced about 2 feet centre to centre. The uprights are supported by 8" $\times$ 3" planks *b, b*, which are tied together with light steel rods *c, c*. After the concrete has set, it is only necessary to cut these rods and the form falls apart. The rods are then clipped off close to the concrete, leaving the construction completed.

14. **Expansion and Contraction.**—When concrete hardens in air, it contracts slightly, but often sufficient to produce cracks

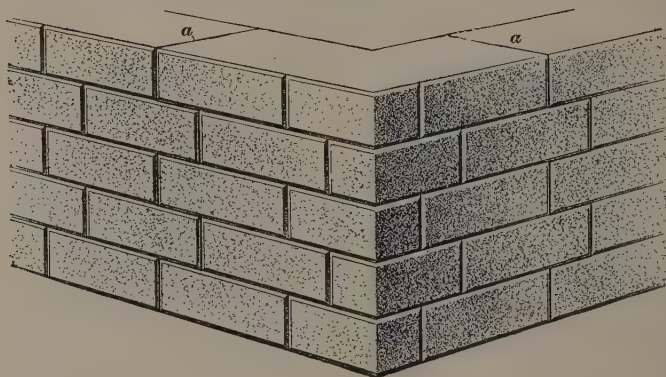


FIG. 6

in the walls and floors. To obviate this, the employment of some means to allow for expansion and contraction is sometimes adopted. This may consist in making planes of least

resistance in the work, which the cracks will follow. These, consequently, will not appear on the face of the wall.

A method of accomplishing this object is illustrated in Fig. 6. As shown at *a*, pieces of tar paper or other heavy paper are laid in the concrete in such a manner as to follow the course of the imitation joints in the facings. The paper should extend to within $\frac{1}{8}$ inch of the surface at each side of the wall, so that, if the concrete contracts, it will crack in these places.

In placing string-courses, or other ornamental features, great care must be taken to have them so arranged that shrinkage in the concrete will not injure them. All sills for windows should be bedded hollow in the centre, so that, if the concrete wall settles, the ends of the sills will remain unbroken. For similar reasons, it is well to have the window lintels made in the form of a flat arch with a through joint at the centre, and a recess for a key, which is put in later.

15. Surface Finish.—Up to within a few years, either plaster or mortar was used for the outside covering of most concrete buildings; this finish, however, does not prove satisfactory and increases the cost considerably. It is now not uncommon to cut the outside faces of concrete walls to closely resemble rough-dressed stonework. The effect of coursed stone or ashlar is produced by

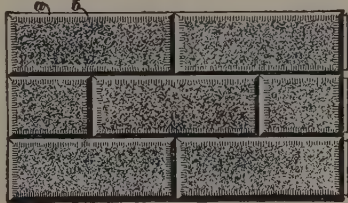


FIG. 7

forming imitation joints in the face of the wall, as shown in Fig. 7, and afterwards either picking or tooling the surface. In imitating rough-dressed work, the form is taken from the wall before the concrete becomes absolutely hard, and the finishing may then be done very rapidly. When an imitation of finer tooled work is desired, the concrete should be allowed to harden for a longer period before being cut.

Fig. 7 represents a method of finishing the face of concrete blocks; *a* shows the joint in the concrete; *b*, a margin, or

draught, cut to imitate tooled work ; and *c*, the picked face. If the strips inserted in the inside of the timbering to form the apparent recessed joints are properly planed and bevelled, such joints will not require dressing.

OTHER USES OF CONCRETE

16. Concrete can be put to innumerable uses. Foundations, floors, partitions, and roofs made of this material are described in other Sections ; the construction of walls has just been mentioned, and concrete fence-walls and chimneys are made in practically the same way. Other uses of plain concrete are found in the construction of the foundation of roadways to be paved with granite sets, wood blocks, or asphalt, in which case the concrete is from 6 inches to 9 inches thick and follows the curve of the road surface, and in the construction of kerbs and the paving of footways. Concrete may either take the form of specially made blocks, or may be constructed in position.

17. Combined Concrete Kerb and Footway.—In Fig. 8 is shown a combined concrete kerb *a* and footway *b*, the whole being laid in position, and not in the form of cast blocks. The thickness of the paving to the footway is 3 inches, of which the lower 2 inches is composed of concrete in the proportions of 1 part of Portland cement to 2 parts of sand and 3 parts of broken stone

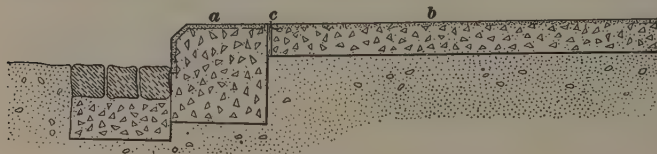


FIG. 8

or ballast, and the top inch of a mixture of 1 part of cement to 2 parts of finely broken stone. A deal spline *c*, from $\frac{1}{4}$ inch to $\frac{3}{8}$ inch thick, is shown placed between the concrete of the footway and the kerb, and similar splines are placed across the footway and kerb, at right angles to the run of the road, at intervals of about 5 feet. The splines thus divide the concrete

into bays and prevent the cracking which would be likely to occur if the footway were laid all in one piece. As shown in Fig. 8, the concrete rests directly on the ground; but if the ground is soft, a layer, about 3 inches thick, of broken brick or of burnt clay, usually termed *burnt ballast*, may be provided before the concreting is commenced. In important work and where the ground is very soft a layer of ballast much thicker than 3 inches is often used. In laying the concrete, the wooden splines are first placed in position and accurately levelled to give a fall of about $\frac{1}{4}$ inch in 1 foot to the footway. Then the concrete is deposited, levelled and consolidated with an ordinary plasterer's steel trowel, and brought to an even face by means of a straightedge running from spline to spline.

MOULDED CONCRETE

CONCRETE BLOCKS

18. Although, as already stated, concrete has been employed for centuries, it is only within the last few years that concrete building blocks have come into use. As a rule, these blocks are hollow in construction, which makes them both lighter and

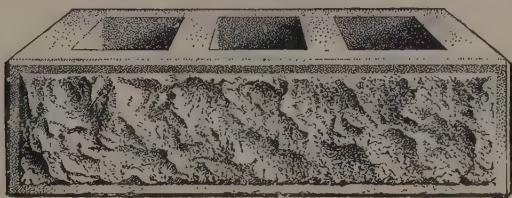


FIG. 9

cheaper, and also has a tendency to keep walls built of them free from moisture.

A typical concrete block is shown in Fig. 9. The makers of these blocks claim that it is safe to use plaster directly on the blocks on the insides of walls, without any special precautions, as the hollow construction will not allow moisture to penetrate

to the inner surface. This is probably true under ordinary conditions, but in the case of a wall in a very exposed position there is a liability that dampness may occur on the inside. In such case the erection of a hollow wall or the facing of the blocks with some less permeable material may be desirable.

19. Manufacture of Blocks.—In making concrete blocks, it is customary in the United States, which may be considered the home of concrete-block construction, to form the blocks in a machine having an iron mould. Dry concrete, that is, concrete made with as little water as possible, is often used, so that the block may be quickly removed from the mould and a new one cast. In this way, the moulding machine will have a larger productive power than if wet concrete were used.

By changing the face of the mould, the blocks can be made to represent rock-faced work or any particular stonework that is desired. The colour of the block can also be made to imitate various kinds of stone by using the proper kind of aggregate. Some makers, however, use artificial colouring.

Almost all building blocks are made of ordinary concrete, but some manufacturers use special ingredients or have a special process. It is common among some makers to cast the blocks under heavy pressure. This method has a tendency to make the blocks more dense, so that they will absorb less water.

20. The architect should exercise caution when selecting concrete building blocks, as many of them are cheaply made in order to compete with natural stone. For instance, some are sandy, absorb water freely, and present a dull regular appearance, which is often very far from the natural stone that is supposed to be represented. However, improvements are being made constantly, and the number of manufacturers who make first-class concrete blocks is very much on the increase. Some of these use the *wet process*; that is, use sufficient water in the mixture to allow the concrete to attain its full strength on setting. The moulds employed in this method are porous, and thus permit any superfluous water to escape. Blocks made in this manner are very dense and do not absorb water readily; they can also be tooled in the same manner as natural stone.

REINFORCED-CONCRETE CONSTRUCTION

INTRODUCTION

21. **Definitions.**—The terms **ferro-concrete**, **concrete-steel**, **armoured concrete**, and **reinforced concrete** have been indifferently applied to concrete in which steel is inserted for the purpose of strengthening the weakest portions of the concrete. Although all these terms are sometimes still employed, the last one, *reinforced concrete*, is superseding all the others. The metal used with the concrete is called the **reinforcement**. The term *reinforced concrete* does not apply to those combinations of steel and concrete, used in ordinary building construction, where the steel is designed to support all the loads. In these combinations the concrete simply protects the steel from corrosion and fire. In reinforced concrete, on the contrary, the proportions of steel and concrete are so designed that the stresses will be distributed properly between the two materials, having regard to their respective safe resistances. In order fully to understand the points at issue, it is necessary to consider the question of the stresses in materials. The forces which tend to pull the particles in the material away from one another produce *tensile stresses*, and those which tend to compress the particles produce *compressive stresses*. The tendency of the particles to slide one on the other produces *shearing stresses*, which result, in the case of beams, in diagonal tension.

22. **Object of Reinforcement.**—The primary object of the reinforcement is to reduce the cost of construction. Concrete has a low tensile strength, and the amount of it that would be required in structures subjected to tension would be very large. It is therefore more economical to use steel to resist tensile stresses. On the other hand, the compressive strength of

concrete is comparatively high, and in many cases it is cheaper to use this material to resist compressive stresses. Consequently, a very economical and comparatively light structure can be obtained by combining the two materials and so designing the structure that the compressive stresses will usually be entirely resisted by the concrete and the tensile stresses by the steel. This is the principle on which reinforced-concrete construction is based.

23. Concrete structures that resist compression only, such as columns and piles, are also commonly reinforced by steel, as it is found that they can better withstand the effects of shocks than when not reinforced. In some cases the steel assists in resisting the compression ; in others, it performs the still more important duty of holding the concrete together in such a way that it can withstand greater stresses.

24. Protection of Steel From Corrosion.—There has been considerable difference of opinion on the subject of the protection of steel embedded in concrete. There are many cases on record in which steel has been found in perfect condition, almost free from corrosion, on being removed from concrete in which it had been embedded for many years. There have been cases in which the steel was found rusted all through. All the evidence available at the present time, however, seems to indicate that, if the concrete is properly made and the steel is a sufficient distance from the surface, the danger of corrosion is slight. Steel embedded in cinder concrete is more likely to rust than steel embedded in stone concrete. This is due to the fact that cinder concrete is more porous than stone concrete, and allows air and moisture to reach the steel. In addition, there are likely to be some ingredients in the cinders that will combine with the moisture in the air, forming acids that corrode the steel. The coating of the steel with a cement wash before it is inserted in the concrete is usually adopted, regardless of the character of the aggregate. The bars should never be painted.

25. Basis of Reinforced Construction.—The whole theory and application of reinforced-concrete construction is based on the

assumption that the concrete and the steel reinforcement are so united that they will act together under all conditions of loading. It has been found that where steel rods are embedded in concrete, the *ultimate* adhesion between the two materials is between 250 to 450 pounds per square inch of surface of steel in contact with the concrete, and the *safe working* adhesion in work of the best character is taken as about 100 pounds per square inch.

In all ordinary forms of reinforced-concrete construction it is not customary to calculate the resistance offered to the sliding of the reinforcement. Experience has shown that, with perfectly sound materials and workmanship, this resistance will be amply sufficient. But as everything depends on the adhesion between the two materials it has latterly become customary to take special precautions against failure in this direction. Where the reinforcement in beams and slabs consists of round bars, it is now the almost invariable custom to split or bend the ends of the bars in order to increase the resistance to sliding. Another practice, and one becoming of very general adoption throughout all branches of reinforced-concrete work, is to use specially designed bars of a form providing a *mechanical bond* with the concrete. In addition to bars of this nature there are those having portions bent upwards, which, in addition to preventing diagonal cracks, offer a very considerable resistance to the sliding of the tensional reinforcement.

26. Uses of Reinforced Concrete.—Reinforced concrete has been considerably used, both on the Continent and in America, for the erection of buildings, but in the British Isles, though utilized in the construction of foundations, floors, and roofs, its complete adoption, until comparatively recently, has been restricted to works of an engineering character. For many structures of this kind it is most strikingly suitable. The erection, for instance, of grain silos or large retaining walls, in any material other than reinforced concrete, will soon be thought to almost constitute a constructional blunder. British building regulations up to the present have rendered the erection of reinforced-concrete buildings a matter of difficulty, if

not impossibility, except by bodies such as railway or dock companies, which, by reason of special Acts of Parliament, are exempt from the local regulations. As, however, the Government, in the additions to the General Post Office and in several other buildings, has shown its appreciation of this form of construction, the statutory restrictions affecting its general adoption are not likely to remain in force very much longer.

As an economical form of construction for such buildings as stores and warehouses, reinforced concrete, when it comes in general use, will probably be without a rival. The scientific placing of the materials so that each does the work for which it is specially well fitted, the concrete resisting compressive stresses and the steel resisting tensile stresses, results in a great saving of material, as compared with ordinary methods of construction, in which masses of materials, weak in tension, such as brickwork, are used without reinforcement. Whether reinforced concrete will be found as suitable for such buildings as offices and dwelling houses, where the external appearance together with the comfort of the occupants of the building has to be considered, the future must prove. If a suitable system of bonding to other walling materials is arrived at, possibly reinforced concrete will take the place of the customary brick-backing in stone-faced walls. In such a case the thickness of the stone facing would possibly be reduced so as to render it little more than a veneer of stone.

27. Resistance to Fire.—It is generally stated that reinforced-concrete structures offer an excellent resistance to fire. This statement may be considered to apply correctly to those reinforced-concrete structures, and to those only, in which the necessary precautions have been taken, by means of a reasonably deep embedding of the reinforcing bars, to ensure proper protection to the steel. Investigations, especially those made at San Francisco after the great fire of 1906, point to the fact that unless the reinforcement is well protected, a high temperature causes the steel to expand to a greater extent than the concrete, and the lower portion of the latter material, being also weakened by the heat, is thus liable to break and drop off, with the result that

the adhesion between the steel and the general body of the concrete is destroyed. The allowance of a distance of about 2 inches from the surface of the concrete to the steel is to be recommended, but, for reasons of economy, this course is rarely adopted in the case of floor slabs. A rule more suitable for general practice is to make the protection to the steelwork at least equal to the diameter of the steel reinforcing bars, and in no case less than 1 inch.

MATERIALS AND WORKMANSHIP

28. Quality of Concrete.—The materials used in plain concrete construction, with the general exception of aggregates lacking in strength, such as coke-breeze, are used throughout reinforced-concrete work, but they must be of the very best quality. Portland cement must invariably be used. The use of a higher class Portland cement specially manufactured for reinforced-concrete work is distinctly desirable. The sand should, of course, be clean and free from any earthy matter. In the Report of the Reinforced Concrete Committee of the Royal Institute of British Architects, a document which contains much information of practical value, it is recommended that the sand should be composed of hard grains the largest of which will pass through a $\frac{1}{4}$ -inch mesh, but at least 75 per cent. should pass through a $\frac{1}{8}$ -inch mesh. Fine sand is considered undesirable. The aggregate, which is usually broken stone, should be varied in size, but should all pass through a $\frac{3}{4}$ -inch mesh.

29. Proportioning the Materials in Concrete.—For the proper proportioning of the materials, it is necessary that the amount of cement added to the sand should be sufficient at least to fill the interstices or voids between the grains of the sand, and that the amount of matrix, consisting of cement and sand, added to the aggregate, should be sufficient to fill the interstices in the aggregate, and leave at least 10 per cent. surplus. The size of the particles of sand and of the aggregate thus have considerable influence in proportioning the materials, for the larger the particles, the larger are the voids between them, as the larger

particles cannot be so closely intermixed and consolidated. It has, however, been found, with the usual materials employed, that a proportion of 1 part of Portland cement, 2 parts of sand, and from 3 to 4 parts of the aggregate will give satisfactory results. Where special water-tightness or strength is required, the proportion of the cement should be increased. As it is so necessary to have the materials in their correct proportions, it is essential, in the use of an aggregate containing sand, to separate the sand from the stone and then to measure the materials and add the proper proportion of cement.

30. Quality of Steel.—The Committee of the Royal Institute of British Architects state, in their Report, that the steel should have a breaking strength of not less than 60,000 pounds per square inch, an elastic limit* of not less than 50 per cent. and not



FIG. 10

more than 60 per cent. of the breaking strength, and an elongation of not less than 22 per cent. of the length, measured generally in a gauge length of 8 diameters. There is a further requirement to the effect that the steel must be able, when cold, to be bent almost double, so that the distance between the portions of a bent bar is equal to the thickness of the bar, without there being any fracture on the outside of the bent portions. This is illustrated in Fig. 10, where the original position of the bar is shown in dotted lines. The bar is bent so that the distance a between the portions of the bent bar is equal to b , the thickness of the bar, and there should then not be any fracture on

the outside of the bent portion c .

31. The use of a steel having a high elastic limit is recommended by many authorities. By using steel of this character, it will be possible to provide either a smaller amount of reinforcement, or, if a concrete of a stronger quality than usual is employed, the amount of the concrete, and consequently the size of the various members, may be considerably reduced.

* The *elastic limit* of a material is that stress per square inch of sectional area which is the utmost stress that the material will withstand, and afterwards return to its original form. The relative elasticity of various materials can thus be measured by the elastic limits of such materials.

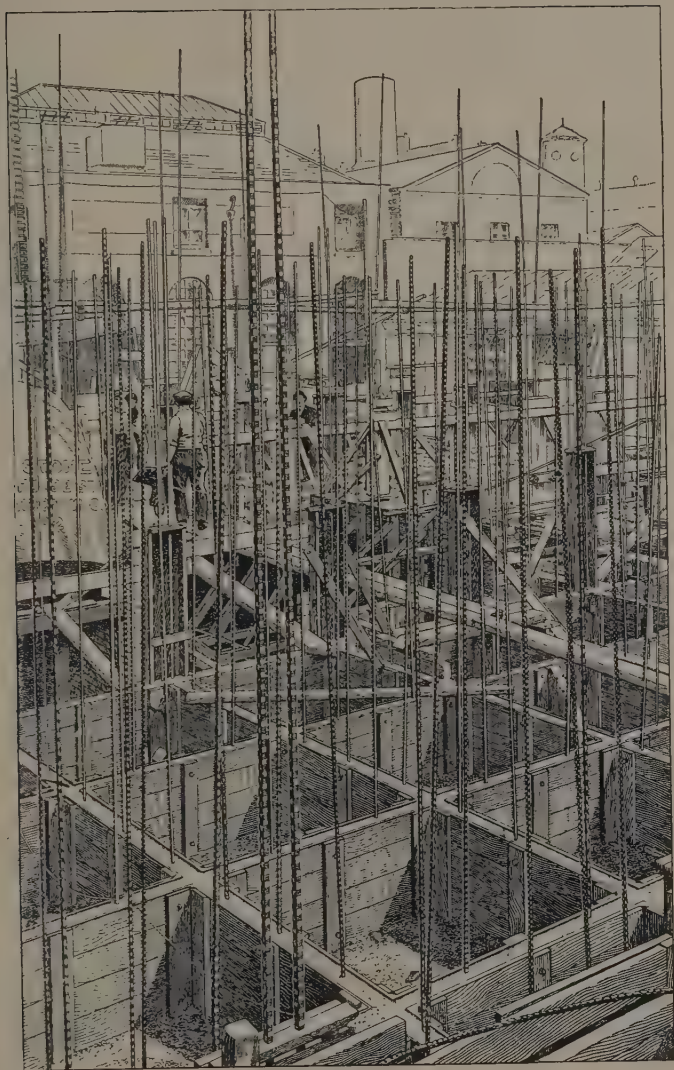


FIG. 11

Several of the special forms of bars used in reinforced concrete have a higher elastic limit than ordinary steel. The indented bar, which has been much used in the British Isles, has an elastic limit as high as 50,000 pounds per square inch.

A building in which the indented bar was used as a reinforcement is shown in Fig. 11. This figure is of considerable interest as illustrating a reinforced-concrete building in course of construction. The building was constructed to contain bins for the storage of plumbago. The vertical bars forming the reinforcement for the sides of the bins are very noticeable in the figure.

32. Workmanship.—It is admitted by all, that no matter how carefully and well a reinforced-concrete structure is designed, its stability may be seriously and even dangerously affected by bad workmanship. In structures of ordinary materials, the factor of safety adopted will cover an appreciable amount of deficiency in the methods of execution. But, in reinforced concrete, careless and bad workmanship may, by the entire misplacement of the reinforcement, create an amount of insecurity in the construction much beyond that which can be covered by any reasonable factor of safety; or if by any faults in workmanship the estimated adhesion between the steel and the concrete is rendered non-existent, the construction will no longer be reinforced concrete, it will merely be plain concrete, with, in the case of beams and slabs, the material, already of very insufficient strength, disrupted in its weakest point by useless steel rods.

33. The absolute necessity of good workmanship is therefore apparent. At present, reinforced-concrete work is chiefly in the hands of a relatively small number of specialist firms, who have a widespread reputation to maintain, and who may therefore be expected to carry out work in a sound manner. But when this work is undertaken by the ordinary contractor, employing workmen who are not familiar with the use of the material, the greatest care will have to be exercised in superintendence, particularly as regards the correct placing of the reinforcing bars.

In all works in which concrete is used the proper mixing of the material is very important, and this is particularly so in the case of reinforced work where the concrete is required to be of increased strength and soundness. Opinions have been, and still are, somewhat divided as to whether concrete should be mixed in a comparatively dry or very wet state. The position may probably be stated as follows: If experienced workmen are employed and great care is exercised, it is possible to produce concrete of very high efficiency by relatively dry mixing, but unless the operation of mixing is conducted with great care a better result will be likely to be obtained by a wetter process. Mechanical rather than hand mixing is usually adopted, and for this a batch concrete mixer is preferred, as it is considered to give more even results than a mixer of the continuous type.

CHARACTERISTICS OF CONSTRUCTION

COLUMNS

34. Methods of Reinforcement.—The methods of reinforcing columns differ in several points of principle from those customary in the case of beams. In the first place, the reinforcement is introduced essentially for the purpose of assisting the concrete, whereas, in the case of beams, the tensional reinforcement is assumed not merely to assist, but to relieve the portion of the concrete which is on the tension side of the beam or slab from the duty of offering any resistance whatever to tension. Methods of column reinforcement, in contradistinction to those applicable to beams, are also divisible into two distinct classes. In one instance, the method of reinforcement which consists in the insertion of plain bars, bound together at intervals by wire ties in order to prevent bending, results merely in the provision of a certain proportion of steel, the resistance of which to compression can be added to that of the concrete. This aspect of the question is illustrated by the fact that some designers place a portion of the reinforcement in the centre of the column. It should, however, be particularly noted that it is not possible to

make use of the full compressional resistance of the steel. The concrete and the steel must both compress equally, and although steel is about thirty times as strong as concrete in direct compression, its stiffness is only about fifteen times that of concrete. Thus, on the necessary assumption that the two materials act together, the resistance of the steel to compression can only be fifteen times the resistance of the concrete.

35. The other method of column reinforcement consists in a spiral winding of steel, throughout the whole length of the

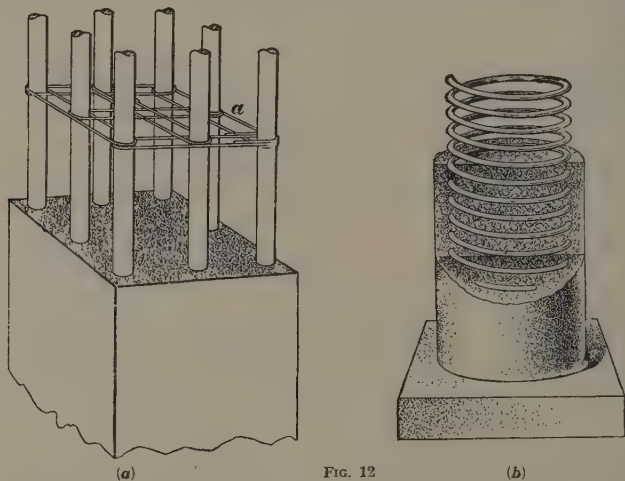


FIG. 12

column. The steel arranged in such a fashion, though in itself able to offer little resistance to compression, is found greatly to increase the strength of the concrete by preventing its tendency to spread when subjected to a load. An illustration of the two methods of column reinforcement is given in Fig. 12 (a) and (b), in which (a) represents the form of reinforcement with vertical bars, and (b) the method of spiral winding. There appears to be no general rule determining the spacing of the groups of wire ties shown at *a* in Fig. 12 (a), but it is most desirable that the vertical bars should be sufficiently well bound together. A very sound arrangement consists in spacing the ties at vertical

distances equal to the diameter of the column, but a very much wider spacing is often adopted.

Although the most general method of column reinforcement consists in this use of vertical bars with wire ties at intervals, the method of spiral winding is being adopted to some considerable extent, and certainly this form of construction, if well executed, produces columns of much greater strength. In most work of the present day, where a spiral reinforcement is adopted, it is customary also to use vertical rods which, in addition to increasing the resistance to compression, are of value in forming a framework for the winding of the spiral portion of the reinforcement. It is considered that, to ensure the best results, the distances between the coils of the spiral should not exceed one-tenth to one-seventh of the diameter of the spiral.

Reinforced concrete has been used to some considerable extent in the construction of piles. The method of construction is very similar to that adopted in the case of columns. Either vertical rods connected together at intervals with wire ties may be used or a spiral system of reinforcement may be adopted.

36. Percentage of Reinforcement.—The question of the amount of reinforcement to be used in any particular case is naturally a point of very practical importance. If the steel is to be taken into account in the calculations, the percentage should not be less than 0·8 per cent. On the other hand, there is a limit to the amount of reinforcement which it is desirable to provide. It is generally considered, in the case of columns, that it is inadvisable to use a greater amount of reinforcement than 6 per cent., for, if a column is very heavily reinforced, there is a possibility that the two materials will not act together, that the load will come on the steel only, and that the concrete will fracture and break away. The percentage of steel used in practice depends very largely on the loading, and varies usually from 1 to 4 per cent.

BEAMS

37. Methods of Reinforcement.—In the case of beams, the primary reinforcement is that which is introduced for the purpose of strengthening the portion of the beam which is in tension.

At one time it was thought that the steel reinforcement itself not only offered a resistance to tension, but that its presence increased the cohesion of the adjoining particles of concrete when subjected to tension. This is now admitted to be incorrect. Plain concrete in tension usually breaks at one section, and the break is plainly visible. In the case of reinforced concrete, the adhesion between the concrete and the steel distributes the elongation throughout the length of the reinforcement, with the result that the concrete breaks simultaneously at a great number of sections, each crack being so small as to be invisible to the naked eye. When extended in this way, the concrete is no more capable of resisting tension than if one large crack were plainly visible. The methods of calculation for beams being based on this fact, sufficient steel is always provided to take up the whole of the tension.

38. Both the concrete and steel in reinforced-concrete beams are usually united to the concrete and steel forming the columns.

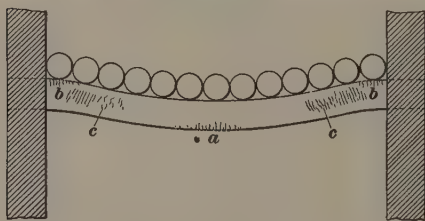


FIG. 13

A beam being thus to some extent fixed at the ends, it has a tendency, when subjected to a load, to assume the form illustrated in an exaggerated manner in Fig. 13. It will be seen that not

only is there tension at *a* in the lower portion of the beam in the centre of the span, with a corresponding compression in the upper portion, but that, in the part of the beam adjoining the supports, the stresses are reversed, and there is tension in the upper portion, as shown at *b, b*. Further, there is what is termed *diagonal tension*, tending to produce cracks, running naturally at right angles to the direction of the stress, as shown at *c, c*. The existence of this diagonal tension, increasing toward the supports of a beam, is common both to supported and fixed beams, and is present in all materials, but it has rarely to be taken into account, as the materials are usually of amply sufficient strength

in this respect. Concrete, however, is so weak in tension as to render it essential that, in addition to the reinforcement at the points where the beam is subjected to ordinary tension, further reinforcement against diagonal tension should be provided.

One method consists of introducing special bars attached to or forming part of the lower tension bars, and inclined toward the supports at an angle of 45° to such bars. Another method, and one very generally adopted, consists in carrying up one or more of the lower tension bars, as shown in Fig. 14 (a). This treatment achieves a double result. The bars, in addition to resisting, in their inclined position, the diagonal tension, also, when carried along the top portion of the beam near the supports, resist the tension in this portion of the beam, and usually render unneces-

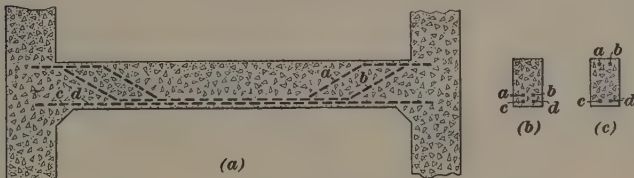


FIG. 14

sary the provision of any further reinforcement at this point. In the cross-section through the centre of the beam shown in Fig. 14 (b), the four bars *a*, *b*, *c*, *d* are shown at the bottom of the beam where the tension is greatest; in (c), which is a cross-section near the supports, two of the bars *a* and *b* are shown to have been brought to the top of the beam by running them up at an angle, as shown at *a* and *b* in (a), the bars *c* and *d* in (a) being carried through horizontally for the entire length of the beam. The previous remarks, as regards reversed stresses at the supports and, of course, as regards diagonal tension, apply equally to beams which are continuous over supports.

39. Compressional Reinforcement.—Concrete offers very considerable resistance to compression, the usual allowable safe stresses being 500 pounds per square inch in ordinary direct compression and 600 pounds per square inch when the material is subject to compression in beams. But the introduction of

one or more light bars in the upper portion of a beam is sometimes found to be desirable. Such bars, by reinforcing the concrete in compression, admit of a reduction in the amount of concrete and a consequent reduction in the size of the beam. Such compressional reinforcement is also of general value, for if, as is sometimes customary, vertical ties or stirrups are adopted to strengthen the concrete, these may be further employed in linking together the tensional and the compressional reinforcements, an arrangement which will materially strengthen the whole construction of the beam.

A compressional reinforcement is also sometimes adopted in the lower portion of fixed or continuous beams adjoining the supports. In the case of beams designed to carry very heavy loads, the provision of a compressional reinforcement at the supports is often exceedingly desirable. In the design of cantilevers, in which tension always occurs in the upper and compression in the lower portion of the member, it is customary to provide both a tensional and a compressional reinforcement. The reinforcing rods of a cantilevered portion should be soundly secured to the reinforcement of the main structure.

40. Percentage of Reinforcement.—The considerations governing the provision of reinforcement in beams are entirely different from those affecting the reinforcement of columns. In the case of columns the steel and the concrete act together in resisting compression, and the provision of a considerable percentage of steel may be justified on several grounds. In the case of beams where there is no compressional reinforcement, the two materials perform entirely separate duties, the concrete resisting the compression and the steel the tension. It can thus be seen that, the safe resistance of each of the two materials being known, there must be a certain economic ratio between the area of steel and the area of the portion of the concrete which is in compression. If this ratio is exceeded, if an excess of steel is provided, the steel cannot be stressed to its full allowable extent without the concrete being stressed beyond safe limits. If, on the other hand, there is an excess of concrete, the concrete cannot be stressed fully without causing too great a stress on the steel.

41. Although the economic ratio between the area of the steel and the area of the portion of the concrete in compression has little immediate application in the designing of beams, it is of great importance to note that the ratio between the area of the steel and the total area of the concrete, or, in other words, the percentage of reinforcement, varies exactly in the same proportion. Thus, there is in the case of beams a fixed percentage of reinforcement, termed the *economic percentage*. This, which is from 0·6 to 0·7 per cent., constitutes one of the bases of reinforced-concrete beam design. This percentage is often exactly adhered to, and in any general case the provision of much more than about 1 per cent. of steel is rather an exceptional occurrence. But, in particular cases, where it is desired to keep the members of the structure as small as possible, larger percentages of steel are adopted. The provision of additional steel increases the strength of a beam in a twofold manner. Not only does the increased amount of steel, as steel, increase the resistance of the beam to tension, but, by lowering the position of the neutral layer,* or layer at which there is no stress, it results in increasing the amount of concrete in compression, and consequently the resistance of the beam to compression.

This aspect of the question is illustrated in Fig. 15 (a) and (b). In (b), owing to an increase in the amount of reinforcement, as shown at *a*, the position of the neutral axis, which is dependent both on the relative stiffness of the materials and also on the ratio of the area of the steel to the area of the concrete, is lowered from *b* in (a) to *c* in (b), and the area of concrete in compression is consequently increased from that shown at *d* in (a) to that at *e* in (b).

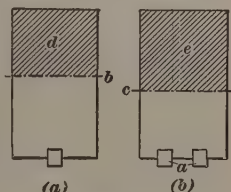


FIG. 15

Occasionally, where the loads are light, the area of the steel

* The *neutral layer* is the portion of a beam, at or near the middle of its depth, which is neither in compression nor tension. In considering the section of a beam, the line indicating the position of this layer is termed the *neutral axis*.

provided is less than 0.6 per cent., but such cases are not of very general occurrence. It is much more usual for the steel to exceed this amount, for it is often true economy to use bars of a standard size throughout, although the area of steel provided may often exceed the economic percentage. The sections of the beams shown in Fig. 15 are the net sections which are taken into account in the calculations. In practice, of course, the concrete extends below the bottoms of the reinforcement in order to encase and protect the steel.

SLABS AND WALL PANELS

42. Methods of Reinforcement.—The remarks already made as regards beams apply practically without exception to the case of slabs. As in beams, it is necessary to provide for the effect of the reversal of the stresses at the supports. The most common instance of this occurs at the junction of a slab with a beam. It is customary to carry every other, or at least every third one, of the reinforcing bars well over to the opposite side of the main beam. An example of this is shown in Fig. 16, the

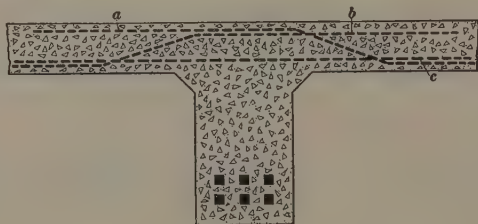


FIG. 16

bars which are carried over being shown at *a* and *b* and those running through horizontally at *c*. In actual practice these bars would be inserted in the concrete in the same horizontal plane at the same distances from the top and lower faces of the concrete, but, for the sake of clearness, in Fig. 16 they have been shown running above and below one another. These bars also, in their inclined position, increase the resistance of the concrete to diagonal tension. Floor slabs are occasionally reinforced in

both directions, and when so constructed are considerably stronger than slabs reinforced in one direction only.

43. The construction of wall panels, which, together with the framework of columns and stanchions, form the enclosures of a building, is governed by considerations very similar to those applicable to slabs. Instead of floor loads, these panels have to resist the loads due to wind pressure, which varies, according to the height and situation of the building, from about 25 pounds to a maximum of 50 pounds per square foot. The reinforcing bars of wall panels are usually vertical, but additional bars running horizontally are sometimes provided. Occasionally the wall panels are formed of purpose-made reinforced-concrete slabs.

SYSTEMS OF CONSTRUCTION

44. The Future of the Systems.—Reinforced concrete largely owes its present position in the structural world to the enterprise of numerous originators of special systems of construction. These systems, with their methods and forms of reinforcement protected by patent rights, still hold the field. It is to be expected, however, that in course of time a state of fusion will result. One special form of reinforcement is particularly suitable for one part of a structure, and another special form suggests itself as very desirable for adoption in another portion of the structure. Thus, it seems unquestionable that to obtain the best results in the erection of a reinforced-concrete structure, the use of more than one form of reinforcement is desirable. Such considerations are bound to have their effect in course of time. Those systems which have received chief application in British practice will now be referred to in brief outline. The use of expanded metal as a reinforcement to foundations and floors is mentioned in other Sections.

45. The Hennebique System.—The system which has been most generally adopted in the British Isles is the Hennebique system. The *columns* in this system are of the general type, reinforced by vertical bars held together at intervals by wire ties. In the construction of the *floors* the tensional reinforcement

consists of ordinary round bars. Half the bars are inclined upwards and brought along the top of the beam over the supports. A compressional reinforcement is rarely adopted. The U-shaped stirrups of hoop-iron, which, passing under the rods, extend from the bottom to the top of the beam, are a special feature of this system. As these stirrups are provided for the purpose of resisting the diagonal tension, the spacing is considerably reduced adjoining the supports. In the case of the slabs the use of stirrups is generally restricted to the portions of the slab next the supports. The *wall panels* are usually reinforced in two directions, the vertical bars are placed so as to resist the tension, and horizontal bars are also provided, for purposes of general stiffness, in the centre of the slab.

46. The construction of a beam at its junction with a column is shown in Fig. 17 (a), (b), and (c). The vertical reinforcing rods of the column with their wire ties are shown at *a*, the

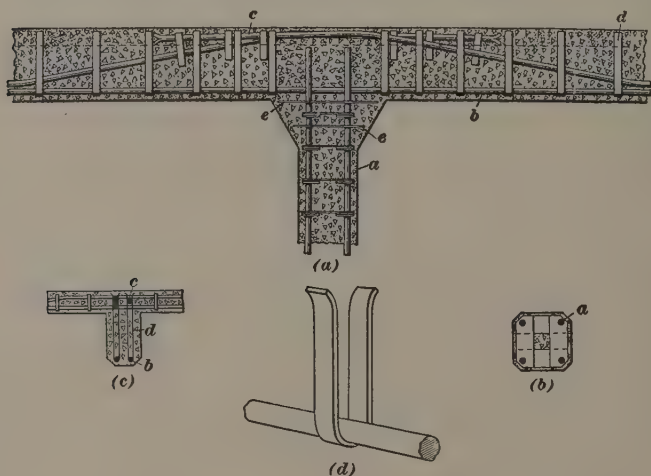


FIG. 17

continuous tensional reinforcement of the beam at *b*, and the portion of the reinforcement which is carried up to strengthen the concrete against diagonal tension and to resist the tension

at the top of the beam over the supports is shown at *c*. The stirrups are shown at *d* in elevation in Fig. 17 (*a*) and in section, adjoining the supports, in (*c*). The splay pieces of concrete which it is customary to provide in most forms of reinforced-concrete construction for the purpose of strengthening a beam at its junction with a column are shown at *e, e* in Fig. 17 (*a*). A sketch of a stirrup passing round one of the bars is given in (*d*).

47. Coignet System.—A method of construction which does not greatly differ from the last mentioned is the Coignet system. *Columns* are constructed in an almost exactly similar manner. In the construction of *beams* the system differs from the Hennebique in that the hoop-iron stirrups are omitted, a compressional reinforcement is adopted, and the reinforcing bars at the bottom and at the top of the beam are united by small bars varying from $\frac{3}{16}$ inch to $\frac{1}{4}$ inch in diameter. The reinforcement for *slabs*

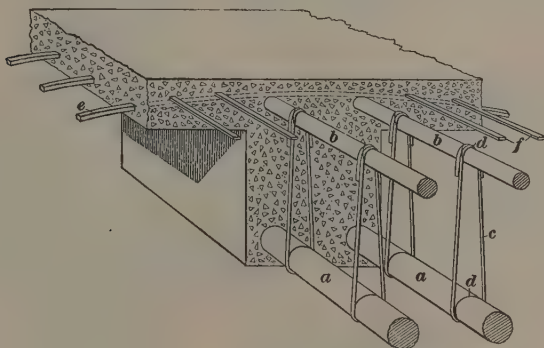


FIG. 18

consists of a meshwork of principal and secondary bars crossing one another at right angles, and bound together at the junctions with annealed wire. The principal bars are calculated to be sufficient to resist the tension, and the secondary bars are introduced only for the purpose of general stiffness. A meshwork of this description is used in the construction of *wall panels*, but in this instance is generally placed in the centre so as to resist pressure from either side. Where the thickness of floor slabs is

more than 6 inches a double meshwork with ties connecting the upper and lower principal bars is usually adopted.

A sketch illustrating the construction of a beam at its junction with a floor slab is given in Fig. 18. The tension bars are represented at *a, a*, the smaller compression bars at *b, b*, the light bars uniting them at *c* and the annealed wire, forming the connection, at *d, d*. Coming to the construction of the floor slab, the principal bars are shown at *e* and the secondary bars, which run at right angles, at *f*. It will be noted that the compressive reinforcement in the beam unites the beam to the floor slab in a very thorough manner.

48. *Considère System*.—The great increase of strength result-

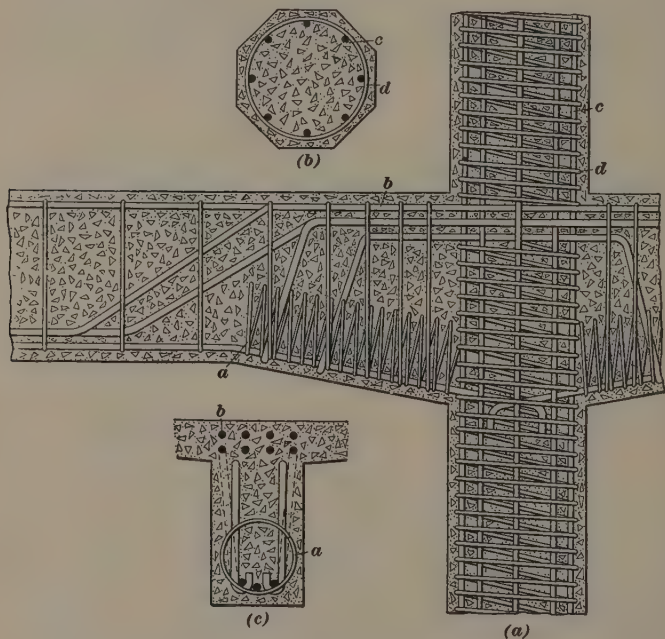


FIG. 19

ing from the use of a spiral form of reinforcement to columns has already been pointed out. In the *Considère* system bars are

provided in a spirally wound form, not only as a reinforcement to columns, but also for the purpose of increasing the compressive resistance of beams adjoining the supports. It is customary in such cases to strengthen the construction by establishing a more thorough continuity than is usually achieved. This is effected by strongly reinforcing the upper portion of the beam over the supports. Such a mode of construction increases the reversed stresses at the supports, and consequently can only be safely adopted when a sufficient compressive reinforcement is also provided. Fig. 19 represents the junction of a beam with a spirally reinforced octagonal column. A plan of the column is given in (b) and a cross-section of the beam adjoining the supports in (c). The spiral reinforcement to the portion of the beam which is in compression is shown in elevation and section, respectively, at *a*, and the heavy tensional reinforcement at the top of the beam at *b*. The vertical reinforcing rods of the column are shown at *c* and the spiral reinforcement to the column at *d*, this being wound round the vertical rods.

SPECIAL FORMS OF REINFORCEMENT

49. **The Kahn Bar.**—A form of reinforcement of special value in strengthening the concrete against diagonal tension is illustrated in Fig. 20 (*a*) and (*b*). This, known as the Kahn bar, consists of a central rib *a*, with a wing *b* on each side, portions of which *c, c* are cut and turned up at an angle of 45° with the central

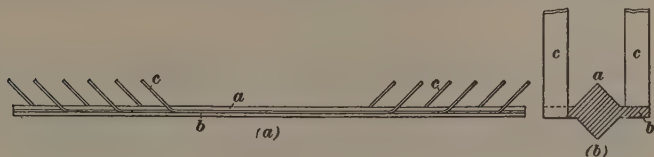


FIG. 20

rib, as shown in Fig. 20 (*b*). These *wings*, as they are termed, are cut so as to occur in the portion of the beam adjoining each of the supports, where they are needed for the purpose of resisting the diagonal tension, while in the centre, where the ordinary beam tension is the greatest, the bar is uncut and of its full

sectional area. Although the form of beam will not admit of full continuity over supports being obtained, yet the concrete at least is continuous, and thus, for the purpose of resisting the tension, which to a greater or less extent will be present at the upper portion of the beam over the supports, it is customary to

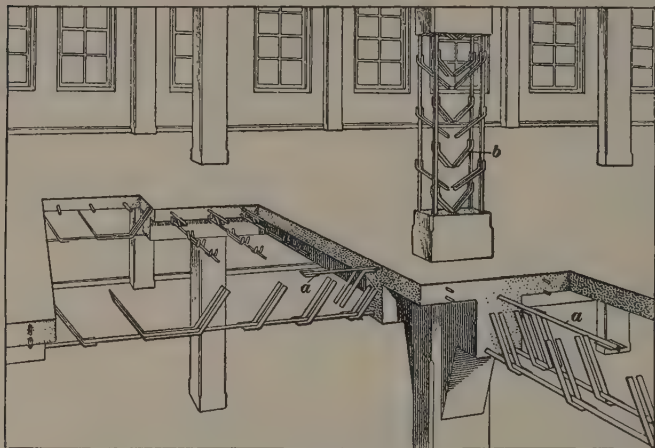


FIG. 21

insert inverted bars at this point, as shown at *a, a* in Fig. 21. This form of bar is also used for columns. In such a case wings are cut throughout the whole length of the bar, and when fixed in position are inclined downwards toward the centre of the column, as shown at *b*.

50. The Kahn bar is often used for reinforcing purposes throughout all the members of a structure; but it is of chief value in connection with the construction of beams, where the inclined wing members offer great resistance to the diagonal tension. As these members also are an actual part of the ordinary tension bars, they are of some considerable further utility in preventing the sliding of such bars through the concrete.

51. **Mechanical-Bond Bars.**—Reference has already been made to the growing tendency in reinforced-concrete construction

not to rely on the adhesion between concrete and plain steel bars. The splitting of the ends of the bars, although desirable as a precautionary measure in all cases where ordinary round bars are adopted, is not considered of very great value, for if the adhesion between the steel and the concrete becomes non-existent, the fact that the bars are held at the ends will not be sufficient to safeguard the construction. The use of **mechanical-bond bars** is thus becoming very general. A bar of this type, which is largely adopted in both British and American practice, is the *indented bar*, an illustration of which is given in Fig. 22. This is a square bar rolled with a series of projections and indentations, of which those on the sides alternate with those on the top and bottom. The indentations are shown by dark lines of shading in the figure. The bar is supplied in the following sizes: $\frac{1}{4}$ inch, $\frac{1}{3}$ inch, $\frac{1}{2}$ inch, $\frac{5}{8}$ inch, $\frac{3}{4}$ inch, $\frac{7}{8}$ inch, 1 inch, and $1\frac{1}{4}$ inches.

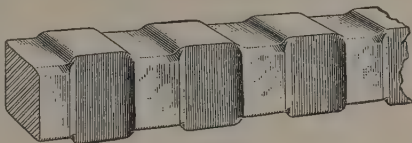
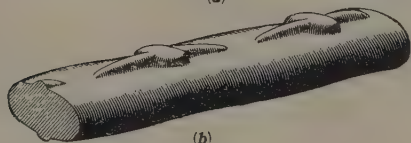


FIG. 22

52. Bars which have been used to a considerable extent in the United States are illustrated in Fig. 23 (a) and (b). In (a)



(a)



(b)

FIG. 23

is shown a bar of square twisted section. This form of bar is used in the Ransome system of reinforced concrete. The bars are twisted when cold, and the twisting is often done on the site of the

work. The twisting, in addition to providing a bar with a mechanical bond to the concrete in which it is afterwards placed, is also found to increase the strength of the steel. Fig. 23 (b) is an illustration of the Thatcher bar, which is elliptical in cross-section and rolled with a series of projections.

METHOD OF CONSTRUCTION

CONSTRUCTION OF FALSEWORK

53. As reinforced concrete, at least as at present executed, is essentially a monolithic form of concrete constructed in position, it is obvious that the question of the timbering, or falsework, to hold the concrete in position until it has set sufficiently is one of very great importance. From the point of view of economy of construction, the arrangement of the falsework is almost of as much importance as the designing of the structure itself. The methods already described for the erection of forms for walls in plain concrete apply equally to reinforced-concrete work. The remarks as to the treatment of the moulds, with a view of preventing the adhesion of the concrete to the woodwork, also apply. It is, however, necessary to emphasize the fact that much greater accuracy and care in the erection of the falsework are necessary in the case of reinforced-concrete construction. The amount of material used is very much less than in plain concrete construction, and consequently the whole workmanship must be much more exact in character. The two chief points to be kept in view in the design and construction of falsework are that it should be sufficiently stiff to obviate any risk of deflection and that it should be easily removable.

54. Falsework for Columns.—The falsework for columns may either be built up piece by piece as the concreting proceeds, or it may be fully constructed before the concreting is commenced, or, in accordance with the custom in the Hennebique system, three of the sides may be constructed and the fourth side built up as the concreting progresses. This last-mentioned method results in a construction of reasonable stiffness, and yet admits of the concrete being properly tamped as the work proceeds, an operation which can only be effected with difficulty by means of long-handled rammers if all four sides of the form are solidly built up before the concreting is commenced.

55. The construction of a column form with a fourth side which can be built up as the work progresses is illustrated in Fig. 24. The three principal sides of the form are shown at *a*, *b*, and *c*. The planks forming these sides are held together by boards *d*, *d*, to which they are nailed, spaced about 3 feet apart, and also by clamps as shown at *e*. The fourth side, which is secured by being nailed to the sides *b* and *c*, is shown at *f*. Triangular strips *g*, *g* are inserted in each corner of the form in order to provide the customary splayed angles to the finished column.

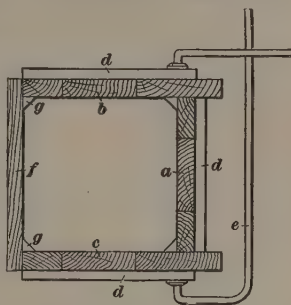


FIG. 24

56. **Centring for Floors.** — In the case of ordinary steel and concrete floors the centring for the construction may either be hung from the ceiling or supported from the floor beneath. In

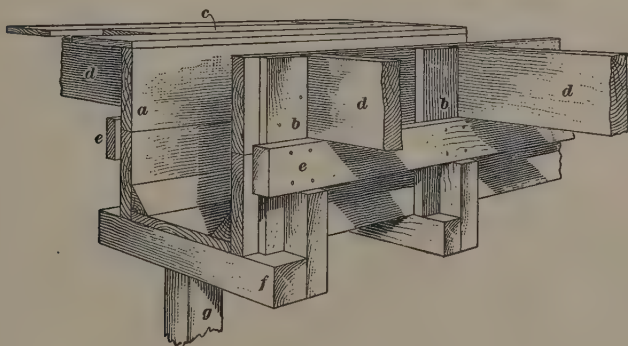


FIG. 25

the case of reinforced-concrete floors there is no alternative, and a supported type of centring must of necessity be adopted. The method of erection of supported centring for floors is the same, whether the floor is of reinforced or ordinary steel joist and

concrete construction. It consists usually of boarding varying in thickness from 1 to 2 inches carried on $8'' \times 2''$ or $9'' \times 2''$ joists, supported at intervals of about 6 feet by posts resting on the floor beneath. A special form of boxing must necessarily be provided for the construction of the main beams.

57. Fig. 25 shows the construction of the centring for a reinforced-concrete floor at the point where one of the main girders occurs. The boards forming the base of the boxing for the beam are shown at *a*, the battens holding together the sides of the beam box at *b, b*, the boarding for carrying the concrete of the floor at *c*, and the joists on which the boarding rests at *d, d*. The ends of the joists next the beam box take their bearing on a horizontal member *e*, which is nailed to the battens, and is also supported by means of blocks attached to the cross-pieces *f* at the bottom of the beam box. One of the posts which carry the whole construction is shown at *g*. These should be well braced together.

COURSE OF CONSTRUCTION

58. **Construction of Columns.**—The vertical bars of the reinforcement with their horizontal ties, or an additional reinforcement of spiral form, having been placed in position, and the falsework erected, the concreting is commenced. If possible, the material should be used fairly dry and in relatively small quantities, and should be well tamped as the work proceeds. If, however, the falsework adopted is such that the concrete has to be inserted from the top, it is most difficult to secure efficient tamping, and consequently, in such a case, it is customary to mix the concrete very wet, so that it will be likely, under the influence of gravity, to properly fill all portions of the form and thoroughly surround the reinforcement. On this account, the adoption of a close form of this character should be avoided. If there is any weight coming on the columns, the forms should be left in position until the concrete has thoroughly set; but if there is no weight on the columns, the forms may, if desired, be removed after the lapse of 24 hours from the completion of the concreting.

59. Construction of Floors and Roofs.—The customary practice in the construction of floors and roofs is first to place a layer of concrete on the top of the centring, which layer will constitute the “cover” beneath the tensional reinforcements of the beams and slabs, and then to proceed with the insertion of the reinforcement and the provision and ramming concrete. Very occasionally the practice is adopted of erecting the whole steel skeleton of the reinforcement before the provision of any concrete.

60. Construction of Walls.—The order of operations in the construction of walls of reinforced concrete is, in effect, a combination of those customary in the erection of columns and in the erection of floors. The framework of reinforced-concrete walls consists of columns and beams, and the wall panels are constructed in a very similar manner to floor slabs, although, of course, the timber falsework is vertical and similar in arrangement to that used in the construction of plain concrete walls.

SURFACE FINISH OF THE CONCRETE

61. As timber, and practically every other material which can reasonably be used in the construction of the falsework, is found to leave its impressions on the face of the concrete, the necessity for some special face treatment is widely recognized. There seems, however, to be very little agreement as to the means by which the concrete shall be rendered more sightly in appearance. If the face is rendered, it is possible, in the case of a sound, well-consolidated concrete, that the rendering will scale off. It has been found that, by going over the surface with a stiff brush immediately the falsework is removed, it is possible to remove the external film of cement and to provide a rough pebbled face; but the adoption of a facing of this nature, which might not be unsuitable for a warehouse or factory, would hardly be justified in the case of shop or office premises in an important thoroughfare. Unless a veneer of stone or tiles or some such treatment is adopted, it seems difficult to conceive how a reinforced-concrete building can be given a very presentable appearance.

FIRE-RESISTING CONSTRUCTION

PLANNING AND ARRANGEMENT

NEED OF FIRE-RESISTING CONSTRUCTION

1. **Increased Risks of Fire.**—As the danger of fire has of late years much increased with the close grouping together of buildings in large towns, consequent on the rise in the value of building land, a knowledge of the best types of fire-resisting construction must form part of the equipment of every competent architect.

2. **Fire-Resisting and Fire-Proof.**—The term **fire-proof** was at one time used to denote this class of construction, and this term is still in use in some countries. But as no material or form of construction is absolutely fire-proof, even firebrick being burnt away when continually subjected to great heat, it is customary in British practice to adopt the expression **fire-resisting** as being more truly illustrative of the real facts.

3. **Need of Protecting the Ironwork and Steelwork.**—It is most important to note that in providing a fire-resisting form of construction it is not sufficient merely to use incombustible materials. Buildings are often filled with inflammable goods, and consequently, although the materials of which a building is constructed are incombustible, a fire may very easily occur. Therefore, if any materials are used in the construction which cannot withstand the effects of a high temperature, they must be properly protected with some suitable fire-resisting non-conducting material.

4. Iron and steel and other similar structural metals are incombustible at ordinary temperatures, but they expand appreciably when subjected to great heat, and steel suffers a considerable reduction in strength under a high temperature. If a structure is to be considered fire-resisting in any real sense of the term, it is essential that all ironwork and steelwork shall be properly protected against the effects of fire.

GENERAL DESIGN

5. **Fire-Tight Divisions.**—In the designing of a large building used for purposes of trade or manufacture, it is not sufficient to consider only the character and disposition of the materials used. There are several questions of general arrangement, which, if properly handled, will have the greatest influence in limiting the spread of fire. One of the most important of these questions is the subdividing of the building into several *fire-tight divisions*, somewhat analogous to the water-tight compartments of a battleship. If a building is thus subdivided by means of brick walls properly carried up above the roof, with all the openings in such walls fitted with double iron or other fire-resisting doors, which should always be closed at night, it will be possible to confine any ordinary outbreak of fire to one division.

6. **Limitation of Cubical Extent.**—The limitation of cubical extent in the case of practically all buildings used for trade or manufacture is required by the London building laws. The general limit of any division is 250,000 cubic feet. The London County Council has power, however, to allow any greater cubical extent which it may consider desirable under the particular circumstances of any case; but a greater cubical extent than 500,000 cubic feet is very rarely allowed.

The building requirements of almost all other British towns do not contain any requirements as to the limitation of cubical extent. This may also be said of the building laws of most American towns, and there is no doubt that many of the big fires in the United States, as, for example, the one which destroyed

a large part of Baltimore in 1904, attained their great proportions by reason of the very large cubical extent of the building in which the original outbreak of fire occurred.

7. Rules of the Fire Offices' Committee.—Although the subdivision of a building may not be required by the local building regulations, yet in all very large buildings used for trade purposes the adoption of such a course is very desirable. The Fire Offices' Committee, in its rules for standard fire-resisting buildings, insurable at very special rates, requires that no compartment shall exceed 60,000 cubic feet. In measuring a compartment, the total floor area and the clear height from floor to ceiling are to be taken. This rule is on the assumption that the floors are fire-resisting and that all staircases, lifts, and floor openings are properly enclosed. The rules provide that no compartments which, taken together, will exceed 60,000 cubic feet are to communicate with one another except by means of a fire-proof communicating compartment, built up from the basement with walls of solid brickwork, and having all openings protected by fire-resisting doors at least 6 feet apart. However, buildings are rarely erected to comply with these rules in every respect.

8. Size of Openings.—The ordinary allowable size of openings in walls separating divisions of a building erected under the London Building Act is 7 feet wide and 8 feet high. The County Council has power to allow openings of a larger size, in which case the fire-resisting doors are usually required to be placed at least 2 feet apart. An opening having a greater area than 120 square feet is very rarely allowed. The limit of size of openings given in the rules of the Fire Offices' Committee, as applying to ordinary buildings, is 5 feet wide by 7 feet high. The length of openings taken together should not exceed one-half the length of the wall of the story in which they occur.

9. Enclosure of Staircases and Lifts.—Whenever an outbreak of fire occurs on one of the lower floors of a building, it is found that the flames and smoke ascend to the floors above. Every open staircase or lift consequently forms a flue for the

conveyance of the fire from the lower to the upper portions of the building, and it is therefore desirable that all such vertical shafts should be properly enclosed. This is at present not often done, except in such buildings as factories and workshops where many persons are employed, and it is necessary to enclose the staircases in order to render them smoke-proof, and consequently available as a means of escape for the workpeople. But there is an increasing tendency, in the erection of trade buildings of the best class, to provide fire-resisting enclosures to the staircases and lifts, and in the course of time this will doubtless come to be a general custom.

Where staircase and lift enclosures are provided, the doors are usually of 2-inch hardwood, which offers but a relatively slight resistance to fire. Iron doors or shutters, or armoured doors of sheathed wood, are, for fire protective purposes, much to be preferred. The staircases are usually constructed of concrete or artificial stone, or occasionally of hardwood. Provided that the staircases are soundly enclosed with fire-resisting materials, the form of staircase construction adopted is not of primary importance, but the use of ordinary deal or other soft wood is naturally most undesirable.

10. Protection From Adjoining Buildings.—If a well-constructed fire-resisting building contains inflammable goods, and adjoins buildings not of a fire-resisting character, its safety may be seriously threatened by the existence of such buildings unless precautionary measures are taken. It is illogical to take every precaution to prevent an outbreak of fire in a building and yet make no effort to guard against the effects of fire in an adjoining building.

11. Where the two buildings are separated by a proper party wall carried up above the roof, no particular harm need be feared. There is no better preventive to the spread of fire than a solid, well-constructed brick wall. But danger exists when the windows or door openings of the two buildings are near one another, as, for instance, when the buildings directly abut on either side of a narrow passageway, or when both are lighted from the same internal court. Under such circumstances, all the fillings to

the openings next the adjoining building are made fire-resisting. The doors and their frames may be of iron or hardwood, the window-frames of hardwood or iron, and the glazing of wired or other fire-resisting glass.

METHODS OF EXTINGUISHING FIRES

12. Fire Brigade.—In large trade buildings in which the goods are of a very inflammable character, the occupiers often train a portion of their staff to act as a fire brigade, in order that any sudden outbreak may be dealt with immediately. But the most serious fires have been found to occur at night, when the premises are vacant; consequently, it has been necessary to devise some further safeguard.

13. Automatic Sprinklers.—A method of extinguishing fires which is fast becoming general is the provision of a system of **automatic sprinklers**. A sprinkler installation consists of rows of water piping spaced about 10 feet apart along the ceilings of all stories. The piping is filled with water under pressure and fitted at intervals of 10 feet with nozzles sealed with a material which melts at a temperature but little above that of summer. The question whether or not a sprinkler installation is proposed is of considerable importance in determining the cubical contents to be allowed in any division of a building about to be constructed.

14. The construction of the Grinnell automatic sprinkler is shown in Fig. 1 (*a*) and (*b*). The glass hemispherical disc, closing the sprinkler, is shown at *a* and the strut holding the disc in position is shown at *b*. This strut is in three pieces joined together by solder which fuses at a temperature of 155° Fahrenheit: When the solder has melted, the strut and the glass disc fall away and the sprinkler is open for

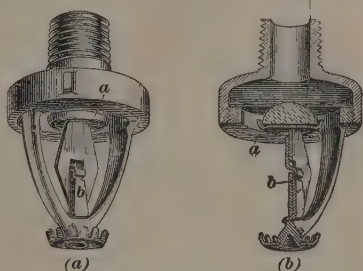


FIG. 1

the discharge of water. As the water pours out it strikes the serrated disc below and is dispersed over an extended area. Fig. 1 (a) shows a general view of the sprinkler and (b) a sectional elevation.

TYPICAL FIRE-RESISTING BUILDING

15. A general outline plan of a fire-resisting building to be used for trade purposes is shown in Fig. 2. The building abuts on the south side on a main street; on the west side on a passageway 10 feet wide, on the opposite side of which there is another building; and on the north and east sides on adjoining buildings, from which it is separated, except at the north-east corner, by proper party walls.

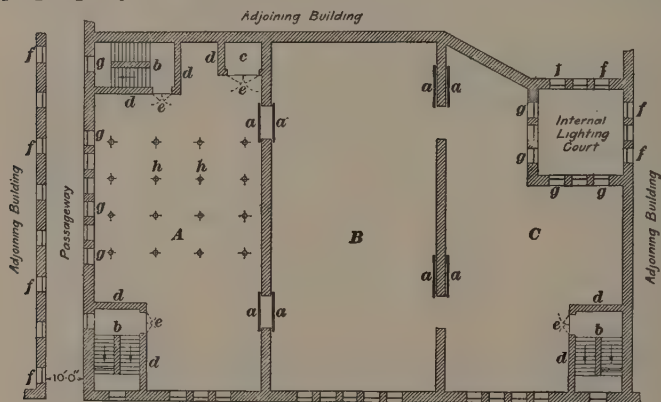


FIG. 2

16. **Arrangement of Divisions.**—The building, which is five stories in height, is cut into three divisions, A, B, and C, by brick walls extending the full height of the building and carried up above the roof in a similar manner to a party wall. The double iron sliding doors closing the openings in such walls are shown at *a, a*. The doors in the openings in the wall between divisions A and B are shown closed; those in the openings in the wall between divisions B and C are shown open. All the double doors are shown to be spaced apart the full thickness of

the wall. This is always necessary, as the greater the space between the doors the greater is the resistance offered by the doors to the spread of fire from one division to another.

The staircases are shown at *b, b* and the lift well at *c*. The staircases and lift are enclosed with 9-inch brick walls, as shown at *d, d*, and are fitted with swing doors of 2-inch teak, as shown at *e, e*. These teak doors are provided with springs, to render them self-closing, and swing either way.

17. Fire Protection and Extinction.—The windows in the walls of the adjoining buildings, which buildings are assumed not to be fire-resisting and therefore constitute a decided element of danger, are shown at *f, f*, against both the passageway and the interior lighting court. The adjoining windows in the fire-resisting building, which are protected by fire-resisting glazing in teak frames, are shown at *g, g*. At *h, h* is shown a portion of the sprinkler installation which extends over the whole building. The stanchions and joists forming the internal construction of the building are, for the sake of clearness, not indicated.

MATERIALS

CHARACTER OF MATERIALS USED

18. Two Classes of Materials.—The materials used in fire-resisting buildings may be grouped in two broad classes: (1) Materials such as iron and steel, which, however useful from the weight-carrying point of view, require to be specially protected from the effects of fire; and (2) materials such as concrete, brick, and terra-cotta, which offer a very good resistance to fire, and not only need no protection themselves, but are suitable materials for protecting the ironwork and steelwork. Timber might possibly be said to constitute a third class, but in the sizes customary in modern construction, even when in the form of hardwood, it is not a true fire-resisting material. In dealing with this question of materials, only the second of the two above-mentioned classes will receive further mention. The iron and

steel used in fire-resisting construction differ in no way from iron and steel used generally in constructional work.

19. Cast Iron and Steel Compared.—If in any particular case it is proposed to sacrifice efficiency to economy and leave the columns or stanchions unprotected, cast iron will constitute a better material than steel. Cast-iron columns have been found to be apparently unharmed under a temperature which has caused steel stanchions to fail entirely; but this fact does not in any way lessen the extreme desirability of properly protecting all structural ironwork and steelwork.

20. Unreliability of Stonework.—The schedule of fire-resisting materials which forms part of the London building law, in addition to including iron, steel, and copper, without any reference being made to the necessity of protecting these materials, also includes granite and other stones suitable for building purposes by reason of their solidity and durability. No building stone, however, offers a good resistance to fire. Limestones and sandstones are decomposed under heat, and granite will fly off in fragments when heated. If a steel-frame building is to be faced with natural stone of any kind, this facing should be additional to the brickwork, terra-cotta, or other materials provided for the protection of the steelwork.

FIRE-RESISTING MATERIALS

21. Portland Cement and Sand.—The matrix or mortar with which the materials of a concrete aggregate are bound together should, in the case of concrete for fire-resisting construction, be formed of Portland cement and sand of the best quality. The usual proportions for such matrix are one of cement to two of sand. Portland cement for all work of a sound character requires to be finely ground, and this is exceptionally desirable in fire-resisting construction. The blowing of the cement due to the presence in the finished work of comparatively large unslaked particles of lime, a defect at all times liable seriously to affect the construction, may so affect the cohesion of the particles of concrete as to render the material valueless for

fire-resisting purposes. The sand, as in all constructional work, should be sharp and free from earthy matter.

22. Concrete Aggregates.—The materials used as aggregates for concrete in fire-resisting construction may be very similar to those used for concrete of a general character, broken stone, ballast, shingle, clinker or furnace slag, and hard broken brick, all constituting reasonably sound materials. It is customary, as regards aggregates of this nature used for general fire-resisting work, to specify that all the material shall pass through $1\frac{1}{2}$ -inch mesh. In the case of floors and partitions, a light concrete is naturally very desirable, and thus coke breeze, a light though by no means strong material, is much used. Pumice stone, a similarly light material, is also occasionally used.

23. Coke Breeze.—Coke breeze, if free from any admixture of ashes or coal, forms a fair aggregate from the fire-resisting point of view, but a considerable amount of trouble has been experienced in the expansion of floors in which this material has been used. The expansion has sometimes been so serious as to occasion the thrusting out of the enclosing walls, even where an allowance has been made for expansion. Thus, coke breeze, though still very largely used, is not the most suitable material for use in fire-resisting construction, and broken stone, furnace clinker, or hard broken brick are being given the preference. A good specification of the size of a coke breeze aggregate is that it shall all pass through a $1\frac{1}{2}$ -inch mesh and none through a $\frac{1}{8}$ -inch mesh.

24. Proportion of Materials in Concrete.—Concrete is used not only for floors, partitions, and roofs, but also, the aggregate being coke breeze or finely broken stone, as a protection to iron and steel columns and stanchions. The proportions of the materials in concrete used in fire-resisting work are usually one of Portland cement, two of sand, and three or four of the aggregate. Where coke breeze is used, no sand is required, and the customary proportions are one of Portland cement to five of coke breeze.

25. Weights of Concrete.—The weight of any particular concrete depends almost entirely on the weight of the aggregate.

The weight of concrete in which the aggregate consists of granite chippings is about 150 pounds per cubic foot. Concrete of other broken stone, ballast, shingle, or clinker is somewhat less in weight. Concrete in which the aggregate is of broken brick weighs about 112 pounds per cubic foot, and coke-breeze concrete, by far the lightest kind in general use, weighs about 90 pounds per cubic foot.

26. Expanded Metal.—A diamond-shaped steel mesh, termed **expanded metal**, is largely used in fire-resisting construction. This is a form of sheet steel, in which slits have been cut and the sheet then stretched out or expanded. Fig. 3 shows a sheet of metal in which the slits have been cut, and a portion of the cut sheet

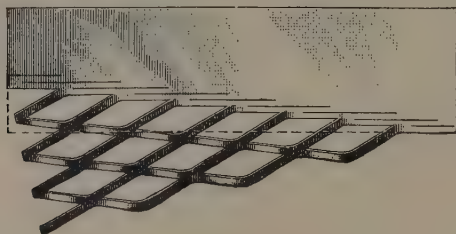


FIG. 3

expanded. When used in the construction of floors and roofs the expanded metal is inserted in such a position as to reinforce the portion of the concrete which is in tension. The material is also very valuable as the basis of the construction of partitions and of hanging ceilings, and, in the finer forms of mesh, it is largely adopted for holding together the concrete used as a protection to columns and stanchions. When thus used in connection with iron cradling, it takes effect in soundly securing the concrete casing to such members.

Expanded metal may be obtained in sheets of various convenient widths, and of lengths up to 16 feet. The stock sizes of mesh, measured the short way of the diamond, are as follows: $\frac{3}{16}$ inch, $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, $\frac{3}{4}$ inch, $1\frac{1}{2}$ inches, 3 inches, and 6 inches, the last being the largest size of mesh procurable. The 3-inch mesh is generally adopted for floors.

27. **Terra-Cotta.**—Terra-cotta is largely used in America for the protection of columns and stanchions, and for the construction of floors, roofs, partitions, and panel walls. In British practice the material is used considerably as a facing to brickwork and as a protection to columns and stanchions. Its use in floors, roofs, and partitions is almost entirely restricted to special systems of construction adopted by a few firms. Two kinds of terra-cotta, known as *dense* and *porous*, are used. The **dense variety** is made of selected refractory clay, or clay to which crushed brick or sand has been added. The **porous variety** is made of clay and sawdust

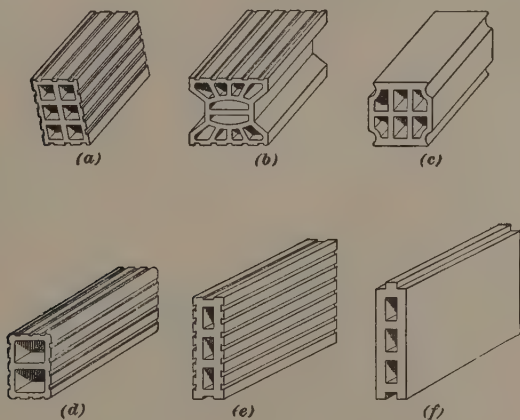


FIG. 4

or other combustible material. The latter compound is thoroughly mixed, moulded into the required shapes, and when sufficiently dry is placed in kilns and subjected to intense heat, which consumes all the combustible material, leaving the material in a porous condition and consequently much lighter than the dense variety. Dense terra-cotta is liable to crack when heated and suddenly cooled by water. Porous terra-cotta, which has not this defect, is thus a much better fire-resisting material, although most of the terra-cotta at present used in the British Isles is of the dense variety.

28. Forms of Terra-Cotta Blocks.—In Fig. 4 are shown several forms of terra-cotta blocks used in building construction : (a), (b), and (c) are forms employed in floor systems ; (d), (e), and (f) are the types used in the erection of fireproof partitions. On account of the hollow construction of the blocks and the uniform thickness of the webs, usually about $\frac{3}{4}$ inch, the clay is uniformly burnt and the likelihood of internal strain in the material is slight. Blocks of porous terra-cotta possess the property of receiving nails readily and holding them with considerable tenacity, and are thus of great value in partition work.

Floor arches carefully constructed of the blocks shown in Fig. 4 (a), (b), and (c) have proved strong enough to sustain any load the floor has been required to carry. Both the dense and the porous blocks are used in floor systems, but the latter has the greater advantage, as it possesses more refractory properties, is as strong as the dense block, and does not conduct heat so readily. Fig 4 (d) and (e) show forms of terra-cotta partition blocks for adoption when the faces of blocks are to be plastered, in which case the provision of a key for the plastering is necessary. Fig 4 (f) shows a block with smooth sides ; this can be used only when the surfaces of the partition are to be left unplastered, or finished with a thin setting coat.

29. Brickwork.—The use of brickwork is very general in the British Isles for the encasing of columns and stanchions. In the majority of cases where a steel-frame construction is adopted, brick walls of the ordinary scheduled thickness are none the less required. Although for weight-carrying purposes such a great thickness of brickwork seems quite unnecessary, yet the brickwork thus provided undoubtedly constitutes a very valuable protection to the columns or stanchions. Even when the schedules of wall thicknesses are amended to admit of the economic use of materials, it is probable that brickwork will still be largely used, not only for the construction of the panel walls, but also for the protection of the vertical members of the framework of the building. Any hard, sound, well-burnt variety of brick is suitable for this purpose.

CONSTRUCTION

PROTECTION OF COLUMNS AND STANCHIONS

PROTECTION BY CONCRETE

30. Concrete Casing for Phoenix Column.—Fig. 5 (*a*) and (*b*) illustrates the section of a Phoenix column completely protected by a concrete casing. Columns of the Phoenix type have been largely used both in the British Isles and in the United States. In (*a*) is shown the concrete surrounded by the mould, which consists of 2-inch battens *c*, held together by heavy iron straps *b*. These straps are made in halves, the same as the moulds, are

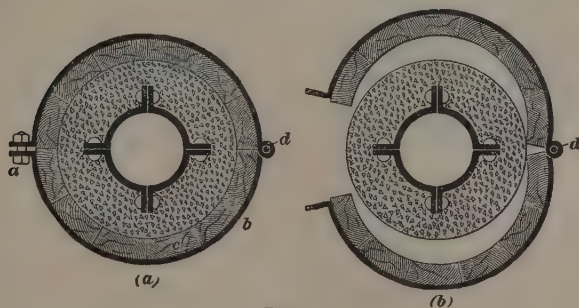


FIG. 5

hinged at *d*, and are bent to form a clamp *a*, being provided with bolts so that the mould can be drawn tightly together. These moulds are made in about 4-foot lengths, and after the concrete has been tamped in place and has had time to set, the bolts are removed and the moulds taken away, as shown in (*b*). They are then applied to another column or another section of the

same column, being supported in the proper position by temporary timbering. By the use of a mould for each column, the protection of all the columns can be carried along simultaneously. Since, however, 24 hours should elapse before the mould is removed, the columns should be dealt with in groups of three or four, so that each group can be laid and tamped consecutively. When the last group is completed, the workmen may return to the first group, in which the concrete has obtained the initial set.

31. Protection of Box Stanchions.—In Fig. 6 is shown a box stanchion protected by concrete. A section of the column, with its concrete covering, is shown in (a). The mould, or form, for the concrete protection, shown in (b), is made in halves that are provided with dowel-pins *e*, so that the two sections will always come together evenly. The side pieces *a* are tapered

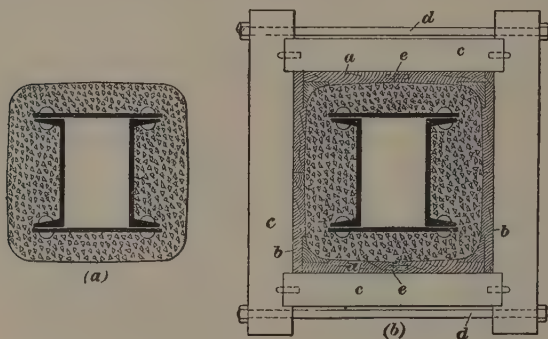


FIG. 6

so that the mould will readily draw away from the concrete when it has set. In order to make the column symmetrical, the other sides *b* of the mould are tapered toward the centre. The mould is held together by clamps *c* composed of 4" $\times$ 3" timbers and provided with bolts *d*. When the concrete has set, the bolts are withdrawn and the mould removed from the column and set up at another place. The moulds are made from 4 to 6 feet in length and are provided with a clamp at each end.

32. Concrete and Expanded Metal.—A still more reliable system of concrete protection to columns and stanchions consists in the holding together of the concrete by means of expanded metal. Sometimes the expanded metal is merely wrapped round the member to be protected, tied together at the joints with galvanized-iron wire, and then covered with plaster. But a much better method consists in the use of an iron cradling by which the expanded metal is kept about 2 inches away from the face of the column or stanchion. The space thus enclosed is then filled in with fine concrete and the expanded metal covered on the outside with a coat of plaster.

This arrangement in connection with both a cast-iron column and a built-up steel stanchion is shown in Fig. 7 (a) and (b),

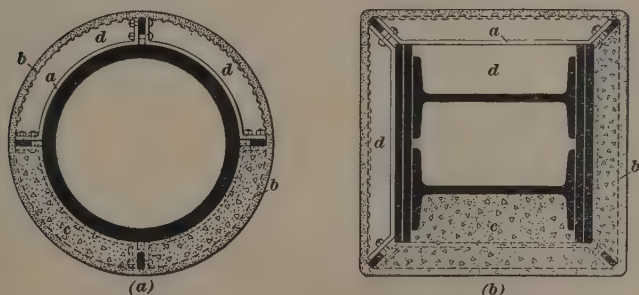


FIG. 7

the wrought-iron cradling being indicated at *a, a*, the expanded metal, covered externally with plaster, at *b, b*, and the fine concrete filling at *c, c*. When iron cradling is used, the expanded metal is kept stiff, and temporary timbering is not required. Sometimes no concrete filling is used, an air space *d, d*, as shown in the figure, being left between the expanded metal and the column or stanchion. This procedure is, however, not desirable, as there is a possibility that the thin steel-and-plaster protection may fail under the heat incidental to a big fire.

33. Advantages of Concrete Protection.—Of the materials used for protecting columns and stanchions, concrete finds the most

favour in British practice. Owing to its refractory properties and to the fact that it can be readily moulded to any shape of column, and also to the fact that the materials of which it is composed can be readily obtained in any locality at moderate cost, concrete is a useful and practical material for protecting columns and stanchions. It is an excellent non-conductor, and protects the member not only from heat, but also from corrosion. When the stanchion is a closed section, the possibility of corrosion occurring on the inside may be obviated by filling the stanchion on the inside with concrete. The concrete is likely, also, to increase the strength of the stanchion, and promotes rigidity throughout the structure. Where concrete is well tamped and is used inside as well as outside of the column, the safe load will be a little in excess of that given by the usual column formulæ. An internal filling of concrete will materially strengthen stanchions loaded eccentrically.

PROTECTION BY TERRA-COTTA

34. **Terra-Cotta Column Protection.**—Terra-cotta is used to a small extent for the protection of columns and stanchions,

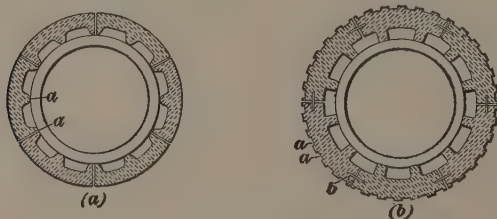


FIG. 8

particularly where appearance is to be considered. Although the use of concrete is much more customary, the method of protecting cast-iron columns by terra-cotta is not infrequently adopted in British practice. The encasing of stanchions with terra-cotta is, however, of rare occurrence, although this method is very general in Canada and the United States.

The simplest forms of **terra-cotta column protection** are illustrated in Fig. 8 (*a*) and (*b*). In (*a*) is shown a covering of terra-cotta blocks protecting a cast-iron column. The inside faces of the blocks have projections *a* that bear against the column and provide air spaces, as shown. These projections also form a support for the blocks and thus prevent them from being displaced.

In (*b*) is shown a similar system in which the terra-cotta blocks are intended to be plastered, and are provided on the outside with vertical grooves *a* to form a key for the plaster. These blocks are set in cement mortar and are held together at the joints by the wire binders, or keys, *b*. The blocks are so arranged that all vertical joints *a* will be broken, as shown in Fig. 9. The principal objection to this system is that the blocks are thin and do not adequately protect the columns from heat; also, the protection is liable to be broken or displaced in case of fire, and, in time, the joints are likely to be affected by moisture and heat.

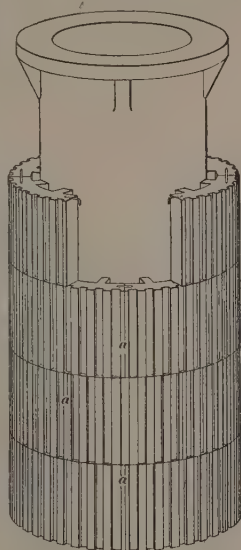


FIG. 9

35. To obviate the difficulties just mentioned, the blocks shown in Fig. 10 may be used. In this the terra-cotta protection is thicker than in the previous example, and the presence of the air spaces *a* tends to render the casing a better non-conductor of heat than one of solid form. Instead of wire binders being provided to unite the blocks, as shown at *b* in Fig. 8 (*b*), the connection of the blocks of the casing is obtained by the curved form of the joints, which is a much better arrangement than that of relying upon metallic ties.

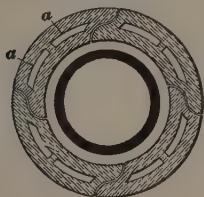


FIG. 10

36. A common system of fireproofing structural-steel columns is shown in Fig. 11. The terra-cotta blocks shown are usually made about 3 inches thick and are provided with air spaces. They are generally backed with rough brickwork filling or concrete. Without such a backing the terra-cotta blocks are hardly sufficiently well connected together to remain in place during a serious conflagration.

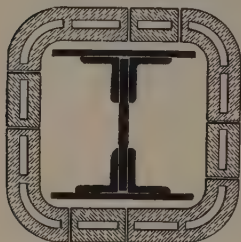


FIG. 11

37. Probably the best system of terra-cotta protection for stanchions is shown in Fig. 12. Here, the irregular shape of the section is filled out by special hollow blocks *a*, *a* to a diameter usually equal to about $1\frac{1}{2}$ inches more than the greatest diagonal of the stanchion. Outside this, a 3- or 4-inch covering of hollow blocks is provided with intervening air spaces *b*. The joints in these blocks are usually arranged so as to break joint with those *c* in the filling tile. The outside blocks are kept the proper distance from the filling blocks by headers of thicker tile or by projecting ribs *d* left on the inner surfaces of the exterior blocks.

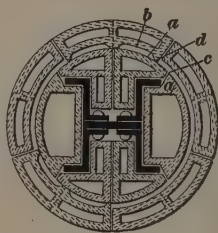


FIG. 12

PROTECTION BY BRICKWORK

38. **Protection of Stanchions by Brickwork.**—At present, when, as is customary in the erection of business premises in the larger towns, a form of steel-frame construction is adopted, the wall stanchions are nearly always protected by brickwork. The invariableness of this custom is due to the fact that brickwork has of necessity to be provided in order to comply with the schedule of wall thicknesses, and as the brickwork in such cases is usually of considerable thickness, a very sound protection is afforded if the workmanship is good.

COMPARISON OF THE METHODS OF PROTECTION

39. If the workmanship is of a thoroughly sound character there is probably little to choose between terra-cotta and concrete when used for the purpose of column and stanchion protection. If the workmanship is bad, for instance, if the jointing of the terra-cotta blocks is not well executed or if the concrete is not well mixed, the use of both materials will constitute an unsatisfactory protection. Probably the best form of protection to stanchions is good brickwork not less than 9 inches thick all round, put together in a substantial manner and well grouted in with cement. This arrangement, however, naturally takes up a considerable amount of floor space, and is rarely adopted except for wall stanchions.

IMPERFORATE FLOORS NEXT STANCHIONS

40. Pipes should never be carried inside the fire-resisting casing and immediately adjacent to the columns or stanchions, because the condensation on the pipes is likely to cause corrosion, and also because it is impossible to inspect or repair the pipes without practically rebuilding the casing. It has been a common practice in the United States to run all piping and electric conduits adjacent to the columns. Such practice cannot be too severely condemned.

The construction shown in Fig. 13 is the method which should be used where it is absolutely necessary to run the pipes and conduits adjoining the stanchions. The stanchion is protected independently of the pipes, which can be replaced or removed without injury to the stanchion casing; in addition, the possibility of moisture penetrating to the column is remote.

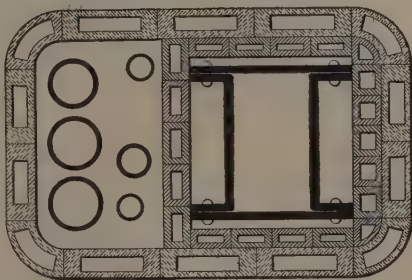


FIG. 13

CONSTRUCTION OF FLOORS AND CEILINGS

EARLY METHODS

41. Formerly, fire-resisting floors were made of concrete supported on brick arches sprung between the lower flanges of rolled-steel joists, although in some cases corrugated-iron arches were substituted for the brick. These forms of construction are, however, undesirable, as the arches and the concrete are extremely heavy, the ceilings are not level, and the bottom flanges of the beams are unprotected. Consequently, a better form of construction has been sought, and the brick- and iron-arch methods have been largely replaced by lighter forms of construction in steel and concrete, or steel and terra-cotta, with the steelwork properly protected from the effects of fire.

STEEL AND CONCRETE FLOORS

42. **Rolled-Steel Joists and Concrete Floors.**—By far the most common type of fire-resisting floor in British practice, and a type extensively used in London, is one formed of small rolled-steel joists, spaced from 3 to 4 feet between centres, with a concrete filling carried by the flanges of and entirely surrounding the joists. A simple floor of this type with 5" $\times$ 3" steel joists may be used across spans up to about 14 feet for office and other similar buildings where the load to be carried is light. If the span is considerable or if a heavy load has to be carried, it will be necessary to use larger joists as main girders, after the manner of binders in an ordinary wooden floor.

A steel joist and concrete floor is often constructed with merely a slight thickness of cement rendering on the under side of the joists, and in such a case it cannot in any way be entitled to the term fire-resisting. The protection is quite insufficient, and, if a floor of this nature is subjected to fire, the joists will expand and twist, and generally cause the failure of the whole construction. A total 2-inch thickness of concrete and plastering below the bottom of the joists, as shown at *a, a* in Fig. 14, is most

desirable, although even in ordinary good-class work a thickness of only 1 inch is not unusual. The flooring is shown in the figure



FIG. 14

to consist of $1\frac{1}{4}$ -inch boarding *b* laid on $2'' \times 2''$ deal fillets *c* spaced 12 inches apart and embedded in the concrete of the floor.

EXPANDED-METAL AND CONCRETE FLOORS

43. Two Classes of Expanded-Metal and Concrete Floors.—The methods of making floors with concrete and expanded metal are innumerable, and it would be impossible to describe the various designs adopted. They can, however, be divided into two general classes, namely, those with ceilings formed by the under side of the floor of the story above and those with *hanging ceilings*.

44. Floors Without Hanging Ceilings.—In Fig. 15 is shown a floor that has no hanging ceiling under it, but the ceiling is

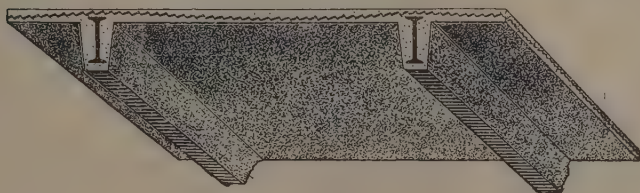


FIG. 15

formed by the under surface of the floor covered with a coat of plaster. This type of floor is suitable for a factory or any building in which the appearance of the ceiling is of no consequence. As can be seen, this floor is extremely simple in construction, and there are no places where dust can accumulate or where vermin can lodge.

45. **Floors With Hanging Ceilings.**—A method of constructing floors with ceilings is illustrated in Fig. 16 (a). As shown in detail in (b), the ceiling is supported by light channels, which are held up by means of wire clips fastened round the lower flanges of the steel joists. The expanded metal is then wired to the channels. As will be noticed, the expanded metal in the

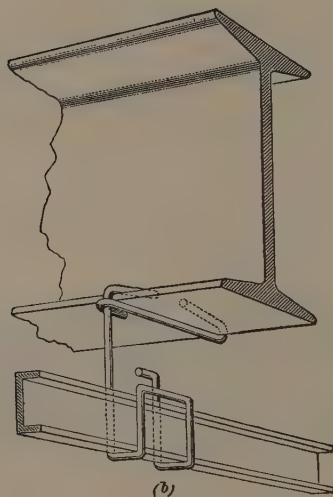
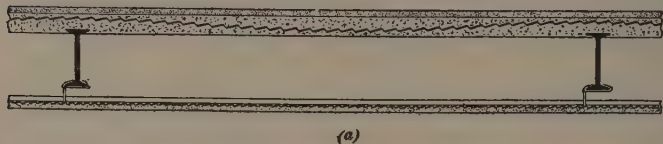


FIG. 16

floor proper is near the bottom of the concrete slab in the centre of the span, and near the top of the slab over the supports, in order to reinforce the concrete in tension, the slab being considered as *continuous over supports*. While a floor with a cement finish is shown, floors of this kind can be made as easily

with wood, tile, or any other kind of top surface. In this form of construction, the floor is made first, and while the concrete is setting the floor is supported by a temporary timber platform. When the concrete is sufficiently hard, the woodwork is removed, and the ceiling is then suspended and the plaster, either of cement or lime, is put on in the ordinary way. Between the floor and the ceiling is an air space through which may be run water or gas pipes, electric wiring, or ventilating and heating ducts.

46. Fig. 17 illustrates a method of supporting an expanded-metal and concrete floor on a wall. As will be noticed, the floor is made just the thickness of two bricks and mortar joints, and sets into the wall, and an allowance for possible expansion has been made as shown at *a*. Instead of fitting the concrete slab into the wall, however, the brickwork may be corbelled out so as to provide a bearing.

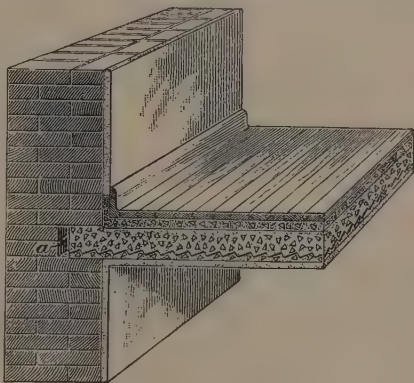


FIG. 17

47. Reinforced-Concrete Floors.

Floors of expanded metal and concrete, in which the expanded metal is inserted, as is customary, on the tension side of the concrete, are entitled to the term *reinforced concrete*, for the steel reinforces the concrete at its weakest point. But the main beams of a floor cannot be economically reinforced with expanded metal. Steel in this form is not of sufficient strength for use in main girders. In reinforced-concrete construction in the wider sense of the term, the reinforcement takes the form of steel rods. This form of construction, which if well executed is considered highly fire-resisting, is dealt with at length in *Concrete Construction*.

48. **The Columbian Floor.**—The Columbian floor has been largely used in London and the large British towns during the past few years. It consists of concrete reinforced and supported in position by special rolled-steel bars, which usually have the sectional form shown in Fig. 18. These bars are either cleated to the steel beams, as shown at *b*, Fig. 19 (*a*), or more usually they are supported by special clips, or stirrups, *c* hung over the top flange of the steel beam, as in (*b*). The stirrup is shown in detail in Fig. 20. Under the steel bars is placed a wooden centre on which the concrete filling is tamped. When the concrete has set, it forms a slab strengthened by the light steel bars which are themselves thoroughly protected by the concrete. Any character of



FIG. 18

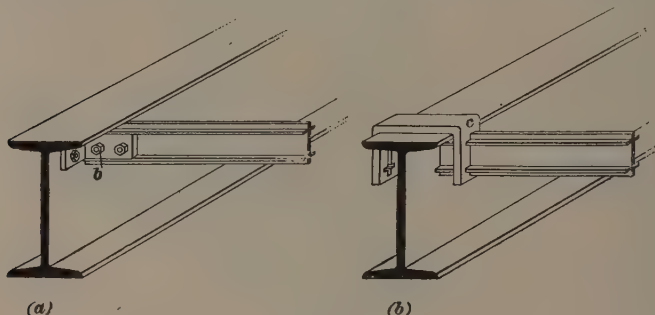


FIG. 19

finished floor can be used. The panelled-ceiling treatment is shown in Fig. 21, but where a flat ceiling is required, the construction shown in Fig. 22 is employed.

49. In the construction shown in Fig. 22, which is known as **double construction**, the floor system is practically the same as in Fig. 21, except that the concrete round the steel joist is omitted, as the suspended ceiling furnishes the desired protection. The ceiling is constructed in the same manner as the floor, that is, special rolled bars are supported on the lower flanges of

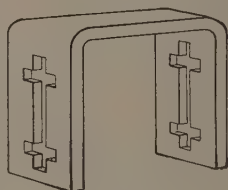


FIG. 20

the steel joists and are embedded in a thin layer of concrete, which they support. The finished ceiling is formed by plastering the bottom surface of the concrete.

In the construction shown in Fig. 21, the steel joists are protected by a specially moulded concrete slab *b*, so arranged as to provide an air space at the lower flange of the joist, thus furnishing additional fire protection, and having embedded in

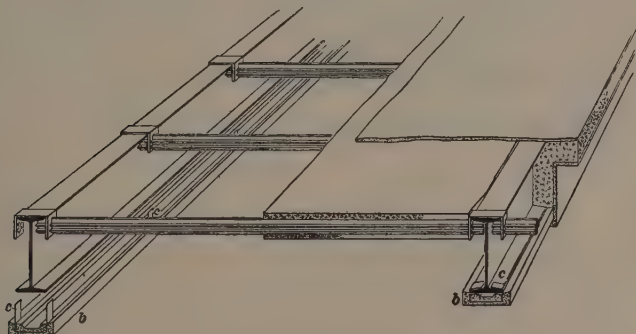


FIG. 21

it iron anchors *c* to fasten it in place. This special slab is shown detached below the left-hand joist in Fig. 21, and in place at the right-hand beam. The embedded anchors at *c* are turned over the lower flange of the joist and hold the slab in position.

50. Centring for Columbian Floors.—All types of Columbian floors require a temporary wooden centring* to support the concrete until it has set. This centring is usually built as shown in Fig. 22, which shows the method employed in the construction of the floor system and the flat ceiling beneath. The centring for the ceiling, which is put up first, is supported on the stringers *a* hung from the lower flange of the steel joist by the wrought-iron clamp and hanger *b*. When the steel bars forming the main support of the ceiling are in position, concrete *c* is dumped on this centring and tamped in place. After the concrete forming the ceiling has obtained its initial set, the sleepers *d*, which carry the wooden supports for the centring on which the floor is

* Often spelt *centering*.

constructed, are put in place. The centring for the construction of the floor is removed through openings provided at intervals in the concrete ceiling, the openings being closed afterwards with a concrete slab. In Fig. 22, *e* shows one of these openings,

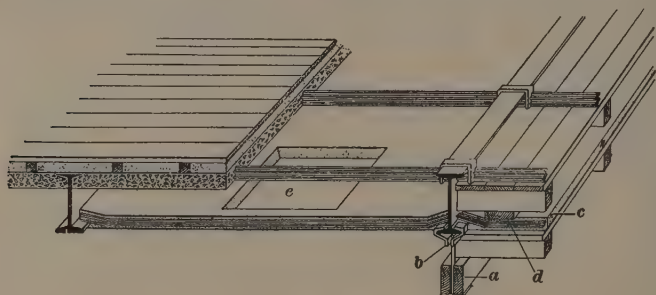


FIG. 22

which is rectangular and is provided with bevelled sides, so that the concrete slab can be introduced through it and, on being lowered into place, make a tight and secure ceiling.

51. Spacing of Joists.—The most economical arrangement for the Columbian floor system is to place the steel joists from 7 to 8 feet apart for the panel construction shown in Fig. 21, and from 6 to 7 feet for the double construction shown in Fig. 22. If the spans are increased beyond these limits, there is little saving by using this system of construction, as ordinary rolled-steel joists can be economically substituted for the special sections. The concrete used is composed of Portland cement, sharp sand, and furnace slag or broken stone, although for light floors coke-breeze concrete is commonly used.

52. Centring Generally for Steel and Concrete Floors.—In the case of all types of steel and concrete floors, one of two methods may be adopted. Either the centring may be hung from the joists after the manner illustrated in Fig. 22, and subsequently in Fig. 31, as regards steel and terra-cotta floors, or it may be propped up from the floor beneath. Where the spacing of the floor joists is comparatively close, the former method will be most convenient.

TABLE I
SAFE LOADS ON FLOORS OF EXPANDED METAL AND CONCRETE

All Sizes Have Mesh 3 Inches Across; A = Size of Metal Strand in Inches; B = Depth of Concrete in Inches

Span Centre to Centre Feet		Safe Load, Hundredweights Per Square Foot											
		1		1½		2		2½		3		4	
		I		A		B		A		B		A	
4		$\frac{1}{8} \times \frac{1}{8}$	3	$\frac{3}{16} \times \frac{1}{8}$	3	$\frac{1}{4} \times \frac{1}{8}$	3½	$\frac{1}{4} \times \frac{1}{8}$	3½	$\frac{3}{16} \times \frac{1}{8}$	3	$\frac{1}{4} \times \frac{1}{8}$	3½
5		$\frac{3}{16} \times \frac{1}{8}$	3	$\frac{1}{4} \times \frac{1}{8}$	3½	$\frac{1}{4} \times \frac{1}{8}$	3½	$\frac{1}{4} \times \frac{1}{8}$	4	$\frac{1}{4} \times \frac{1}{8}$	4	$\frac{5}{16} \times \frac{1}{8}$	4½
6		$\frac{3}{16} \times \frac{1}{8}$	3	$\frac{1}{4} \times \frac{1}{8}$	4	$\frac{1}{4} \times \frac{1}{8}$	4	$\frac{5}{16} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	4½	$\frac{5}{16} \times \frac{1}{8}$	5
7		$\frac{1}{4} \times \frac{1}{8}$	3	$\frac{1}{4} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	5½	$\frac{1}{4} \times \frac{1}{8}$	5	$\frac{1}{4} \times \frac{1}{8}$	6
8		$\frac{1}{4} \times \frac{1}{8}$	3½	$\frac{5}{16} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	5	$\frac{1}{4} \times \frac{1}{8}$	6	$\frac{1}{4} \times \frac{1}{8}$	6	$\frac{5}{16} \times \frac{1}{8}$	7
9		$\frac{5}{16} \times \frac{1}{8}$	4	$\frac{5}{16} \times \frac{1}{8}$	5	$\frac{1}{4} \times \frac{1}{8}$	6½	$\frac{1}{4} \times \frac{1}{8}$	6½	$\frac{5}{16} \times \frac{1}{8}$	7	$\frac{5}{16} \times \frac{1}{8}$	7½
10		$\frac{5}{16} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	5½	$\frac{1}{4} \times \frac{1}{8}$	7	$\frac{1}{4} \times \frac{1}{8}$	7	$\frac{1}{4} \times \frac{1}{8}$	7½		
11		$\frac{5}{16} \times \frac{1}{8}$	4½	$\frac{1}{4} \times \frac{1}{8}$	6	$\frac{1}{4} \times \frac{1}{8}$	7½	$\frac{1}{4} \times \frac{1}{8}$	7½	$\frac{1}{4} \times \frac{1}{8}$	8		
12		$\frac{5}{16} \times \frac{1}{8}$	5	$\frac{1}{4} \times \frac{1}{8}$	6½	$\frac{1}{4} \times \frac{1}{8}$	8	$\frac{1}{4} \times \frac{1}{8}$	8	$\frac{1}{4} \times \frac{1}{8}$	8½		
13		$\frac{1}{4} \times \frac{1}{8}$	5½	$\frac{1}{4} \times \frac{1}{8}$	7	$\frac{1}{4} \times \frac{1}{8}$	9	$\frac{1}{4} \times \frac{1}{8}$	9	$\frac{1}{4} \times \frac{1}{8}$	9½		
14		$\frac{1}{4} \times \frac{1}{8}$	6	$\frac{1}{4} \times \frac{1}{8}$	7½	$\frac{1}{4} \times \frac{1}{8}$	10	$\frac{1}{4} \times \frac{1}{8}$	10	$\frac{1}{4} \times \frac{1}{8}$	10½		
15		$\frac{1}{4} \times \frac{1}{8}$	6½	$\frac{1}{4} \times \frac{1}{8}$	8	$\frac{1}{4} \times \frac{1}{8}$	11	$\frac{1}{4} \times \frac{1}{8}$	11	$\frac{1}{4} \times \frac{1}{8}$	11½		

53. Strength of Steel and Concrete Floors.—The safe loads on floors of expanded metal and concrete are given in Table I, and the safe loads on floors of the Columbian system in Table II. As in each case the results are based on tests conducted by the manufacturers on floors whose materials and construction were presumably of very good quality, it will be well to allow some little further margin, in addition to that afforded by the factor of safety adopted in the compilation of the tables.

54. The strength of floors of rolled-steel joists and concrete is usually ascertained by means of the ordinary beam formulæ. The slight additional strength afforded by the concrete protection of the main joists, if such occurs, is not taken into consideration in the calculations. In the case, however, of the concrete which, with the embedded small joists, forms the floor itself, the circumstances are different. The concrete here undoubtedly constitutes a very important factor in the strength of the construction, and enables steel joists to be safely used with a ratio of depth to span which in ordinary cases would result in a deflection much beyond allowable limits. But calculations in which correct allowance is made for the respective strength afforded by the concrete and by the steel are very complicated. In practice, therefore, it is customary to allow a safe stress of 9 to 10 tons per square inch in calculating the steel joists, and to assume that this takes into account the full strength afforded by the concrete, which consequently is not further considered in the calculations.

CONCRETE FLOORS WITHOUT CENTRING

55. Special Types of Concrete Floors.—Since the introduction of reinforced concrete for floor construction, several special types of floors have been put on the market, which can be erected over wide spans without centring. The basis of the construction of such floors consists of beams of reinforced concrete, of special shape, usually cast in moulds and hoisted into position. Sometimes the whole floor is composed of these specially formed reinforced-concrete beams. Floors of this class offer a good resistance to fire, and appear to have an important future.

TABLE II
SAFE LOADS ON COLUMBIAN FLOORS, IN POUNDS PER SQUARE FOOT

Short-Span System										Long-Span System									
Span Centre to Centre Feet	1-Inch Bar			2-Inch Bar			2½-Inch Bar			Span Centre to Centre Feet	3½-Inch Bar			4½-Inch Bar			5-Inch Bar		
	Depth of Concrete Inches			Depth of Concrete Inches			Depth of Concrete Inches				Depth of Concrete Inches			Depth of Concrete Inches			Depth of Concrete Inches		
	3	3½	4	3½	4	4½	4	4½	5		5	5½	6	5½	6½	6½	7	7½	8
4	390	425	500	425	480	530	600	650	7	525									
5	250	335	350	300	335	265	425	470	8	450	500	525	600	650					
6	175	225	235	200	215	235	300	330	9	380	450	475	520	550	575				
7	110	145	160	150	155	165	200	220	10	325	400	420	450	475	500	550			
8		100	125	110	115	125	150	165	11	290	350	370	400	420	430	475			
9			75	75	95	105	120	135	12	250	300	315	350	365	375	415	500		
10					70	75	100	115	13	220	250	265	300	320	330	360	450		
11								90	14	190	200	215	250	270	290	320	400	450	
12								100	15	160	165	175	200	225	240	265	350	400	
									16			120	150	175	200	220	300	350	
									17				120	135	175	195	250	300	
									18					100	125	135	200	250	
									19						100	110	150	200	
									20						75	90	100	130	

56. **The United Kingdom Floor.**—The United Kingdom floor is constructed by the United Kingdom Fireproofing Company and has been largely used. There are several forms used by the Company, one of which is illustrated in Fig. 23. The

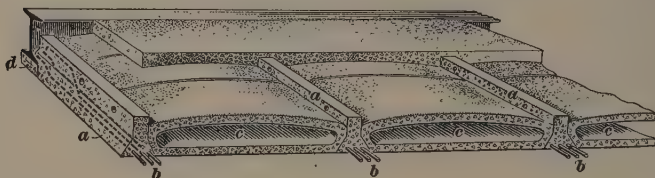


FIG. 23

reinforced-concrete beams, spaced about 3 feet apart, are shown at *a*, the steel reinforcement of such beams, which is shown projecting, for the sake of clearness, at *b*, and the hollow concrete tubes, extending between the beams, at *c*. The method of protecting the under side of the steel joists by means of a specially cast block with clips at intervals extending on to the bottom flange of the joist is shown at *d*. The use of steel joists in connection with a floor of this description is often not necessary, as the beams may be constructed to extend across spans of as much as 30 feet. The beams may either be cast and hoisted into position, or they may be cast in place.

57. **The Siegwart Floor.**—A floor which differs from the one mentioned above in several points of principle is the Siegwart

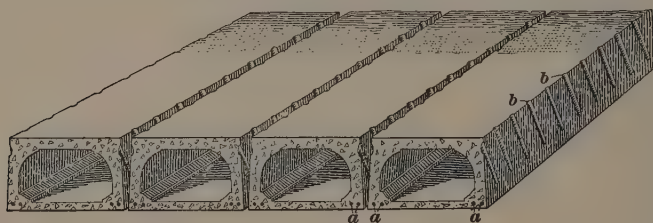


FIG. 24

floor. The construction of this is shown in Fig. 24. The floor consists of hollow reinforced-concrete beams, 10 inches wide, placed side by side, and the spaces between the beams grouted

in cement. The beams are supplied ready for use by the manufacturers. The reinforcement of the beams is shown at *a, a*, and the small cavities left at the sides to receive the cement grout are shown at *b, b*. The beams are made in spans up to 20 feet, and vary in depth from 4 inches to 8½ inches. Where joists have to be used it is customary, in order to secure continuity, to provide steel stirrups extending over the top of the joists and running into the joints between the beams, as shown at *a, a* in Fig. 25. These stirrups occur at every second or third joint.

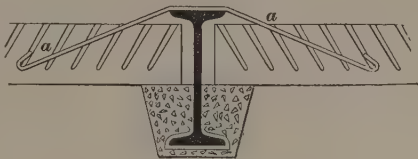


FIG. 25

TERRA-COTTA FLOORS

58. In the British Isles, with the exception of a few forms of lintel construction, terra-cotta has not been much adopted for the construction of floors. The subject will be treated here at some length, however, because the successful use of this material in Canada and the United States warrants the belief that terra-cotta in time will come to be of more importance in England as a material for floor construction than it is at present.

59. **Flat Arches.**—At present three methods of constructing flat terra-cotta floors are in use in Canada and the United States. The first is known as the *side method*, because in each transverse row the sides of the blocks abut against those of the adjacent ones; the second is called the *end method*, the blocks being set end to end, at right angles to the beams; the third is a combination of the first and second, the length of the skewbacks being parallel with the beams, while the other blocks are set at right angles thereto.

60. **Side-Method Arches.**—The first terra-cotta arch was built about 1872 on the lines of what is known as the *side method*, and was somewhat similar in appearance to the one shown in

Fig. 26. Although an improvement over the brick arch, this arch left much to be desired. The bottom was smooth and would not hold the plaster firmly; the bottom flanges of the



FIG. 26

joists were not protected except by the plaster, which in case of fire was inadequate; and the entire thrust of the arch was taken up by the top and bottom web of each block.

61. The latest side-method arch is constructed as shown in Fig. 27. The toothed faces of the blocks shown at *e* serve to give them a better grip on the cement placed between them, on

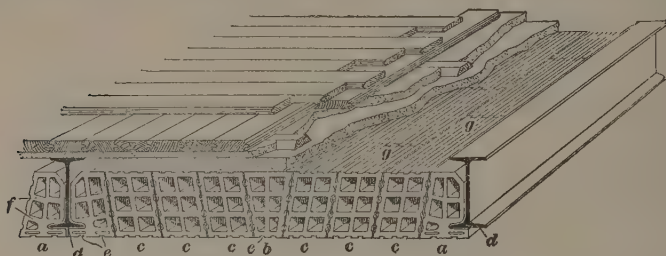


FIG. 27

the concrete above, and on the plaster below. The blocks marked *a, a* are called *skewbacks*; those marked *c, c*, *voussoirs*; and *b* is the key. The skewbacks are made to protect the steel



FIG. 28

joist, having a lip, as shown at *d, d*. Sometimes, instead of using a lipped skewback, the blocks are bevelled and a *soffit block*, as shown at *a*, Fig. 28, is employed. It will be noticed in Fig. 27 that the blocks are strengthened by two horizontal webs, as shown at *f*. These blocks are laid with their joints *g* *staggered*, so as to give additional strength to the arch.

When using a terra-cotta arch, it is necessary to fill the space above the blocks to the top of the joists with some incombustible

material, concrete being usually employed for the purpose. The fillets to which the floor is nailed are generally made so as to run at right angles to the joists, as shown in the illustration. These nailing fillets are bevelled and embedded in a layer of concrete equal in depth to their thickness, less $\frac{1}{4}$ -inch clearance, and are thus held securely in place. This type of floor has been adopted to some extent in British practice.

62. There is one objection to the side method of arch construction, however, and that is that the vertical surfaces do not take up any of the stress; consequently, there is always a certain amount of dead weight to be carried. To overcome this objectionable feature and devise an arch in which all the material would assist in resisting the stress, the *end-method arch* was invented.

63. End-Method Arches.—Arches formed by the end method of construction are usually made of rectangular blocks that are divided by vertical and horizontal webs into four or more sections, and have bevelled end joints. It is not customary to make the blocks in one transverse row break joints with those in the next, as the expense of setting would thereby be increased. Fig. 29 illustrates a very common type of the end-method arch. At *a* is shown the skewback, having a protective flange *b*; at *c*, one

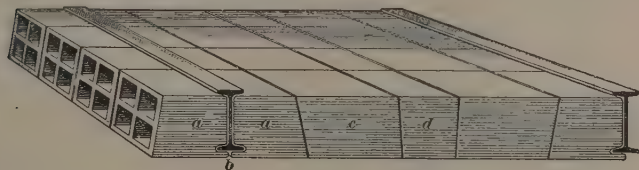


FIG. 29

of the inside blocks; and at *d*, the key block. This form of construction is economical and simple, as all the blocks in each longitudinal row of the arch are made alike.

64. Combination Arches.—The combination method, which is the improved system of flat-tile arch construction, is shown in Fig. 30. The air spaces in the hollow blocks run at right angles to the steel joists, except in the skewbacks, where they run

parallel with them. The skewbacks *a, a* are practically the same as those employed in the side-method construction, but the voussoirs *c, c* and key block *b* are considerably different, as they take a bearing on the end section and not on the side, and are of such a form that hollow spaces are provided between the block in a direction at right angles to the joist, as shown at *d*. These spaces, besides diminishing the weight of the system considerably,

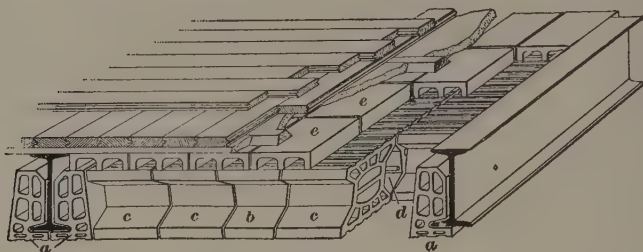


FIG. 30

provide a means of passing the tie-rods through the arch from beam to beam. At *e, e* are shown pieces of terra-cotta which take the place of a concrete filling and make the entire construction lighter. The nailing fillets are placed directly on top of these slabs, and are held in place as usual by concrete.

The combination system is considered superior to the side or end method of construction, because, for a given weight, it is stronger, and is consequently a lighter system of flooring. The openings between the blocks readily permit the insertion of the tie-rods and allow any variation in their spacing; thus, the cutting of the blocks is avoided and any risk of breakage likely to occur with the side method of construction is eliminated.

65. Tie-rods are necessary in all floor arches to prevent the joists from spreading. They are placed from 5 to 7 feet apart, and are generally $\frac{3}{4}$ -inch rods, threaded at the ends and provided with nuts to take up the thrust of the arch. These rods should be placed in the web of the joist, as near as possible to the lower flange.

66. Centring for Flat Arches.—To support the blocks during laying, a firm centring is needed. A good form, shown in Fig. 31,

is made of 1-inch boards *a*, dressed and set close together and resting on a $4'' \times 4''$ or $6'' \times 4''$ piece *b* which extends parallel with the joists and midway between them. The piece *b* is suspended by T-headed bolts *c* from similar pieces *d*, which are laid across the top of the joists. The blocks forming the protection of the joists are put in place first, and if they are separate

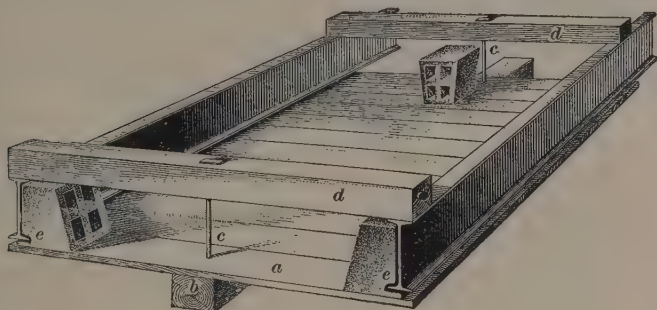


FIG. 31

pieces, are laid on the planks directly beneath the beams, or, if the joists are protected by the skewbacks, as at *e*, the latter are set. The bolts are then tightened so as to bend the planks slightly, forming the camber, which should be about $\frac{1}{4}$ inch for a 6-foot span.

In setting the blocks, the pieces should be adjusted so that the surfaces will abut properly. Only the best cement mortar should be used, and the joints should be made thin, but care should be taken that the mortar does not become pressed out. The centring should be kept in place until the mortar between the blocks has set, which takes from 12 to 36 hours. When the centring is removed, the arch should have a level surface, showing no open joints or projecting blocks.

67. Size of Blocks for Arches.—Terra-cotta blocks are usually made the same depth as the steel joists, and as they extend about $1\frac{1}{2}$ inches below the lower flange of the joists, the upper surface of the flat arch is about $1\frac{1}{2}$ inches below the top flange. The camber in flat arches is very slight and never over $\frac{1}{8}$ inch per foot of span.

68. **Weight and Strength of Arches.**—The weight of arches varies with the depth and span. The depth ranges from 6 to 12 inches in arches constructed by the side method, and from 6 to 15 inches in those built by the end method. For offices and shops, a common depth is 10 inches, the floor joists being spaced from 5 to 7 feet apart. Table III gives, approximately,

TABLE III
APPROXIMATE SAFE LOAD ON FLAT TERRA-COTTA
ARCHES, IN POUNDS PER SQUARE FOOT

Depth of Arch Inches	Distance Between Beams				
	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet
6	150	100			
7	200	150			
8	275	175	125		
9	300	200	140		
10	325	225	150	100	
12	400	250	200	125	100

the safe allowable load for arches of flat terra-cotta block construction. In the table, the approximate safe loads of flat arches, as ordinarily constructed, have been deduced from recent experiments. Since the arches are often carelessly set, the margin of safety is greater than would otherwise be necessary.

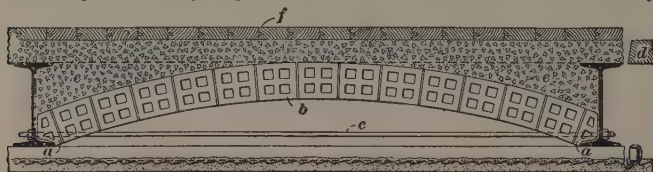


FIG. 32

69. **Segmental Terra-Cotta Arches.**—Segmental terra-cotta arches of the forms shown in Fig. 32 are frequently used in warehouses in the United States, the plaster being applied to expanded

metal secured to the joists. In Fig. 32, *a* are the skewbacks, *b* the hollow inside blocks, and *c* the tie-rod taking up the thrust of the arch. At *e* is shown the concrete filling, at *d* the nailing strip, and at *f* the flooring. The bottom flanges of the joists are protected either by plaster laid on expanded metal or by properly shaped skewbacks. This kind of arch makes a very strong floor, and is much cheaper than a flat arch. Segmental arches have been built in spans up to 20 feet, thus saving considerable weight in the columns and beams.

SPECIAL FORMS OF TERRA-COTTA FLOORS

70. Although the arched type of terra-cotta floor has not been much adopted in the British Isles, there are a few special systems of terra-cotta lintel construction, such as the *Fawcett floor* and the *Homan and Rodgers floor*, which have been used to a very considerable extent. Among the modern types of floors, the *Kleine floor*, of hollow bricks strengthened by iron tension bands, appears to be meeting with considerable favour.

71. The Fawcett Floor.—The Fawcett floor bears a similarity to all floors using lintels as a means of carrying the construction between the steel joists. This floor, however, is distinguishable from other systems in that it endeavours to provide a circulation of air round the steel joists for the purpose of protecting them from the heat of a conflagration.

The Fawcett system, as shown in Fig. 33, consists of hollow terra-cotta lintels supported on the lower flange of the steel joists. These lintels *a*, which have the cross-section shown in (*a*), are placed in such a manner that their shortest diagonal *b b* in (*b*) is at right angles to the web of the steel joists. Since the lintels are laid in this manner, a special form is needed at the end of each bay. This lintel, shown in section in (*a*), consists of the ordinary form cut on the diagonal in the moulding. It is provided with a lip *c* that rests on the adjacent lintel, and the lower part has dovetailed grooves so as to give a key to the plaster

72. The lintels are composed of dense terra-cotta moulded to form, and are burnt to a considerable degree of hardness, so as to

obtain the required transverse strength. After they are laid in position between the steel beams, concrete is tamped on top of them, as shown at *d*, Fig. 33 (*a*). The concrete takes a bearing on the lower flanges of the steel joists. One of the purposes of the tubular form of the lintel is to provide a circulation of cold

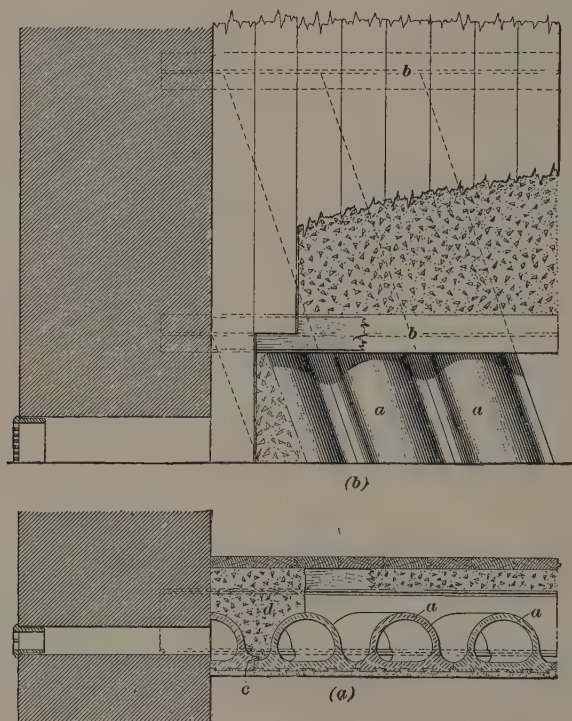


FIG. 33

air round the steel joists, which is done by introducing in the wall a hollow block *b*, Fig. 34 (*a*), (*b*), and (*c*). This block is simply an iron box, provided with a grating to prevent clogging with dust or rubbish. The cold air from the outside circulates through this opening and through the tubular lintel, as shown in (*a*) and (*b*), passing through the various lintels and along the entire length

of the lower flange of the steel beam through the openings *g* in the ends of the lintels.

Where the ends of the lintels abut on a wall, a steel channel *f* must be used as a support, as shown in Fig. 34 (*a*). Care should be taken, in such a case, that the channels do not entirely close up the ventilation flues, the openings of which must be dropped somewhat, as shown at *h*. The use of a channel may be obviated by building into the brickwork a $6'' \times \frac{1}{4}''$ plate *i*, as shown in (*b*), which should project a sufficient distance to provide a bearing for the end lintels. At the end of the bay, where the walls run approximately

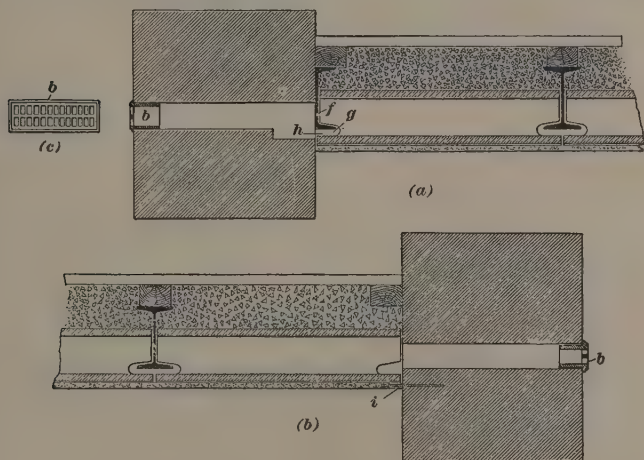


FIG. 34

parallel with the length of the lintel, no channel or plate is required, for the lintel is cut on the diagonal. The end rests on the lower flange of the steel beam and is further supported by the lip resting on the adjacent member.

73. Homan and Rodgers Terra-Cotta Floor.—The construction of the Homan and Rodgers floor is shown in Fig. 35. This floor does not differ much from the Fawcett floor, the chief differences consisting in the shape of the lintels and the fact that they are run at right angles to the steel joists. The lintels are of hollow

triangular section, as shown at *a*, and are keyed all round, in order to give a good hold both to the plaster at the bottom and the concrete *c* at the sides. The steel joists *b, b* carrying the lintels

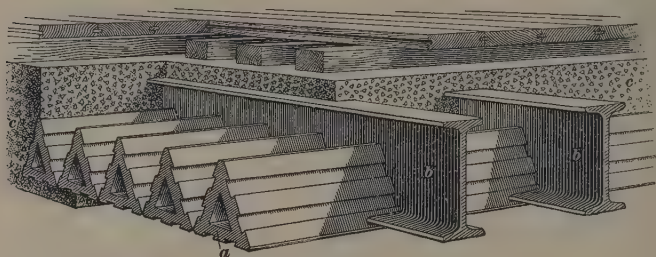


FIG. 35

are spaced 1 foot 6 inches apart, a closer spacing than that customary in the Fawcett floor, in which the spacing is usually 2 feet.

74. It is to be noted that both in this and in the preceding type of floor the joists are spaced very much closer than in an ordinary steel and concrete floor. This naturally increases the cost of the floor, although the fact that no centring is required has some counterbalancing effect.

75. **The Kleine Floor.**—The **Kleine hollow-brick floor**, in addition to the variation in the material used, differs in construction very much from the two previously mentioned systems in that, owing to the strength afforded by the iron tension bands, the supporting steel joists may be very widely spaced. Spans of from 10 to 14 feet are quite general, and where special means are adopted for securing the tension bands to the joists a distance of as much as 20 feet may be covered in one span.

The thickness of the hollow bricks *e*, Fig. 36, varies in accordance with the span and the load to be carried. In the case of ordinary floors for light loads, the total thickness of the floors, including the concrete covering to the bricks, is one twenty-second of the span. Details of the general construction of a Kleine floor, including the coke-breeze concrete covering, are given in Fig. 36. The iron tension bands situated in the cement mortar joints are shown in section at *a, a* and one of them in elevation at *b*. The

ballast-concrete haunch, on which the floor takes its bearing, is shown at *c*, and the cement-mortar protection to the steel joist at *d*. At *f* is shown the coke-breeze concrete filling, and at *g* the wooden fillet bedded in the concrete, to which are nailed the floor

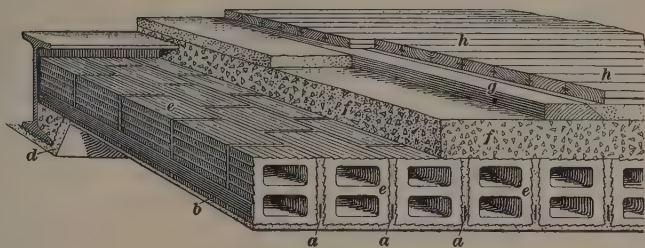


FIG. 36

boards *h*. This type of floor has been found to possess good fire-resisting qualities, but, from the point of view of strength and durability, it can hardly compare with the more conservative forms of floor in which the steel joists are spaced close together.

STEEL-PLATE FLOORS

76. There are several forms of floor construction in which bent, crimped, or corrugated steel plates constitute the supporting medium between the steel joists. The plates, however, will not stand great heat, and unless protected beneath by a suspended ceiling, they are not satisfactory. Further, the moisture in the warm air of the room underneath condenses on the steel plate to such an extent as to cause water of condensation to drip, which is not only liable to damage any goods in the room, but also corrodes the plates.

77. **Corrugated-Iron Arches.**—The corrugated-iron arch, which is the simplest of the steel-plate systems, is shown in Fig. 37. This arch consists of corrugated sheet iron of No. 18 S.W.G., bent or sprung between the lower flanges of the steel joists, as shown at *a*. The corrugated sheet bent in this way possesses great strength, and not only assists in supporting the floor load,

but acts as a centre on which the concrete *b* may be dumped and tamped. The finished floor may be laid on the concrete.

Tie-rods between the steel beams are not necessary where the arches are placed side by side and where the thrusts counteract

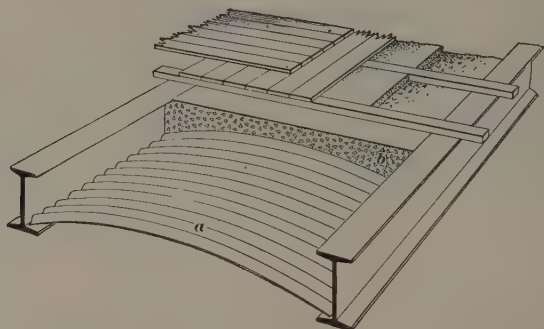


FIG. 37

each other. However, they should be used to hold the beams from any side deflection during construction. When tie-rods are used, they should lie in a corrugation, so as to enter the steel

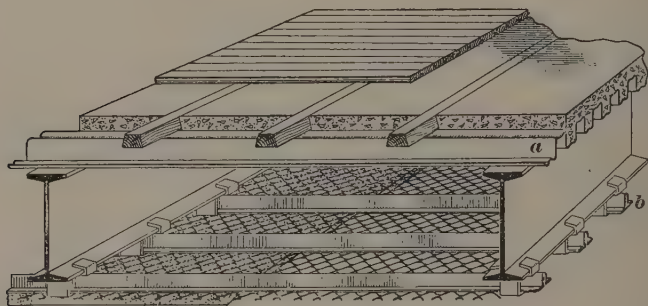


FIG. 38

joists as near the lower flange as possible. The weight of such a floor is from 25 to 40 pounds per square foot. Floors of this kind were at one time largely used, but unless a suspended ceiling is provided, or unless at least the bottom flanges of the

steel joists are in some manner protected, such a floor cannot be termed fire-resisting.

78. Multiplex Steel-Plate Floors.—The multiplex steel-plate floor construction, shown in Fig. 38, a form used to some little extent in the United States, consists of plates *a*, bent as shown, which rest on the steel floorbeams and provide a means of supporting the concrete and the floor load. The steel plates are usually galvanized to protect them from corrosion, which is otherwise likely to occur in this type of construction. A suspended ceiling is shown at *b*.

SELECTION OF A FIRE-RESISTING FLOOR

79. The question of choosing the best form of floor construction out of the many of nearly equal merit hinges largely on the cost, as any of those that have been described will be found reasonably satisfactory if properly designed and constructed. Other things being equal, the floor that is lightest is the best, as the weight and size of beams, girders, columns, etc., may then be considerably lessened, which is an important factor in the cost of the structure. Where strength is the object sought, the circumstances will warrant a very careful consideration of the several types, with a view of ascertaining the most truly economic form consistent with the soundness of construction required. In all floors, the character of the workmanship and the quality of the materials used have a great deal to do with the durability and efficiency, and hence should receive careful attention during construction. It is not only in the case of floors, but it is in all forms of fire-resisting construction most necessary to hold in view the extent to which any form of construction is likely to be damaged by fire. When the question of reconstruction after fire has to be considered, the great importance of this aspect of the subject becomes very strongly emphasized.

PUGGING OF WOODEN FLOORS

80. Fire-Resistance of Balk Timber.—It is quite usual in inspecting the remains of an ordinary non-fire-resisting building

after a fire to find that the large wooden floor girders have suffered very little from the flames. The surface has usually charred, and has then constituted a protection to the remainder of the beam. The fire-resisting capabilities of balk timber are fully recognized in Germany, where, in districts where timber is plentiful, whole floors are often constructed throughout of heavy beams. A floor of this kind, though heavy, cumbersome, and costly to construct, will be very satisfactory from the point of view of ordinary fire-resistance.

81. Pugging of Floors.—Floor construction of heavy timber is not customary in the British Isles, but the pugging of wooden floors with concrete so as to render them of a fire-resisting character is of very general occurrence, particularly in the case of existing buildings. If the pugging is carried out in a satisfactory manner, and the flames prevented from attacking the sides of the joists, a pugged floor constructed of timber throughout offers a very fair resistance to fire. A composite floor, with the main girders of steel, is less satisfactory, unless very exceptional precautions are taken in the protection of the steelwork. Concrete and expanded metal, or sometimes terra-cotta blocks, are used for this purpose.

82. Wooden floors pugged with concrete at least 5 inches thick are recognized as fire-resisting by the London building law if 1-inch square fillets spiked to the joists are provided in the centre of the concrete, as shown in



FIG. 39

Fig. 39, or if the concrete is in the form of blocks laid between the joists on fire-resisting bearers secured to the sides of the joists. The concrete pugging is shown at *a, a* and the 1-inch square wooden fillets are shown at *b, b*. A ceiling of expanded metal and plaster is shown at *c*, but compliance with the London building law may be obtained without the construction of a ceiling, the bottoms of the joists being left exposed.

One of the disadvantages of the method of pugging shown in Fig. 39 is that, the concrete having necessarily to be applied

in a wet state, there is the risk of dry rot being occasioned in the joists. If the pugging consists of concrete blocks, there is not this disadvantage; also, the pugging may be inserted from the under side of the joists, without the removal of the floor boards, but if the joists are irregularly spaced there are obvious difficulties in the adoption of this method, as blocks of standard size cannot be used.

FIRE-RESISTING PARTITIONS

CONCRETE PARTITIONS

83. Concrete Partitions Constructed in Position.—Coke-breeze concrete has already been alluded to in the construction of floors, and it is also sometimes used in a similar manner for the construction of partitions in position. But the double rows of *shuttering* required render this form of construction rather expensive unless there is a large amount of work to be done and the same shuttering can be used a number of times.

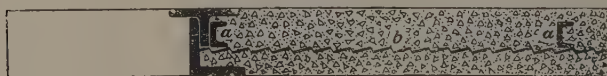
84. Concrete-Slab Partitions.—Partitions of concrete slabs are coming into rather common use. Both partitions of concrete built in place and those formed of concrete slabs are usually too thin to offer a very great resistance to fire.

EXPANDED-METAL-AND-PLASTER PARTITIONS

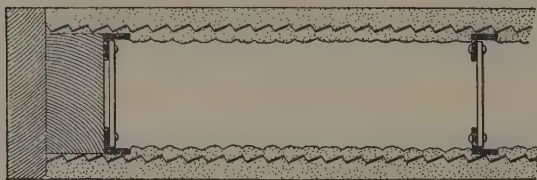
85. Partitions of expanded metal and plaster are frequently used, and if constructed in a sound and substantial manner and of a total thickness, in the case of solid partitions, of at least 2 inches, they offer a moderate resistance to fire. The construction of a solid partition, as arranged in connection with a steel door frame, is shown in Fig. 40 (*a*), while the construction of a hollow partition is shown at (*b*).

86. Solid Construction.—The solid partition is shown to be stiffened by small channel irons *a* which have the top and bottom ends bent at right angles, so that they can be nailed or spiked to the floor or ceiling, and the expanded metal *b* is

secured to the channels with galvanized wire. Both surfaces of the steel are plastered in the usual manner. Instead of the channels, vertical iron rods spaced about 12 inches apart are often used. This single construction is frequently adopted, but it does not offer so much resistance to fire as the double form of partitions shown at (b), although the existence of a cavity is to be objected to on hygienic grounds.



(a)



(b)

FIG. 40

87. **Hollow Construction.**—The hollow partition shown at (b), which has a finished thickness of 4 inches when plastered, is constructed of two rows of expanded metal plastered on the outside and connected together and stiffened by braced angle irons. Neither the hollow nor the thin solid form of expanded-metal-and-plaster partition possesses the fire-resisting qualities of a thick, well-constructed terra-cotta partition of hollow blocks.

TERRA-COTTA PARTITIONS

88. The usual terra-cotta partition is shown in Fig. 41. Pipe chases are readily provided by the use of recessed hollow blocks *a, a*. The other blocks are also hollow, but the spaces usually run horizontally. Sometimes, the blocks are set in lime mortar containing about 25 per cent. of cement, and are grooved on the face so as to provide a key for the plaster and

thus avoid the necessity of using any form of lathing. Some quick-setting plaster is generally used for finishing the faces of the blocks.

89. Bedding of Partitions.—In building partitions, the practice of setting terra-cotta or any other type of fire-resisting material on a wooden sill cannot be too strongly condemned. This method of construction has led to considerable fire losses from the burning away of the sill and the ultimate destruction of the partition. The best practice requires that all fire-resisting partitions rest directly on the concrete of the floor system.

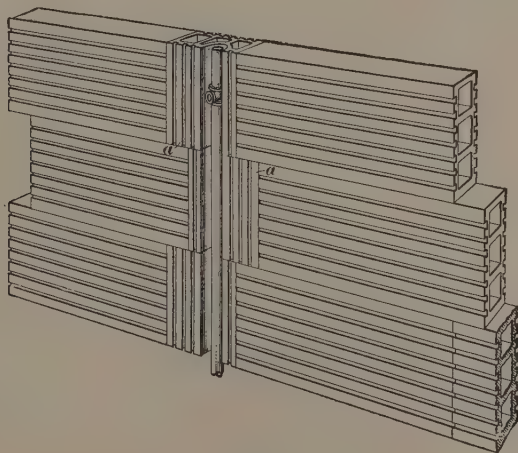


FIG. 41

90. Advantages of Terra-Cotta Partitions.—The hollow terra-cotta partition combines considerable strength with lightness and is also vermin-proof. The partitions, being hollow, do not readily transmit sound, and being made of a porous non-conducting material and provided with air spaces, they do not transmit heat, in consequence of which the enclosed rooms are warm in winter and cool in summer. The average weight of hollow terra-cotta block partitions, exclusive of plaster, is from 15 to 28 pounds per square foot.

DOOR FRAMES AND FINISHINGS

91. In first-class work, the door frames and finishings for all fire-resisting partitions should either be made of iron or be covered with a metallic casing, or, if the doors are of hardwood, the frames and finishings should be of a similar material.

LIGHT PARTITIONS

92. **Plaster-Slab Partitions.**—Partitions of plaster slabs of various kinds are used very largely throughout all classes of buildings. These, however, offer no very great resistance to fire, and for that reason are not dealt with here. Reference to partitions of this kind is made in *Plastering*.

CONSTRUCTION OF FIRE-RESISTING ROOFS

FLAT ROOFS

93. **Concrete Roofs.**—It is a very common practice in the erection of factory and similar buildings to form the flat roofs adopted in a similar manner to the floors, the only important difference being the relative lightness of the construction. The columns or stanchions supporting the floors are often carried up in order to form the supports to the roof, and the general arrangement of the main floor and roof joists in such a case will often be exactly the same. Where steel joists and concrete, or expanded metal and concrete, have been used in the construction of the floors, the roof, if of the flat variety, is almost invariably constructed of the same materials. The roof covering usually consists of $\frac{3}{4}$ inch of asphalt laid in two layers. Before commencing to lay the asphalt the concrete should be floated to a smooth face, for, if there are inequalities in the surface, an extravagant use of asphalt will be occasioned.

94. **Terra-Cotta Roofs.**—The above general remarks in relation to the construction of flat roofs of concrete apply equally to

flat terra-cotta roofs, although these are not at all customary in British practice. As there will be little weight coming upon the roof, the depth of the joists and of the arch blocks may be very materially reduced, and the thickness of the blocks need not exceed 3 inches to 4 inches. The covering, as in the case of concrete roofs, is usually of asphalt.

MANSARD AND OTHER PITCHED ROOFS

95. Fire-Resisting Pitched Roofs.—It is only of comparatively recent date that pitched roofs of a fire-resisting character have been erected in the British Isles to any appreciable extent. Ten years ago pitched roofs were practically invariably of the timber-framed, slate, or tile-covered variety. Now that particular attention has been directed to the great desirability of fire-resisting construction in the case of trade or business premises erected in the larger towns, the absurdity of allowing the roof to be the only non-fireproof portion of the building is being realized. Buildings constructed *throughout* in a fireproof manner are consequently becoming very general.

96. Pitched Roofs of Concrete.—All the forms of ordinary steel and concrete construction used in floors are suitable for roof work. For ordinary pitched roofs the lighter forms of construction, such as expanded metal and concrete or the Columbian system, are preferable. The latter, by reason of its combination of rigidity with comparative lightness of construction, is particularly suitable for roof work.

97. In the construction of mansard roofs, steel joists or channels spaced from 3 to 4 feet apart are often used. These should be soundly connected together, at the ridge and at the junction of the two roof surfaces, by means of angle plates *a, a*, as shown in Fig. 42, where the construction consists of a pair of channels, placed side by side, with room for angle plates between. It is also most desirable that the feet of the steel members of the lower slope should be connected to the members of the floor, as shown at *b, b*. The concrete in which the

steelwork skeleton is embedded stiffens the construction in a longitudinal direction. Slate or tile roofing is usually adopted, the covering being nailed direct to the concrete, or to battens similar to those used in the construction of wooden roofs.

98. Pitched Roofs of Terra-Cotta.—The use of terra-cotta for pitched roofs is not of very frequent occurrence in British practice, but in Canada and the United States it is very general. The general construction consists of steel joists spaced from 5 to 7 feet apart, carrying 3-inch hollow terra-cotta blocks similar to those used in partition work. If slate is to be used as

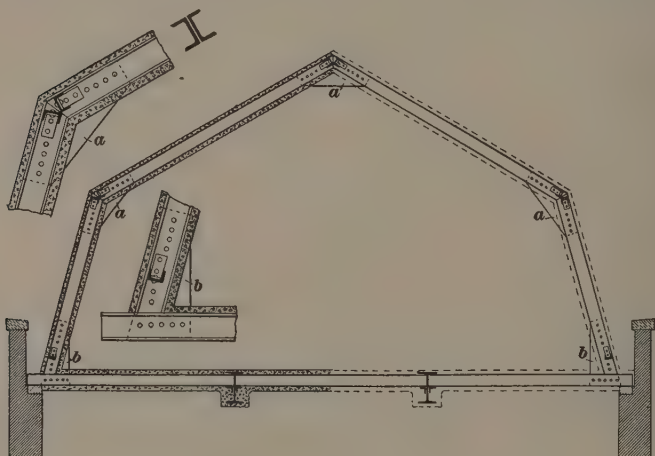


FIG. 42

a covering, wood battens are fastened to the tile and the surface is brought up flush with cement mortar, on which the slates are laid. The use of battens is not necessary if the blocks are of porous terra-cotta. For roofs having less than 45° pitch, slabs of porous terra-cotta, supported on $3'' \times 3''$ tee beams, have been found satisfactory. A roof of this kind weighs about 20 pounds per square foot. All parts of the steelwork should be well protected by casing them with porous terra-cotta, or with expanded metal covered with a thick coating of plaster.

99. **The Kleine Roof.**—A roof constructed of hollow bricks and iron tension bands in a similar manner to the previously described Kleine floor is shown in Fig. 43. One of the steel joists constituting the general framework of the construction, and in this instance also the trimming for the dormer, is shown at *a*.

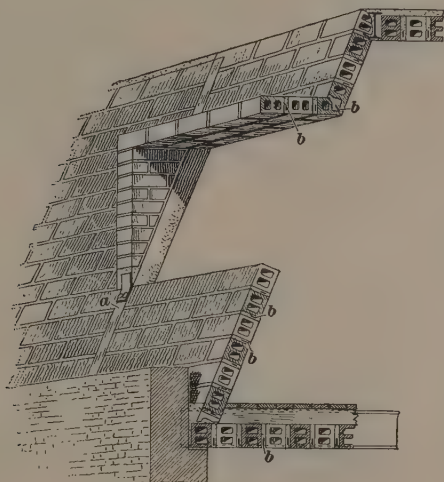


FIG. 43

The iron tension bands, which occur at every joint, both of the general roof and of the roof of the dormer, are shown at *b, b*. This form of construction constitutes a light and efficient fire-resisting roof.

CEILING CONSTRUCTION

100. In large buildings where steel roof trusses occur, it is not sufficient merely to form the general construction of the roof in a fire-resisting manner. Either the members of the truss must also be protected by some suitable material, such as terracotta, or, as is more customary in British practice, a ceiling must be suspended from the trusses. Such a ceiling may either follow the line of the ties of the truss or may be horizontal or

slightly coved. A portion of the construction of a horizontal

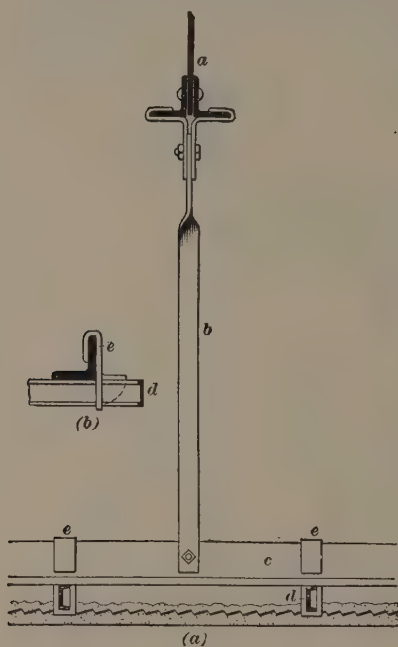


FIG. 44

ceiling is shown in Fig. 44. Such a ceiling is hung from the ties of the trusses at each joint, as at *a*, and is supported by metal straps *b*, which are secured at their lower extremities to angles *c* running at right angles to the roof trusses. Immediately underneath angles *c* are fastened small channels *d*, running perpendicular to the angles *c*, being held in place by $\frac{1}{2}$ -inch malleable-iron hooks *e*, a detail of which is shown in Fig. 44 (*b*). The expanded metal is fastened to these channels by annealed wire, and the cement plaster is applied in the usual manner.

PROTECTION OF OPENINGS

METHODS OF PROTECTION

101. **Party-Wall Openings.**—Openings in party walls, or in walls separating divisions of a building, are likely to be subjected to a much more severe fire attack than those in external walls, and thus they require to be very carefully protected, so that the passage of fire may be prevented. Double doors, of fire-resisting construction, placed as far apart as possible, should always be provided to party-wall and similar openings.

102. External Openings.—Openings in external walls are usually upon a different basis. It is only when there is the risk of the passage of fire from a closely adjoining building that protection is generally provided, and then the provision of single doors to the doorway openings is usually quite sufficient. Although iron doors or shutters or armoured doors are to be preferred, doors of 2-inch solid hardwood are sometimes used. In Canada and the United States, window openings are often protected by fire-resisting shutters, which are closed at night ; but in British practice, the use of fire-resisting glass in hardwood or iron frames is much more general.

FIRE-RESISTING DOORS

103. Iron Doors.—Doors of wrought iron have been very largely used in London for the protection of party-wall openings, although of late years armoured doors have been used to some considerable extent. An iron door to a party-wall opening is shown in Fig. 45. The thickness of the iron panels should not be less than $\frac{1}{4}$ inch, and the stiles and rails are 4 inches wide by $\frac{1}{4}$ inch thick. There is, of course, a similar door on the other side of the party wall. Hinged doors are to be preferred, and they should be hung in a wrought-iron frame, attached directly to the brickwork of the opening.

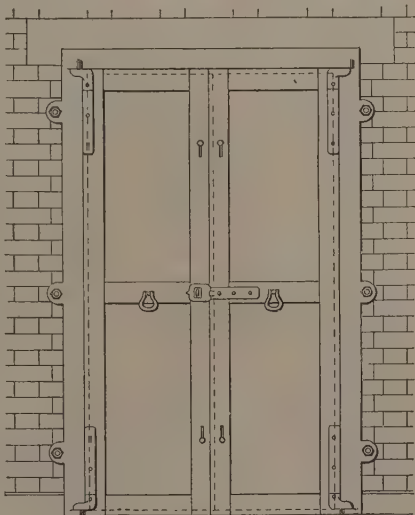


FIG. 45

104. Armoured Doors.—A door constructed of two or three thicknesses of deal, covered on either side with tinned-steel sheets,

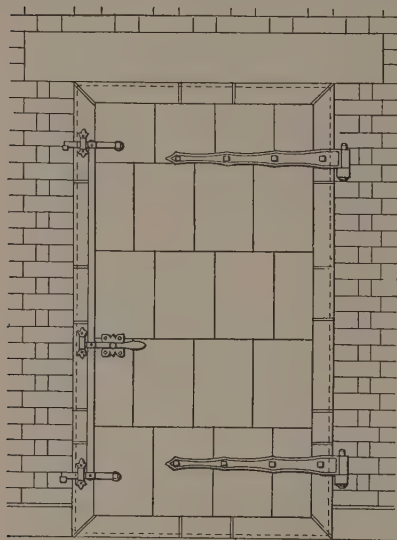


FIG. 46

lapped and nailed at the joints, is shown in Fig. 46. The total thickness of the door is about $2\frac{3}{4}$ inches. Doors of this construction have been found to offer a good resistance to fire for a considerable period, and do not buckle and twist under heat like an iron door; but it has been found that when subjected to a fire for any great length of time the tinned sheets are liable to buckle and become displaced.

Armoured doors may be constructed to slide vertically or horizontally

or to swing on hinges, and should have a 3-inch overlap at the top and at the sides.

105. Steel Rolling Shutters.—A form of construction not yet very largely adopted for party-wall openings in British practice is illustrated in the Kinnear steel roller shutter, shown in Fig. 47 (a), (b), and (c). The shutter is formed of a series of specially constructed horizontal steel strips, curved in section, and linked to one another in such a manner as to admit of expansion and contraction. A section of the strips is given to a larger scale in Fig. 47 (c), the points at which the shutter is able to expand being denoted by *a, a*.

The distance between the two shutters *b, b* should be preferably about 2 feet, although in ordinary walls this will necessitate a thickening of the brickwork immediately adjoining the opening

The rollers of the shutters are protected by iron hoods *c, c*, and, both at the sides and the bottom, the shutter fits into iron grooves *d, d*. All bolt holes are slotted to allow for expansion. The shutters appear to offer a very good resistance to fire.

106. Hardwood Doors.—Though hardly fire-resisting in the highest sense of the term, doors of solid 2-inch oak or teak have been largely used for openings in the walls or partitions enclosing staircases and for external openings, where such are required

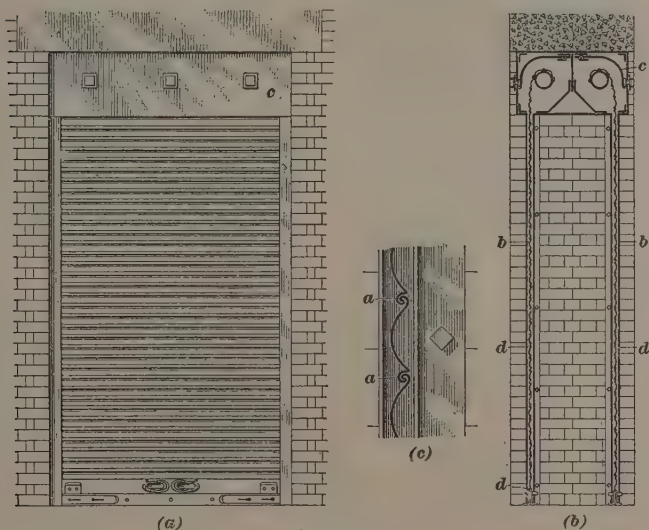
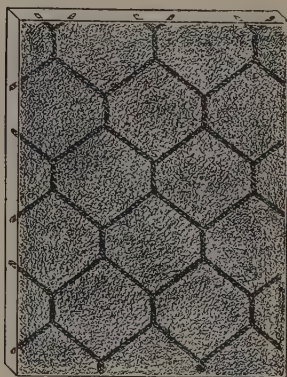


FIG. 47

to be protected in a minor degree. The construction of such doors is exactly similar to that of an ordinary panelled door, with the exception that the panels are made solid, and of the full thickness of the stiles and rails throughout.

107. Fire-Resisting Glazing.—Ordinary glass containing embedded wire netting, as shown in Fig. 48 (*a*), has long been recognized to possess very good fire-resisting characteristics, and

it is still largely used in factory and similar buildings. In buildings where the question of appearance has to be considered, the use of *electro-glazing* is becoming very customary. A casement window fitted with this kind of glazing is shown in Fig. 48 (b).



(a)



(b)

FIG. 48

The glass is about $\frac{1}{4}$ inch thick and is used in squares not exceeding 4 inches in height or width. The copper divisions, which are in the first instance of plain rectangular section, hold and secure the glass by means of additional copper electrically deposited.

Whatever the class of fire-resisting glazing adopted, it is desirable that the distance between the framing in which the glazing is fixed should not be more than 2 feet, either horizontally or vertically. The glazing may fail if constructed in large areas unassisted by framing.

A SERIES
OF
QUESTIONS AND EXAMPLES
RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

The following Examination Questions are divided into sections corresponding to the text, so that each section has a head-line which is the same as that of the section to which the questions refer.

No attempt should be made to answer the questions or to solve the examples until the corresponding part of the text has been carefully studied.

ARITHMETIC

(PART 1)

EXAMINATION QUESTIONS

- (1) What is arithmetic ?
- (2) In what three ways may numbers be represented ?
- (3) What is a number ? A unit ?
- (4) What is the difference between a concrete expression and an abstract number ?
- (5) What is the reading of a number called ?
- (6) Express, in both Arabic notation and Roman notation, seven thousand five hundred and three.
- (7) For what purpose are ciphers used ?
- (8) Write each of the following numbers in words : (a) 980 ; (b) 605 ; (c) 28,284 ; (d) 9,006,042 ; (e) 850,317,002 ; (f) 700,004.
- (9) Express in Roman notation the following numbers : (a) 76 ; (b) 353 ; (c) 1,732 ; (d) 1,496 ; (e) 1,888.
- (10) Represent in figures the following expressions : (a) seven thousand six hundred ; (b) eighty-one thousand four hundred and two ; (c) five million four thousand and seven ; (d) one hundred and eight million ten thousand and one ; (e) ten million and six ; (f) thirty thousand and ten.
- (11) Find the sum of $83,027 + 46,928 + 4,769 + 81,987 + 46,729 + 479,897 + 627 + 14,896 + 987,649$. Ans. 1,746,509
- (12) Multiply 29,800 by 390. Ans. 11,622,000
- (13) A man owes £1,000 ; he pays £329 the first year and £438 the second year. How much does he still owe ? Ans. £233

(14) Find the difference between $23,896 + 4,982 + 96,875 + 59,674$ and $31,627 + 54,892 + 6,925 + 8,976$. Ans. 83,007

(15) Multiply 8,765 by 987, and from the product subtract $4,695 \times 823$. Ans. 4,787,070

(16) The greater of two numbers is 1,004, and their difference is 49. What is their sum? Ans. 1,959

(17) A drover bought 36 oxen at £5 each and 23 cows at £19 each. How much did they all cost? Ans. £617

(18) Divide $2,937 \times 864$ by 923. Ans. $2,749\frac{241}{923}$

(19) Multiply 8,976 by 4,298, and subtract 98,765 from the product. Ans. 38,480,083

(20) An engine and boiler in a manufactory are worth £3,246; the building is worth three times as much, plus £1,200; and the tools are worth twice as much as the building, plus £1,875.

(a) What is the value of the building and tools? (b) What is the value of the whole plant? Ans. $\begin{cases} (a) \text{ £34,689} \\ (b) \text{ £37,935} \end{cases}$

(21) The divisor is 1,389, the quotient is 748, and the remainder is 1,263. What is the dividend? Ans. 1,040,235

(22) The multiplicand is 4,896, and the product 3,862,944. Find the multiplier. Ans. 789

(23) A man left £25,375 to his widow and £12,450 to each of his seven children. How much did they all receive? Ans. £112,525

(24) If a farm is sold for £75 an acre, the price received would be £6,396 more than if it were sold for £49 an acre. How many acres are there in the farm? Ans. 246 acres

(25) Solve the following by cancellation:

$$(a) \quad \frac{231 \times 96 \times 192 \times 45}{11 \times 24 \times 48 \times 15 \times 7} = ?$$

$$(b) \quad \frac{42 \times 64 \times 125 \times 98 \times 144}{14 \times 16 \times 25 \times 49 \times 36 \times 20} = ?$$

$$\text{Ans. } \begin{cases} (a) \text{ 144} \\ (b) \text{ 24} \end{cases}$$

ARITHMETIC

(PART 2)

EXAMINATION QUESTIONS

- (1) Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$. Ans. $4\frac{1}{24}$
- (2) What is the value of $3\frac{3}{4} + 6\frac{7}{8} - 4\frac{5}{6}$? Ans. $5\frac{19}{24}$
- (3) A man bought $12\frac{3}{4}$ tons of coal. How much was left after he had burned $5\frac{4}{5}$ tons? Ans. $6\frac{19}{20}$ tons
- (4) A coil of wire is $53\frac{1}{3}$ yards long. How many pieces of wire, each $6\frac{2}{3}$ yards long, can be cut from it? Ans. 8 pieces
- (5) Find the product of $83\frac{2}{3} \times 67\frac{4}{5}$. Ans. $5,672\frac{3}{5}$
- (6) Divide $4\frac{1}{2} \times 6\frac{3}{4}$ by $1\frac{4}{5}$. Ans. $16\frac{7}{8}$
- (7) At $12\frac{7}{8}$ shillings a hundredweight, what must be paid for 24 cwt. of steel shafting? Ans. 309 shillings
- (8) Simplify $\frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{5}}{1\frac{5}{8} \times 4\frac{2}{5}}$. Ans. 8
- (9) Multiply $8\frac{1}{20}$ by $\frac{1}{40}$. Ans. $\frac{161}{800}$
- (10) Subtract from 100 the sum of $8\frac{3}{4}$, $7\frac{5}{6}$, and $10\frac{1}{2}$. Ans. $72\frac{11}{12}$
- (11) A cask containing 40 gallons leaks at the rate of $3\frac{3}{4}$ gallons an hour. In how many hours will it be empty? Ans. $10\frac{2}{3}$ hours
- (12) Find the value of $\frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2}$. Ans. $19\frac{43}{60}$
- (13) The circumference of a wheel is $\frac{355}{113}$ times its diameter. What is the circumference of a wheel whose diameter is $8\frac{2}{3}$ feet? Ans. $27\frac{77}{839}$ feet

(14) A company of soldiers consumes $165\frac{3}{4}$ pounds of meat, which is an allowance of $1\frac{5}{8}$ pounds each. How many soldiers are in the company? Ans. 102 soldiers

(15) A bundle of steel rods, weighing $4\frac{4}{5}$ cwt., is cut up into 96 pieces for studs. How many pieces are there to the hundredweight? Ans. 20

(16) How much is $4\frac{3}{4}$ times $\frac{2}{3}$ of $6\frac{1}{2}$? Ans. $20\frac{7}{12}$

(17) Find the cost of $24\frac{3}{4}$ tons of nickel-steel forgings at £23 $\frac{3}{4}$ a ton. Ans. £587 $\frac{1}{8}$

(18) Three men in partnership gain £3,696. In dividing the profits, one gets $\frac{1}{3}$ of the amount, the second gets $\frac{2}{5}$ of the remainder, and the third gets what still remains. Find the share of each. Ans. £1,232; £985 $\frac{3}{5}$; £1,478 $\frac{2}{5}$

(19) A man paid £357 $\frac{1}{2}$ for $9\frac{3}{4}$ acres of land. What did he pay per acre? Ans. £36 $\frac{2}{3}$

(20) The sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$. One of the numbers is $12\frac{3}{4}$. Find the other number. Ans. $3\frac{1}{7}$

(21) If $12\frac{7}{8}$ times the difference between two numbers is $41\frac{1}{5}$, and one of the numbers is 21, what is the other number? Ans. $24\frac{1}{5}$ or $17\frac{4}{5}$

(22) What is the circumference of a wheel that goes 108 feet in turning $5\frac{3}{4}$ times? Ans. $18\frac{1}{2}\frac{2}{3}$ feet

(23) From $67\frac{5}{8}$ take $48\frac{2}{3}$. Ans. $18\frac{2}{3}\frac{3}{4}$

(24) If a man travelled $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles the next day, and $125\frac{1}{3}\frac{7}{5}$ miles the third day, how far did he travel in the three days? Ans. $289\frac{2}{4}\frac{1}{2}\frac{1}{6}$ miles

(25) A man bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ pence per pound and sold a part of it for $2\frac{1}{2}$ pence per pound, receiving for it the same amount as he paid for the whole. How many pounds did he have left? Ans. $52\frac{1}{8}\frac{3}{6}$ pounds

ARITHMETIC

(PART 3)

EXAMINATION QUESTIONS

(1) Change $2\frac{1}{2} \times 3\frac{1}{3} \div 8$ to a decimal, giving the result correct to four places. Ans. 1.0416

(2) Reduce 2.0416 to a mixed number. Ans. $2\frac{26}{25}$

(3) How many tons of coal at £1.4375 a ton should be received in exchange for 128 cords of wood at £.8125 a cord and £218 in money? Ans. 224 tons

(4) A dealer bought peaches at £1.875 a case, and sold them at a loss of £12.5 on 100 cases. How much did he receive per case? Ans. £1.75

(5) Reduce to decimals and add the following fractions:
 $\frac{3}{4}, \frac{7}{8}, \frac{11}{16}, \frac{17}{32}$. Ans. 2.84375

(6) If the diameter of the earth is 7,912 miles and its circumference is 3.1416 times as much, how many miles are there in its circumference? Give the result to four places of decimals.
Ans. 24,856.3392 miles

(7) Find the value of $\frac{2}{5}$ of $\frac{2}{3}$ of .0168. Ans. .00672

(8) The sum of .37 of a number and .23 of the same number is 33.6. Find the number. Ans. 56

(9) If $\frac{2}{3}$ of a number be subtracted from .8 of the number, the remainder is 87.8. What is the number? Ans. 658.5

(10) A plot of land cost £3,300, and this was .165 of the cost of the house erected upon it. Find what was paid for the house. Ans. £20,000

(11) A man paid $\cdot 143$ of his annual salary to his butcher, $\cdot 347$ of it to his grocer, $\cdot 256$ of it for clothing, and $\cdot 154$ for other expenses. How much did he save, if his butcher received £65.494 ? Ans. £45.8

(12) A farm was bought at £17.5 per acre and sold at £22.5, by which there was a gain of £734. How many acres were there ? Ans. 146.8 acres

(13) If $\cdot 17$ of a quantity of oil amounts to 424.5 gallons, how many gallons are there in all ? Ans. 2,497.06 gal.

(14) If a cubic inch of water weighs $\cdot 03617$ pound, what will be the weight of 231 cubic inches ? Ans. 8.355 + pounds

(15) What decimal part of a foot is $\frac{3}{16}$ inch ? Ans. $\cdot 015625$

(16) From 1 plus $\cdot 001$ take $\cdot 01$ plus $\cdot 000001$. Ans. $\cdot 990999$

(17) Solve the following :

$$(a) \left(\frac{7}{16} - \cdot 13 \right) \times \left(\cdot 625 + \frac{5}{8} \right).$$

$$(b) \left(\frac{19}{32} \times \cdot 21 \right) - \left(\cdot 02 \times \frac{3}{16} \right).$$

$$(c) \left(\frac{13}{4} + \cdot 013 - 2 \cdot 17 \right) \times 13\frac{1}{4} - 7\frac{5}{16}.$$

$$\text{Ans. } \begin{cases} (a) \cdot 384375 \\ (b) \cdot 1209375 \\ (c) 6.4896875 \end{cases}$$

(18) Express (approximately) $\cdot 7292$ as a common fraction whose denominator is 40. Ans. $\frac{29}{40}$

(19) Find the value of the following expression when the result is carried to three decimal places :

$$\frac{74.26 \times 3.1416 \times 19.5 \times 19.5 \times 350}{33,000 \times 12 \times 4}. \quad \text{Ans. } 19.601 +$$

(20) Add together 5.2467, 17.5346, 26.1347, $\cdot 0328$, 9.00562, 54.67325. Ans. 112.62767

(21) Work out the following correct to four decimal places :

(a) 83.7621×2.3542 ; (b) $76.321 \times \cdot 06572$; (c) 7.93016×43.2013 .

$$\text{Ans. } \begin{cases} (a) 197.1927 \\ (b) 5.0158 \\ (c) 342.5932 \end{cases}$$

(22) Divide 17.892 by $\cdot 231$, and carry the result to four decimal places. Ans. 77.4545 +

(23) Express (approximately) (a) $\cdot 7928$ in 64ths; (b) $\cdot 1416$ in 32nds; (c) $\cdot 47915$ in 16ths.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \frac{51}{64} \\ (b) \frac{5}{32} \\ (c) \frac{8}{16} \end{array} \right.$$

(24) State the difference between a vulgar fraction and a decimal.

(25) Add 17 thousandths, 2 tenths, and 47 millionths

$$\text{Ans. } \cdot 217047$$

ARITHMETIC

(PART 4)

EXAMINATION QUESTIONS

(1) Reduce: (a) £8 17s. 5d. to pence. (b) £160 17s. 6d. to half-crowns. (c) 7 tons 8 cwt. 1 qr. 15 lb. to pounds. (d) 13 gal. 3 qt. 1 pt. to pints.

Ans. $\left\{ \begin{array}{l} (a) \text{ 2,129 pence} \\ (b) \text{ 1,287 half-crowns} \\ (c) \text{ 16,619 lb.} \\ (d) \text{ 111 pt.} \end{array} \right.$

(2) Reduce: (a) 8,262 pence to £, etc. (b) 81,324 oz. to hundredweights, etc. (c) 567,432 in. to perches, etc. (d) 4,876 pt. to bushels, etc.

Ans. $\left\{ \begin{array}{l} (a) \text{ £34 8s. 6d.} \\ (b) \text{ 45 cwt. 1 qr. 14 lb. 12 oz.} \\ (c) \text{ 2,865 per. 4 yd. 1 ft. 6 in.} \\ (d) \text{ 76 bush. 1 gal. 2 qt.} \end{array} \right.$

(3) Add together: (a) 5 tons 16 cwt. 2 qr., 30 tons 10 cwt. 1 qr. 20 lb., 3 tons 14 cwt. 27 lb., 48 tons 2 cwt. 1 qr. 5 lb., 16 cwt. 16 lb. (b) £208 15s. 6d., £27 13s. 7½d., £16 14s. 9¼d., £86 3s. 10¼d., £35 2s. 10½d.

Ans. $\left\{ \begin{array}{l} (a) \text{ 88 tons 19 cwt. 2 qr. 12 lb.} \\ (b) \text{ £374 10s. 8d.} \end{array} \right.$

(4) Subtract the sum of £23 4s. 10d., £11 13s. 9½d., £15 1s. 2d. from £114 0s. 2¼d.

Ans. £64 0s. 4¾d.

(5) Multiply: (a) 3 yd. 2 ft. 10 in. by 26. (b) £6 3s. 2½d. by 29. (c) 7 tons 8 cwt. 1 qr. 11 lb. by 63.

Ans. $\left\{ \begin{array}{l} (a) \text{ 102 yd. 1 ft. 8 in.} \\ (b) \text{ £178 13s. 0½d.} \\ (c) \text{ 467 tons 5 cwt. 3 qr. 21 lb} \end{array} \right.$

(6) Divide : (a) £690 4s. 2½d. by 12 and 42. (b) 3,000 tons 15 cwt. 2 qr. 16 lb. by 330 and 108.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \left\{ \begin{array}{l} \text{£57 10s. 4.21d., nearly} \\ \text{£16 8s. 8.06d., nearly} \end{array} \right. \\ (b) \left\{ \begin{array}{l} \text{9 tons 1 cwt. 3 qr. } 12\frac{52}{55} \text{ lb.} \\ \text{27 tons 15 cwt. 2 qr. } 22\frac{4}{9} \text{ lb.} \end{array} \right. \end{array} \right.$$

(7) The floor area of a certain passage $\frac{7}{8}$ of a yard wide is equal to that of another passage 28 yards long and $\frac{3}{4}$ of a yard wide. What is the length of the first passage? (The area is equal to the length multiplied by the width.) Ans. 24 yd.

(8) What must be paid for a rectangular block of granite, its dimensions being 6ft. $\times$ 4½ ft. $\times$ 3 ft., at 4s. 7½d. per cubic foot? (The contents of the block are found by multiplying together the length, breadth, and thickness.) Ans. £18 14s. 7½d.

(9) Find to four decimal places the number of times that 2 ft. 8 in. must be repeated to be equal to the circumference of a fly-wheel 12 feet in diameter. (The circumference equals 3.1416 times the diameter.) Ans. 14.1372

(10) A cubical cistern is 8 ft. 6 in. in each of its three dimensions. How many gallons will it hold? (See example 8.)

Ans. 3,824.68 gal., nearly

(11) A family used a quantity of sugar at the rate of $8\frac{3}{4}$ ounces per day and it lasted 80 days. How many pounds were there in all? Ans. 43¾ lb.

(12) Find the difference between 25 ft. square and 75 sq. ft.

Ans. 550 sq. ft.

(13) A rectangular beam 36 ft. long and 10 in. thick contains 40 cu. ft. How wide is it? (See example 8.) Ans. 16 in.

(14) (a) How many square feet are there in a passage-way 18.75 ft. long and 2 ft. 10 in. wide? (See example 7.) (b) What will it cost to board it over at 4d. a square foot?

$$\text{Ans. } \left\{ \begin{array}{l} (a) \text{ 53.125 sq. ft.} \\ (b) \text{ 17s. } 8\frac{1}{2} \text{d.} \end{array} \right.$$

(15) How many bushels will a bin contain whose dimensions are 8 ft. $\times$ 6½ ft. $\times$ 6 ft.? Ans. 242.9 bush., nearly

- (16) (a) What fractional part of 63 gallons is 18 gal 3 qt. 1 pt.?
 (b) What decimal part?

$$\text{Ans. } \begin{cases} (a) \frac{151}{504} \\ (b) .2996 + \end{cases}$$

- (17) How many kilos of water will a tank contain that is
 3 m. $\times$ 3 m. $\times$ 2 m. ?

$$\text{Ans. 18,000 Kg.}$$

- (18) Change 56.8 metres to yards. Ans. 62.1 yd.

- (19) If a cubic foot of marble weighs 168.75 lb., what will
 be the weight of a marble block, the dimensions of which are
 3.75 m. $\times$ 2.8 m. $\times$ 2.5 m. ?

$$\text{Ans. 156,434 lb., nearly}$$

- (20) From £21 5s. 3d. take £13 9s. 6d. Ans. £7 15s. 9d.

- (21) Reduce 4,763,254 links to miles.

$$\text{Ans. 595 mi. 32 ch. 2 rd. 4 li.}$$

- (22) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails
 be, if placed end to end ?

$$\text{Ans. 53 rd. 1 yd. 2 ft. 3 in.}$$

- (23) Multiply 7 tons 15 cwt. 10.5 lb. by 1.7.

$$\text{Ans. 13 tons 3 cwt. 73.85 lb.}$$

- (24) From 20 rd. 32 ft. 9 in. take 300 ft. Ans. 3 rd. 13 ft. 3 in.

- (25) Divide 358 ac. 57 sq. rd. 6 sq. yd. 2 sq. ft. by 7.

$$\text{Ans. 51 ac. 31 sq. rd. 0 sq. yd. 8 sq. ft.}$$

ARITHMETIC

(PART 5)

EXAMINATION QUESTIONS

- | | | |
|------|---|---------------------------------------|
| (1) | What is the square of 108 ? | Ans. 11,664 |
| (2) | Find the fifth power of 9. | Ans. 59,049 |
| (3) | What is the value of $\cdot 0133^3$? | Ans. $\cdot 000002352637$ |
| (4) | In what respect does evolution differ from involution ? | |
| (5) | Extract the square root of 90. | Ans. 9.4868 |
| (6) | Find the value of $(3\frac{3}{4})^3$. | Ans. $52\frac{47}{64}$, or 52.734375 |
| (7) | What is the cube root of 92,416 ? | Ans. 45.211 |
| (8) | Find the value of $\sqrt{502,681}$. | Ans. 709 |
| (9) | What is the value of $\sqrt[3]{\frac{27}{64}}$? | Ans. $\frac{3}{4}$ |
| (10) | From the cube of 4 take the cube root of 8. | Ans. 62 |
| (11) | What is the value of $\sqrt[3]{\frac{8}{27}}$? | Ans. $\frac{2}{3}$ |
| (12) | Extract the square root of $\cdot 3364$. | Ans. $\cdot 58$ |
| (13) | Find the square root of $\cdot 7854$. | Ans. $\cdot 88623$ |
| (14) | What number multiplied by itself equals 114.9184 ? | Ans. 10.72 |
| (15) | Extract the square root of 3,486,784. | Ans. 1,867.3 |
| (16) | Find the square root of $\cdot 00041209$. | Ans. $\cdot 0203$ |
| (17) | Find the fourth root of 2,490.31. | Ans. 7.0642 |
| (18) | Find the fifth root of 6,039,065,434. | Ans. 90.405 |
| (19) | Find the fifth root of $\cdot 127$. | Ans. $\cdot 66185$ |
| (20) | $\sqrt[5]{72.415} = ?$ | Ans. 2.3548 |

ARITHMETIC

(PART 6)

EXAMINATION QUESTIONS

- (1) Divide: (a) the ratio of $\frac{3}{4} : \frac{4}{5}$ by 5; (b) the ratio of $2 : .05$ by 4; (c) the inverse ratio of $12.5 : 125$ by $\frac{3}{2}$.

$$\text{Ans.} \begin{cases} (a) & \frac{3}{16} \\ (b) & 1 \\ (c) & 6\frac{2}{3} \end{cases}$$

- (2) Express in integers the ratio of 2 lb. 3 oz. to 5 lb. $1\frac{1}{2}$ oz.

$$\text{Ans. } 70 : 163$$

- (3) Multiply: (a) the ratio of $\frac{4}{3} : \frac{3}{4}$ by 3; (b) the ratio of the volume of a gallon to a cubic foot by 27; (c) the inverse ratio of $33\frac{1}{3} : 187\frac{1}{2}$ by 4.

$$\text{Ans.} \begin{cases} (a) & 5\frac{1}{3} \\ (b) & 4.335 + \\ (c) & 22\frac{1}{2} \end{cases}$$

- (4) Which is greater, and by how much, $3 : 8$ or $5 : 12$?

$$\text{Ans. The latter, by } \frac{1}{24}$$

- (5) Find the value of x in the following:

$$(a) 20 + 7 : 10 + 8 = 3 : x; (b) 12^2 : 100^2 :: 4 : x.$$

$$\text{Ans.} \begin{cases} (a) & 2 \\ (b) & 277.8 - \end{cases}$$

- (6) If a railway train runs 444 mi. in 8 hr. 40 min., in what time can it run 1,060 mi. at the same rate of speed?

$$\text{Ans. } 20 \text{ hr. } 41\frac{1}{2} \text{ min., nearly}$$

(7) Find the value of x in the following :

(a) $x : 35 :: 4 : 7$.

(b) $\sqrt[3]{1,000} : \sqrt[8]{1,331} = 27 : x$.

(c) $64 : 81 = 21^2 : x^2$.

Ans. $\begin{cases} (a) & 20 \\ (b) & 29\cdot7 \\ (c) & 23\cdot62 + \end{cases}$

(8) If 100 gal. of water runs over a dam in 2 hr., how many gallons will run over the dam in 14 hr. 28 min. ? Ans. $723\frac{1}{3}$ gal.

(9) Explain : (a) why the following is not a correct proportion, $20 : 23 = 39 : 45$; (b) how much must be added to 23 to make it correct ? Ans. (b) $\frac{1}{18}$

(10) If 5 men in 9 days of 10 hr. each can build a wall 100 ft. long, 6 ft. high, and 18 in. thick, in how many days of 9 hr. each will 8 men build a wall 120 ft. long, 8 ft. high, and 20 in. thick ? Ans. 11 days 1 hr.

(11) Find the value of x in

$$\begin{array}{c|c} \frac{1\frac{2}{5}}{1\frac{5}{8}} & \frac{\frac{3}{5}}{\frac{5}{9}} = x \\ \hline \frac{2}{3} & 20. \end{array} \quad \text{Ans. } x = 22\cdot4$$

(12) The temperature being constant, the volume of a given weight of air varies inversely as the pressure of the air. If a pound of air has a volume of 10 cubic feet at a pressure of 17·8 pounds per square inch, what will be the volume if the pressure is 67·2 pounds per square inch, the temperature remaining the same ? Ans. 2·65 cu. ft., nearly

(13) What added to the first term, or what subtracted from the second term, will form from $3 : 7 = 8 : 12$ a correct proportion ? Ans. $1\frac{2}{3}$; $2\frac{1}{2}$

(14) The total load which a steel floor-girder will safely carry varies directly as the square of its depth and inversely as its length. If the safe load for a girder 15 ft. long and 9 in. deep is 14·2 tons, what load will another girder safely carry which is 20 ft. long and 12 in. deep, but otherwise similar ?

Ans. 18·9 tons, nearly

(15) (a) What is 25% of 8,428 lb. ? (b) What is $\frac{1}{2}\%$ of £35,000 ? (c) How much per cent. is £176 8s. of £2,520 ?

Ans. $\begin{cases} (a) & 2,107 \text{ lb.} \\ (b) & £175 \\ (c) & 7\% \end{cases}$

(16) The builders of a torpedo-boat destroyer guaranteed a speed of 32 knots, but on the trials a speed $7\frac{1}{2}\%$ greater than this was obtained. What was the speed reached in the trials?

Ans. 34.4 knots

(17) A man receives a salary of £250; he pays 24% of it for board, $12\frac{1}{2}\%$ of it for clothing, and 17% of it for other expenses. How much does he save in a year? Ans. £116 5s.

(18) The speed of a mill engine suddenly increases by 4%. If its increased speed is 140 revolutions per minute, what was its original speed? Ans. 134.6 revolutions per minute

(19) What sum diminished by 35% of itself equals £4,810?

Ans. £7,400

(20) A certain train performs a journey in 3 hours 35 minutes; if its speed is increased so that the journey is performed in 3 hours 25 minutes, what per cent. of the original time is the time saved? Ans. 4.65%

APPLICATION OF FORMULÆ

EXAMINATION QUESTIONS

(1) Work out the following formula, taking $A = 5$, $B = 10$, $D = 120$, $h = 200$, and $x = 12$:

$$T = \sqrt{\frac{A^2 \left[490 + \frac{(hx)^2}{D^2} \right]}{h + \frac{x}{D} (A^2 - B^2)}} \quad \text{Ans. } T = 10$$

(2) Using the formula given in Art. 20, what is the area of a triangle whose sides are 28 feet, 39 feet, and 37 feet long?

Ans. 493.311 sq. ft.

(3) In the formula $l = \sqrt{\pi^2 d^2 + t^2}$, assume that $d = 5$, $t = 2$, and $\pi = 3.1416$. What is the value of l ?

Ans. $l = 15.84$

(4) In the formula, $P = \frac{S A}{1 + q \frac{l^2}{G^2}}$, find the value of P when

$S = 70,000$, $A = .7854$ ($10^2 - 7^2$), $l^2 = 186^2$, $q = .00004$, and $G^2 = \frac{10^2 + 7^2}{16}$. Ans. $P = 2,441,127$

(5) Solve the formula in example 3 for t , when $d = 6$ and $l = 25$. Ans. $t = 16.42$

(6) In the formula, $P = F \frac{700 + 15c}{700 + 15c + c^2}$, find the value of P when $F = 5,000$, and $c = \frac{18 \times 12}{10}$. Ans. $P = 3,435$

(7) Given the formula, $I = \frac{A d^2}{6.66}$, find the value of d when $I = 8.9$ and $A = 6.9$. Ans. $d = 2.93$

(8) Transform the formula $d = D \sqrt{\frac{P}{nS}}$, so as to obtain a formula for S , and find the value of S when $d = .85$, $n = 15$, $P = 165$, and $D = 17$. Ans. $S = 4,400$

(9) Using the formula given in Art. 19, what is the value of A when $r = 12$, $E = 150$, $c = 23.182$, and $h = 8.894$? Ans. 152.4944

(10) In the formula $d_1 = d \left(1 + \frac{.08}{n} + \frac{.4}{\sqrt{n}} \right)$, if $d = 1.625$ and $n = 3$, what is the value of d_1 ? Ans. $d_1 = 2.0436$

GEOMETRY AND MENSURATION

(PART 1)

EXAMINATION QUESTIONS

(See note at end of Art. 64)

(1) What is the area in square feet of a parallelogram whose base is 129 inches long, if the shortest distance between the base and the side opposite is 7 feet?

Ans. 75.25 sq. ft.

(2) The parallel sides of a trapezoid are 15 feet 7 inches and 21 feet 11 inches long, and the altitude is 7 feet 8 inches; what is the area of the trapezoid?

Ans. 143.81 sq. ft.

(3) In Fig. I, find the number of degrees in the angle EOA .

Ans. 123°

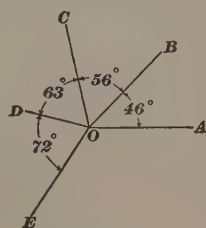


FIG. I

(4) If in triangle ABC , Fig. II, one-third of angle A is $14^\circ 47' 10''$, what are the angles, B being two and one-half times angle A ?



FIG. II

$$\text{Ans. } \begin{cases} A = 44^\circ 21' 30'' \\ B = 110^\circ 53' 45'' \\ C = 24^\circ 44' 45'' \end{cases}$$

(5) The area of a circle is 89.42 square inches; find (a) its diameter and (b) its circumference.

$$\text{Ans. } \begin{cases} (a) 10.67 \text{ in.} \\ (b) 33.52 \text{ in.} \end{cases}$$

(6) In a right triangle, one of the acute angles is 37° . (a) What is the other acute angle? (b) What is the angle formed by extending one side of the given angle from the vertex?

$$\text{Ans. } \begin{cases} (a) 53^\circ \\ (b) 143^\circ \end{cases}$$

(7) ADB , Fig. III, represents an arch of 3 feet radius.

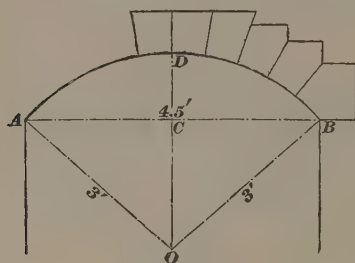


FIG. III

(a) The span AB being $4\frac{1}{2}$ feet, what is the rise CD ?

(b) If the radius OA and the rise CD were given, show how AB would be found.

Ans. (a) $1.02 \text{ ft.} = 1 \text{ ft. } \frac{1}{4} \text{ in.}$

(8) What is the entire floor space of a five-story building, each floor being in the form of a triangle whose sides are 52 feet 6 inches, 59 feet, and 48 feet 6 inches, respectively?

Ans. $6,032 \text{ sq. ft.}$

(9) A rafter has a rise of 12 feet 6 inches and a run of 8 feet; find the rise to the foot.

Ans. $18\frac{3}{4} \text{ in.}$

(10) Explain how to set out a line perpendicular to another line by measurement.

(11) In staking out a building, the points A, B, C, D, E , and F are marked with pegs, the distances being as shown in Fig. IV. To check these measurements, it is desired to measure the diagonals FB, FD, EA , and EC . What are their lengths?

Ans. $\begin{cases} FB = 30 \text{ ft.} \\ FD = 33.11 \text{ ft.} \\ EA = 38.42 \text{ ft.} \\ EC = 18.44 \text{ ft.} \end{cases}$

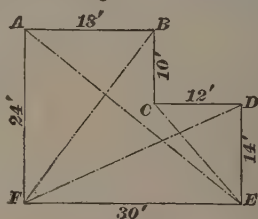


FIG. IV

(12) Show, by a sketch, how to divide a line 6 inches long into seven equal parts.



FIG. V

(13) In Fig. V, what is the length of the ridge plate AB , which is 10 feet above CD , the latter being 36 feet long, and the angles DCA and CDB being 45° ?

Ans. 16 ft.

(14) Fig. VI represents a roof whose span AB is 22 feet. The slopes are half pitch, and the distance from E to D , under C , is 11 feet. (a) Find the length of a common rafter, as HG . (b) Find the length of a hip rafter, as BC .

Ans. { (a) $15.56 \text{ ft.} = 15 \text{ ft. } 6\frac{3}{4} \text{ in.}$
 (b) $19.05 \text{ ft.} = 19 \text{ ft. } \frac{5}{8} \text{ in.}$

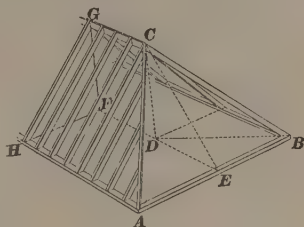


FIG. VI

(15) Fig. VII represents the plan and end views of a roof.

Knowing that the ridges AB and CD are 10 feet above the

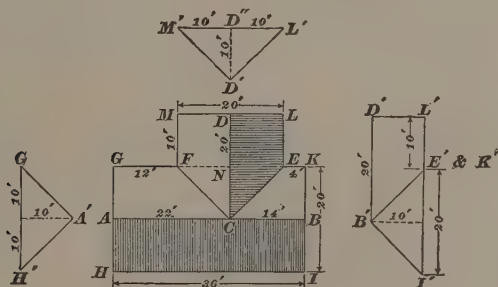


FIG. VII

eaves, find: (a) the total area of the roof; (b) the length of the valleys FC and EC .

Ans. { (a) $1,300.88 \text{ sq. ft.}$
 (b) $17.32 \text{ ft.} = 17 \text{ ft. } 3\frac{7}{8} \text{ in.}$

(16) The top of the frame of a window $3\frac{1}{2}$ feet in width is to be in the form of a circular arc, the rise of which is 3 inches; what will be the length of the piece required?

Ans. $42.57 \text{ in.} = 3 \text{ ft. } 6\frac{9}{16} \text{ in.}$

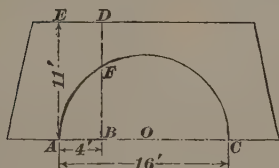


FIG. VIII

(17) What is the angle included between two adjacent sides of a regular nonagon, or nine-sided polygon?

Ans. 140°

(18) In Fig. VIII, the span AC of a semicircular arch is 16 feet, and the distance AE to the top of the masonry is 11 feet; how thick is the stonework at DF , 4 feet from the vertical line AE ? Ans. 4.07 ft.

(19) The span AB of the arch shown in Fig. IX is 6 feet 8 inches, and the rise CD is 8 inches. (a) Find the radius. (b) If there are thirteen ring stones, what is the bottom width of each? Ans. $\begin{cases} (a) & 8 \text{ ft. } 8 \text{ in.} \\ (b) & 6.32 \text{ in.} \end{cases}$

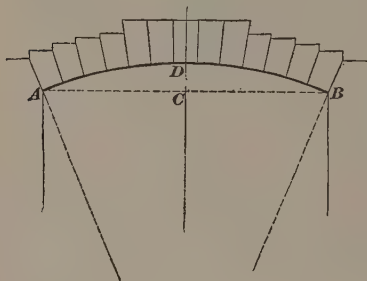


FIG. IX

(20) In a roof truss, one of the tie-rods must sustain a load of 27,500 pounds. The allowable load on each square inch of cross-section being 10,000 pounds, what must be the diameter of the rod? Ans. $1\frac{7}{8}$ in., nearly

GEOMETRY AND MENSURATION

(PART 2)

EXAMINATION QUESTIONS

(1) A square pyramidal monument is 3 feet square at the base, 1 foot square at the top, and 18 feet high. It is capped by a pyramid 1 foot square and 2 feet high. If the stone is granite, weighing 170 pounds per cubic foot, what is the total weight?

Ans. 13,373 $\frac{1}{2}$ lb.

(2) How many gallons, at 6.24 gallons per cubic foot, will be discharged per minute through a 4-inch pipe, if the water flows 5 feet per second? Take the area of the pipe as .09 square foot.

Ans. 168.48 gal.

(3) Taking the weight of lead as .41 pound per cubic inch, what length of lead pipe, measuring 1 $\frac{3}{4}$ inches outside diameter and $\frac{1}{8}$ inch thick, will be needed to make a 3-pound weight?

Ans. 11.6 in.

(4) It is required, in a question of water supply, to determine the area of the water section in a 12-inch pipe flowing 9 inches deep; that is, what is the area below the surface AB , Fig. I?

Ans. 91 sq. in., nearly

(5) The length of a building with a plain roof is 32 feet, and the width is 24 feet. The height of the gables is five-eighths the width. The roof projects over the walls 15 inches (measured on the roof) at ends and eaves. What is the area of the slating in the roof area?

Ans. 1,412 sq. ft.



FIG. I

(6) The dome of a cupola is hemispherical, and its diameter is 3 feet; deducting 5 square feet for windows, etc., what is the area of the remaining surface? Ans. 9·14 sq. ft.

(7) Estimating the weight of cast iron at 450 pounds per cubic foot, what is the weight per 12-foot length of pipe, 10 inches inside diameter, the thickness being $\frac{1}{2}$ inch? Ans. 616·5 lb.

(8) The outside of a circular tower 28 feet high and 8 feet in diameter is to be slated. (a) What will be the area of the slating? (b) If the slates average 6 inches in width and are laid 5 inches to the weather, how many will be required, not allowing for waste?

Ans. $\begin{cases} (a) & 704 \text{ sq. ft.} \\ (b) & 3,380 \text{ slates, nearly} \end{cases}$

(9) At ·26 pound per cubic inch, what is the weight of the cast-iron base shown in Fig. II? Take the four ribs as plain, with no allowance for swollen parts or deduction for holes. The corners of the base are rounded to a 2-inch radius. Ans. 82·32 lb.

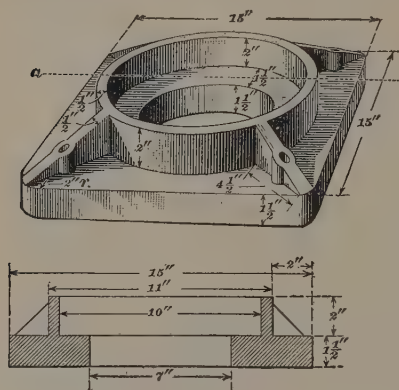


FIG. II

(10) A bridge pier 28 feet high is 30 ft. $\times$ 12 ft. at the base and 22 ft. $\times$ 7 ft. at the top. (a) What is its cubical contents? (b) What will be the cost of the masonry at £5 per cubic yard? Use the prismatic formula.

Ans. $\begin{cases} (a) & 7,009\cdot33 \text{ cu.} \\ & \text{ft., or } 259\cdot6 \\ & \text{cu. yd.} \\ (b) & \text{£1,298} \end{cases}$

(11) A cast-iron ball that will weigh 25 pounds is required; taking the weight of cast iron at ·26 pound per cubic inch, what will be the diameter of the ball?

Ans. 5·68 in., or $5\frac{11}{16}$ in., nearly

(12) Estimating steel at .283 pound per cubic inch, what will be the weight per foot of the stanchion shown in Fig. III, in which (a) is an enlarged section of a channel and (b) is

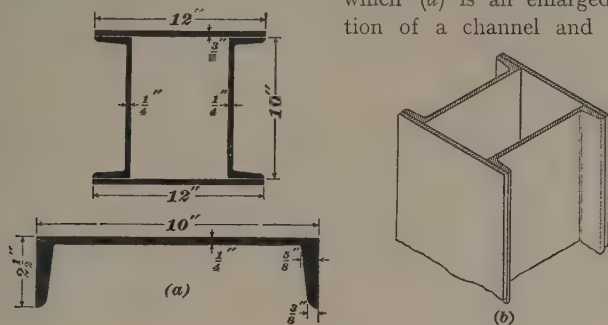


FIG. III

a perspective view of the end of the stanchion? Neglect the curved corners and edges on the channels. Ans. 62.83 lb.

(13) The load on a column resting on a square pier of brickwork is 107,520 pounds. Allowing a safe load of 3 tons per square foot on the soil, and 8 tons per square foot on the brick pier, what must be: (a) the dimensions of the foot of the square pier in order that the soil may safely support the load? (b) the dimensions of the square column base?

Ans. $\begin{cases} (a) & 4 \text{ ft. square} \\ (b) & 2.45 \text{ ft. square} \end{cases}$

(14) Fig. IV represents a cross-section of a wrought-iron column, of which a strip 1 foot long and 1 inch square in section weighs 3.33 pounds. (a) What is the area of the cross-section, disregarding curved corners and edges? (b) Adding 2 per cent. to the weight of the iron for rivet heads, what is the weight of the column per foot of length?



FIG. IV

Ans. $\begin{cases} (a) & 13.21 \text{ sq. in.} \\ (b) & 44.87 \text{ lb.} \end{cases}$

(15) Fig. V represents the plan of a house that is to be faced with pressed brick. The walls are 22 feet high, and there are two gables, one 14 ft. $\times$ 7 ft. high, and the other 12 ft. $\times$ 7 ft. high. Deducting ten windows, each 7 ft. $\times$ 3 ft.; ten windows, 6 ft. $\times$ 3 ft.; and four doors, 8 ft. $\times$ 3 ft. 6 in., and allowing seven bricks to each square foot of surface, how many bricks will be required?

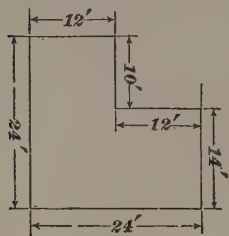


FIG. V

Ans. 11,907

(16) The major and minor axes of an elliptic window glass are 7 feet 4 inches and 4 feet 4 inches, respectively. What is the area of the glass?

Ans. 24.93 sq. ft.

(17) A culvert is required to have an area of 88 square feet; how much more than this area is the cross-section shown in Fig. VI, the arch being semi-elliptic?

Ans. 3.42 sq. ft.

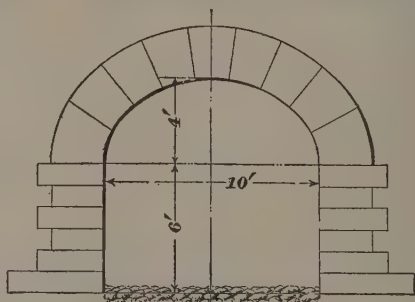


FIG. VI

(18) Fig. VII shows a cross-section of a retaining wall 16 feet long. (a) What is the area of cross-section? (b) How many cubic yards are there in the wall?

Ans. $\left\{ \begin{array}{l} (a) 71.49 \text{ sq. ft.} \\ (b) 42.36 \text{ cu. yd.} \end{array} \right.$

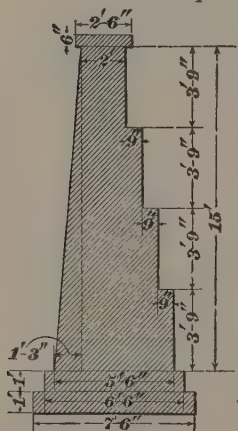


FIG. VII

(19) Fig. VIII represents a centre—half a regular hexagon—for a 5-foot

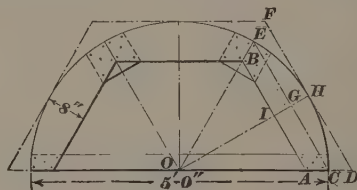


FIG. VIII

semicircular arch. (a) What is the angle of bevel, DFO , to which the ends of the planks should be cut? (b) What length of plank, as FD , is needed for each piece? (c) What is the length of the inner side, as AB , of each plank, after cutting, if the width is 8 inches?

$$\text{Ans.} \begin{cases} (a) 60^\circ \\ (b) 2.89 \text{ ft.} = 2 \text{ ft. } 10.7 \text{ in.} \\ (c) 2.12 \text{ ft.} = 2 \text{ ft. } 1\frac{1}{2} \text{ in., nearly} \end{cases}$$

(20) In Fig. IX, the base ai is 20 feet and the perpendicular lines at b, c, d, e, f, g, h measure 9.3, 12.6, 14.2, 14.8, 14.2, 12.6, and 9.3 feet, respectively; what is the area of the figure?

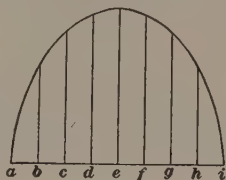


FIG. IX

Ans. 217.5 sq. ft.

PRELIMINARY BUILDING OPERATIONS

EXAMINATION QUESTIONS

- (1) What qualities as a foundation does rock possess ?
- (2) What is meant by wedging ?
- (3) What is a lewis ? Sketch one example.
- (4) What are line boards and what is their use ?
- (5) Describe quicksand and its characteristics.
- (6) What is shoring ?
- (7) Without attempting to describe a tower gantry, what is its characteristic use ?
- (8) Into what divisions is the soil supporting the foundations of a building divided ?
- (9) How deep are cellars usually dug ?
- (10) Describe the method of needling.
- (11) What is an independent scaffold, and why is it necessary ?
- (12) What qualities as a foundation does clay possess ?
- (13) What is the substructure of a building ?
- (14) What is underpinning ?
- (15) What are fans, and how and where are they used ?
- (16) Describe the methods adopted in timbering trenches.
- (17) In the absence of mechanical power, how may heavy material be taken to the various stages of a scaffold ?
- (18) How is the nature of the soil on a building site determined ?

(19) Describe the precautions taken to avoid collapse at the window openings when a wall is being shored, and illustrate by sketches.

(20) What considerations affect the width of concrete foundations?

FOUNDATIONS AND FOOTINGS

EXAMINATION QUESTIONS

- (1) Explain how foundations are placed on sloping ground.
- (2) What is a caisson and what kinds are there in use ?
- (3) In what part of a foundation should the centre of the load fall ?
- (4) What is a screw pile ?
- (5) Explain how a foundation built in a sloping clay bank is protected from water.
- (6) Explain why concrete is sometimes reinforced with steel.
- (7) How are wooden bearing piles driven ?
- (8) When is an inverted arch used ?
- (9) Explain how to bond corners in stone foundation walls.
- (10) Name the materials used in constructing foundations.
- (11) What is expanded metal ?
- (12) Explain why the load per square foot under columns is made greater than that under walls.
- (13) Give a rule for determining the thickness of stone foundation walls.
- (14) Describe and sketch a plank foundation.
- (15) Explain why only a fraction of the superimposed load is considered when proportioning foundations in certain cases.
- (16) Describe a method of placing a foundation on quicksand.
- (17) Explain one method of damp-proofing a cellar wall.
- (18) A ram weighs 13 hundredweights and drops 12 feet, or 144 inches ; the pile sinks $\frac{1}{2}$ inch at the last blow. What safe load will it carry ?
Ans. 468 cwt.
- (19) Name some materials used as damp-proof courses.
- (20) What is a grillage foundation ?

AREAS, VAULTS, AND RETAINING WALLS

EXAMINATION QUESTIONS

- (1) How are the tops of area walls finished ?
- (2) Give a short rule for finding the thickness of retaining walls, and illustrate it with the example of a wall 15 feet high.
- (3) What is the purpose of an area wall ?
- (4) Outline generally the rules which should be observed in building pavement vaults.
- (5) Explain the construction of a stone pavement.
- (6) What important consideration governs the size of window areas ?
- (7) Sketch and explain the construction of a reinforced-concrete retaining wall.
- (8) Explain the construction of a concrete pavement.
- (9) Explain what is meant by a relieving arch in connection with a retaining wall.
- (10) What is a flagstone ?

LIMES, CEMENTS, AND MORTARS

EXAMINATION QUESTIONS

- (1) What are hydraulic limes ?
- (2) How are cements classified ?
- (3) What will underburnt lime do if it is incorporated in a wall ?
- (4) What quality of sand should be used for making mortar ?
- (5) What is the difference between fat lime and poor lime ?
- (6) State one or two rough tests by which the quality of cement may be ascertained.
- (7) Name the three principal uses of lime and cement.
- (8) What is meant by grouting ?
- (9) Briefly outline the method of mixing cement mortar.
- (10) Describe the tests required by the British Engineering Standards Committee in its specification for Portland cement, for both fineness and strength.
- (11) Name some of the conditions on which the setting of a cement depends.
- (12) State, briefly, the method of manufacture of English Portland cement.
- (13) (a) What is a cement ? (b) What are its uses in building construction ?
- (14) What is calcined slag cement ?
- (15) Compare natural cement with Portland cement.

- (16) How can the setting of Portland cement be regulated ?
- (17) Give some particulars of hydrated lime.
- (18) What is : (a) ganister ? (b) parget ?
- (19) What is meant by the air-slaking of lime ?
- (20) Is there any sharp line of distinction between lime and hydraulic cement ?

CONCRETE CONSTRUCTION

EXAMINATION QUESTIONS

(1) Describe the construction of a spirally reinforced-concrete column.

(2) What is the difference between a continuous concrete mixer and a batch concrete mixer?

(3) Describe, with sketches, the Hennebique system of reinforced-concrete construction.

(4) How may steel in concrete be protected from corrosion?

(5) Sketch and describe a Kahn bar.

(6) Describe the characteristics of a good concrete aggregate.

(7) Sketch and describe the method of reinforcing a concrete beam, fixed at the ends, so as to strengthen the concrete at every point where it is subject to tensile stresses when loaded.

(8) Describe the construction of a form for the erection of a concrete wall.

(9) What are the advantages of mechanical-bond bars?

(10) How should forms be treated to prevent the adhesion of the concrete to the material of the form?

FIRE-RESISTING CONSTRUCTION

EXAMINATION QUESTIONS

(1) Describe the method of supporting an expanded-metal and concrete floor on a brick wall.

(2) Sketch the cross-section of type of bar used in the Columbian system, and describe how the bars are attached to the steel joists.

(3) Why is it bad practice to carry pipes inside the fire-resisting casing of columns?

(4) What is meant by the side method and the end method in terra-cotta flat-arch construction?

(5) Into what two broad divisions may the materials used in fire-resisting construction be divided?

(6) Sketch and describe a steel and concrete mansard roof

(7) For what purpose is fire-resisting glazing used?

(8) Sketch and describe a steel and concrete floor which can be erected without centring.

(9) What is the advantage of subdividing a large building by brick walls?

(10) Sketch and describe the construction of a fire-resisting ceiling suspended from a steel roof truss.

(11) What are the points to be considered in the selection of a fire-resisting floor?

(12) Sketch the best arrangement for protecting a steel stanchion with expanded metal and concrete.

(13) Describe the arrangement of a sprinkler installation

(14) Sketch a terra-cotta segmental arch of the largest size desirable and figure the span.

- (15) Sketch and describe a terra-cotta partition.
- (16) What are the disadvantages of a corrugated-iron arch floor?
- (17) Sketch and describe a floor of terra-cotta lintel construction.
- (18) What are the most suitable aggregates for concrete used in fire-resisting construction?
- (19) Describe an armoured fire-resisting door
- (20) What is porous terra-cotta?

ANSWERS
TO THE
QUESTIONS AND EXAMPLES
INCLUDED IN THE
EXAMINATION QUESTIONS

Each Key, or set of Answers, has the same section number as the corresponding set of Examination Questions. All references are to the portion of the text that bears the same section number as the Key in which the references occur, unless the title of some other part of the text is mentioned.

To be of the greatest value, the Key should be consulted sparingly, much in the same manner as a pupil would go to a teacher for assistance in solving a problem he is unable to master unaided. The Key should not be referred to except as a last resource, when the student has failed, after using every reasonable effort, to solve the example or answer the question. If used in this way, the Key will be a help and a source of encouragement in studying the various subjects comprising the Course.

ARITHMETIC

(PART 1)

- (1) See Art. 1.
- (2) See Art. 6.
- (3) See Arts. 2 and 3.
- (4) See Arts. 4 and 5.
- (5) See Art. 8.
- (6) Seven thousand five hundred and three is expressed in Arabic notation by 7,503; in Roman notation by $\overline{\text{V}}\text{MMDIII}$ (see Arts. 9 to 21).
- (7) See Art. 16.
- (8) (a) Nine hundred and eighty.
- (b) Six hundred and five.
- (c) Twenty-eight thousand two hundred and eighty-four.
- (d) Nine million six thousand and forty-two.
- (e) Eight hundred and fifty million three hundred and seventeen thousand and two.
- (f) Seven hundred thousand and four.
- (9) In the Roman notation,
 - (a) 76 is written LXXVI.
 - (b) 353 „ „ CCCLIII.
 - (c) 1,732 „ „ MDCCXXXII.
 - (d) 1,496 „ „ MCCCXCVI.
 - (e) 1,888 „ „ MDCCCLXXXVIII.
- (10) (a) Seven thousand six hundred is written 7,600.
- (b) Eighty-one thousand four hundred and two is written 81,402.
- (c) Five million four thousand and seven is written 5,004,007.
- (d) One hundred and eight million ten thousand and one is written 108,010,001.
- (e) Ten million and six is written 10,000,006.
- (f) Thirty thousand and ten is written 30,010.

(11)	$ \begin{array}{r} 8\ 3\ 0\ 2\ 7 \\ 4\ 6\ 9\ 2\ 8 \\ 4\ 7\ 6\ 9 \\ 8\ 1\ 9\ 8\ 7 \\ 4\ 6\ 7\ 2\ 9 \\ 4\ 7\ 9\ 8\ 9\ 7 \\ 6\ 2\ 7 \\ 1\ 4\ 8\ 9\ 6 \\ 9\ 8\ 7\ 6\ 4\ 9 \\ \hline \end{array} $	
	<i>sum</i> $1\ 7\ 4\ 6\ 5\ 0\ 9$	Ans.
(12)	<i>multiplicand</i> <i>multiplier</i> <i>1st partial product</i> <i>2nd partial product</i> <i>product</i>	$ \begin{array}{r} 2\ 9\ 8\ 0\ 0 \\ 3\ 9\ 0 \\ \hline 2\ 6\ 8\ 2\ 0\ 0 \\ 8\ 9\ 4\ 0\ 0 \\ \hline 1\ 1\ 6\ 2\ 2\ 0\ 0\ 0 \end{array} $
		Ans.

As the multiplier, 390, ends with a nought, we write it under the multiplicand so that its right-hand digit, 9, comes under the units figure of the multiplicand. (See Art. 52.)

First multiply 29,800 by 9 as follows : 9 times 0 units is 0 units ; write 0 under the units figure of the multiplicand. Again, 9 times 0 tens is 0 tens ; write 0 under the tens figure of the multiplicand. 9 times 8 hundreds is 72 hundreds, or 7 thousands and 2 hundreds ; write the 2 hundreds under the hundreds figure of the multiplicand and reserve the 7 thousands to add to the product of the thousands. Next, 9 times 9 thousands is 81 thousands and this with the 7 thousands reserved comes to 88 thousands or 8 ten-thousands and 8 thousands ; write the 8 thousands under the thousands figure of the multiplicand and reserve the 8 ten-thousands. Lastly, 9 times 2 ten-thousands is 18 ten-thousands, and this with the 8 ten-thousands reserved makes 26 ten-thousands ; write the 6 under the ten-thousands figure of the multiplicand and the 2 to the left of it. This gives for the first partial product 268,200.

Now multiply 29,800 by the 3 of the multiplier in an exactly similar way to that in which the first partial product was obtained, placing the right-hand figure under the tens figure of the multiplicand. The second partial product thus obtained is 89,400.

Add the two partial products together. The resulting sum is 1,162,200 ; to the right of this must be placed one more nought, since the nought in the multiplier 390 was not used in obtaining the partial products.

This gives for the final result 11,622,000. Ans. (See Arts. 48 and 49.)

(13)	$ \begin{array}{r} \text{Amount paid the first year} = \pounds\ 3\ 2\ 9 \\ \text{Amount paid the second year} = \pounds\ 4\ 3\ 8 \\ \hline \text{Whole amount paid} = \pounds\ 7\ 6\ 7 \\ \text{Amount of debt} = \pounds\ 1\ 0\ 0\ 0 \\ \hline \text{Amount paid} = \pounds\ 7\ 6\ 7 \\ \hline \text{Amount still owing} = \pounds\ 2\ 3\ 3 \end{array} $	Ans.
------	--	------

(14) The numbers connected by plus (+) signs must first be added, Thus,

$$\begin{array}{r}
 23896 \\
 4982 \\
 96875 \\
 59674 \\
 \hline
 185427 \text{ sum}
 \end{array}
 \qquad
 \begin{array}{r}
 31627 \\
 54892 \\
 6925 \\
 8976 \\
 \hline
 102420 \text{ sum}
 \end{array}$$

Subtracting the less number (102,420) from the greater (185,427),

$$\begin{array}{r}
 185427 \\
 102420 \\
 \hline
 83007 \text{ Ans.}
 \end{array}$$

(15)

$$\begin{array}{r}
 8765 \\
 987 \\
 \hline
 61355 \\
 70120 \\
 \hline
 78885 \\
 8651055
 \end{array}
 \qquad
 \begin{array}{r}
 4695 \\
 823 \\
 \hline
 14085 \\
 9390 \\
 \hline
 37560 \\
 3863985
 \end{array}$$

To obtain the difference of the products, subtract the second product from the first, thus :

$$\begin{array}{r}
 8651055 \\
 3863985 \\
 \hline
 4787070 \text{ Ans.}
 \end{array}$$

(16) The greater of the two numbers is 1,004, and it is greater than the smaller one by 49.

Hence the smaller number is less than 1,004 by 49; that is, it is equal to 1,004 — 49 or 955,

$$\begin{array}{r}
 \text{greater number} \quad 1004 \\
 \text{difference} \quad \quad 49 \\
 \hline
 \text{less number} \quad \quad 955
 \end{array}$$

$$\begin{array}{r}
 \text{greater number} \quad 1004 \\
 \text{less number} \quad \quad 955 \\
 \hline
 \text{sum of the numbers} \quad 1959 \text{ Ans.}
 \end{array}$$

(17) 36 oxen at £5 each would cost $36 \times £5 = £180$
 23 cows at £19 each would cost $23 \times £19 = £437$

Therefore cost of 36 oxen and 23 cows = £617 Ans.

$$\begin{array}{r}
 36 \qquad 23 \\
 5 \qquad 19 \\
 \hline
 180 \qquad 207 \\
 \qquad 23 \\
 \hline
 \qquad 437
 \end{array}$$

$$(18) \quad 2,937 \times 864 \div 923 = 2,749 \frac{241}{923}. \quad \text{Ans.}$$

$ \begin{array}{r} 2937 \\ \times 864 \\ \hline 11748 \\ 17622 \\ 23496 \\ \hline 2537568 \end{array} $	$ \begin{array}{r} 923 \overline{) 2537568} \quad (2749 \frac{241}{923} \\ \underline{1846} \\ 6915 \\ \underline{6461} \\ 4546 \\ \underline{3692} \\ 8548 \\ \underline{8307} \\ 241 \end{array} $
--	--

$$(19) \quad 8,976 \times 4,298 = 38,578,848.$$

$ \begin{array}{r} 8976 \\ \times 4298 \\ \hline 71808 \\ 80784 \\ 17952 \\ \hline 35904 \\ \hline 38578848 \text{ product} \end{array} $	$ \begin{array}{r} 38578848 \text{ minuent} \\ \underline{98765 \text{ subtrahend}} \\ 38480083 \text{ remainder. Ans.} \end{array} $
--	---

- (20) (a) If the engine and boiler are worth £3,246, and the building is worth 3 times as much plus £1,200, the building

£ 3 246 is worth $£3,246 \times 3 + £1,200 = £9,738 + £1,200$ £ 10 938
 3 = £10,938. 2

£ 9 738	£ 2 18 76
£ 1 200	£ 18 75
£ 10 938	£ 23 75 1

If the tools are worth twice as much as the building plus £1,875, they are worth

$$\begin{aligned}
 &£10,938 \times 2 + £1,875 \\
 &= £21,876 + £1,875 = £23,751. \\
 &\text{Value of building} = £10,938 \\
 &\text{Value of tools} = £23,751
 \end{aligned}$$

Value of building and tools = £ 34 68 9 Ans.

(b) Value of engine and boiler = £ 3 246

Value of building and tools = £ 34 68 9

Value of whole plant = £ 37 93 5 Ans.

- (21) The divisor (1,389) and the quotient (748) are given, and it is required to find the dividend. Place these in the usual

1 389	form of division, 1,389) — (748, in which the dash (—)
748	represents the dividend, or number sought. Leaving the
1 11 12	remainder out of account, the dividend is
5556	evidently the product of 1,389 and 748, which
9723	is 1,038,972. But there is a remainder of
1038972	1,263, so that the actual dividend or number is
	1 040 235

$$1,038,972 + 1,263 = 1,040,235. \quad \text{Ans.}$$

- (22) Arrange the numbers in the usual form of multiplication, thus :

$$\begin{array}{r} 4896 \\ \hline 3862944 \end{array}$$

in which the dash (—) represents the multiplier or number that is to be found. 4,896 multiplied by this number 4896) 3862944 (789 gives the product, or 3,862,944. Then, if we divide 3,862,944 by 4,896, the quotient will be the multiplier required.

$$3,862,944 \div 4,896 = 789. \text{ Ans.}$$

$$\begin{array}{r} 4896) 3862944 (789 \\ 34272 \\ \hline 43574 \\ 39168 \\ \hline 44064 \\ 44064 \\ \hline \end{array}$$

- (23) Amount left to each child = £ 12450
7

$$\text{Amount left to seven children} = £ 87150$$

$$\text{Amount left to widow} = £ 25375$$

$$\text{Amount all received} = £ 112525 \text{ Ans.}$$

- (24) If the farm is sold for £75 an acre instead of £49 an acre, I gain £75 — £49 = £26 on one acre. If the whole amount gained is £6,396, then dividing this total gain on the farm by the amount gained on one acre will give the number of acres.

$$\begin{array}{r} 26) 6396 (246 \\ 52 \\ \hline 119 \\ 104 \\ \hline 156 \\ 156 \\ \hline \end{array}$$

$$6,396 \div 26 = 246$$

Hence the number of acres in the farm is 246. Ans.

- (25) See Arts. 71 and 72.

$$(a) \quad \begin{array}{ccccccc} & 3 & & 16 & & & \\ 21 & & 8 & & 32 & & 3 \\ 21 \times 8 \times 32 \times 3 & = & 3 \times 16 \times 3 & = & 144. & \text{Ans.} \\ 11 \times 21 \times 48 \times 16 \times 7 & = & 1 & & & \end{array}$$

$$(b) \quad \begin{array}{ccccccc} & 3 & 4 & 5 & 2 & 4 & \\ 42 \times 64 \times 125 \times 98 \times 144 & = & 3 \times 4 \times 2 & = & 24. & \text{Ans.} \\ 14 \times 16 \times 25 \times 49 \times 35 \times 20 & = & 1 & & & \end{array}$$

ARITHMETIC

(PART 2)

(1) The denominators are not alike, so before adding, a common denominator for the denominators of all the fractions must be found. Find the least common denominator as in Art. 28. Place the denominators in a row, separated by commas.

Divide by some prime number that will divide at least two of the denominators without a remainder, bringing down to the row below those denominators that will not contain the divisor without a remainder.

$$\begin{array}{r} 2 \overline{) 3, 4, 6, 8, 12} \\ 3 \overline{) 3, 2, 3, 4, 6} \\ 2 \overline{) 1, 2, 1, 4, 2} \\ 1, 1, 1, 2, 1 \end{array}$$

Thus, using 2 as a divisor, the second row becomes 3, 2, 3, 4, 6, the first 3 being brought down as it stands, because 2 will not divide it without a remainder. Using 3 as a divisor the result is 1, 2, 1, 4, 2. Again, using 2 as a divisor, for the third row the result is 1, 1, 2, 1, 1. The product of these numbers and the divisors is $2 \times 3 \times 2 \times 2 = 24$, the least common denominator. All the fractions must now be reduced to 24ths, and their numerators added. $\frac{2}{3} = \frac{16}{24}$, $\frac{3}{4} = \frac{18}{24}$, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, $\frac{11}{12} = \frac{22}{24}$.

To reduce a fraction to a fraction having 24 for the denominator, multiply both terms of the fraction by some number that will make the

denominator 24. Thus, $\frac{2 \times 8}{3 \times 8} = \frac{16}{24}$; $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$; $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$; $\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$; $\frac{11 \times 2}{12 \times 2} = \frac{22}{24}$.

Or divide the least common denominator by the denominator of the given fraction, and multiply both terms by the quotient.

The fractions now have a common denominator, 24; hence, the sum of them can be found by adding the numerators and placing their sum over the common denominator; thus,

$$\frac{16}{24} + \frac{18}{24} + \frac{20}{24} + \frac{21}{24} + \frac{22}{24} = \frac{16 + 18 + 20 + 21 + 22}{24} = \frac{97}{24} = 4\frac{1}{24}. \text{ Ans.}$$

(2) The common denominator is readily seen to be 24.

$$\begin{aligned} 3\frac{3}{4} &= 3\frac{18}{24} \\ 6\frac{7}{8} &= 6\frac{21}{24} \\ 9 + \frac{39}{24} &= 9 + 1\frac{15}{24} = 10\frac{15}{24} \\ 4\frac{5}{6} &= 4\frac{20}{24}; 10\frac{15}{24} - 4\frac{20}{24} = 5\frac{19}{24}. \text{ Ans. See Art. 43.} \end{aligned}$$

(3) Weight of coal bought $12\frac{3}{4} = 12\frac{15}{20}$ tons.
 Weight of coal burned $5\frac{4}{5} = 5\frac{16}{20}$ tons.
 remainder $= 6\frac{19}{20}$ tons.

On reducing the fractions to a common denominator, 20, it is found that $\frac{16}{20}$ in the subtrahend is greater than $\frac{15}{20}$. Borrow 1 from 12 (as in subtraction of whole numbers) calling the 1, $\frac{20}{20}$. $\frac{20}{20} + \frac{15}{20} = \frac{35}{20}$. $\frac{35}{20} - \frac{16}{20} = \frac{19}{20}$. 1 was borrowed from 12; hence, 5 from 11 is 6. The remainder is $6\frac{19}{20}$ tons. Ans.

(4) Pieces of wire $6\frac{2}{3}$ yards long are to be cut from a length of $53\frac{1}{3}$ yards. The number of pieces that can be cut will be equal to the number of times $6\frac{2}{3}$ is contained in $53\frac{1}{3}$.

Therefore, number of short lengths cut

$$= 53\frac{1}{3} \div 6\frac{2}{3} = \frac{160}{3} \div \frac{20}{3} = \frac{160}{3} \times \frac{3}{20} = \frac{160}{20} = 8. \text{ Ans.}$$

113

$$(5) \quad 83\frac{2}{3} \times 67\frac{4}{5} = \frac{251}{3} \times \frac{339}{5} = \frac{251 \times 339}{3 \times 5} = \frac{28363}{5} = 5672\frac{3}{5}. \text{ Ans.}$$

First reduce the mixed numbers (see Art. 12) to improper fractions (see Art. 11), as, $83\frac{2}{3} = \frac{251}{3}$ and $67\frac{4}{5} = \frac{339}{5}$ (see Art. 22). Then, after cancelling, divide the product of the numerators by the product of the denominators.

$$(6) \quad 4\frac{1}{2} \times 6\frac{3}{4} \div 1\frac{4}{5} = \frac{9}{2} \times \frac{27}{4} \div \frac{9}{5} = \frac{9}{2} \times \frac{27}{4} \times \frac{5}{9} = \frac{135}{8} = 16\frac{7}{8}. \text{ Ans.}$$

Reducing the mixed numbers to improper fractions, inverting the divisor (see Art. 54), cancelling and multiplying, the result is $16\frac{7}{8}$.

$12\frac{7}{8}$ (7) If 1 cwt. of steel shafting costs $12\frac{7}{8}$ shillings, 24 cwt. will cost $12\frac{7}{8} \times 24 = 309$ shillings.

21
48
24
309

First multiply $\frac{7}{8}$ by 24. $\frac{7}{8} \times \frac{24}{1} = 21$. Then multiply 12 by 24 in the usual way; add the 21 already obtained to the partial product, getting 309 for the final product. (See Art. 48,—III.)

$$(8) \quad \frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{5}}{1\frac{5}{8} \times 4\frac{2}{5}} = ?$$

Reducing the mixed numbers to improper fractions, the expression becomes

$$\frac{\frac{39}{8} \times \frac{11}{3} \times \frac{16}{5}}{\frac{13}{8} \times \frac{22}{5}}$$

Transfer the denominators of the fractions above the line to below it, and those of the fractions below the line to above it. (See Art. 57.)

$$\frac{\frac{39}{8} \times 11 \times 16 \times 8 \times 5}{13 \times 22 \times 8 \times 3 \times 5} = 8. \text{ Ans.}$$

2

(9) Reducing $8\frac{1}{20}$ to an improper fraction,

$$8\frac{1}{20} = \frac{161}{20}. \text{ Multiplying, } \frac{161}{20} \times \frac{1}{40} = \frac{161}{800}. \text{ Ans.}$$

(10) First add $8\frac{3}{4}$, $7\frac{5}{8}$, and $10\frac{1}{2}$. The common denominator is 12. Then $8\frac{3}{4} = 8\frac{9}{12}$; $7\frac{5}{8} = 7\frac{10}{12}$; $10\frac{1}{2} = 10\frac{6}{12}$.

$$\begin{array}{r} 8\frac{9}{12} \\ 7\frac{10}{12} \\ 10\frac{6}{12} \\ \hline 25\frac{25}{12} = 25 + 2\frac{1}{12} = 27\frac{1}{12} \end{array} \qquad \begin{array}{r} 100 \\ 27\frac{1}{12} \\ \hline 72\frac{11}{12} \end{array}$$

Subtracting $27\frac{1}{12}$ from 100, the result is $72\frac{11}{12}$. Ans.

(11) If in 1 hour $3\frac{3}{4}$ gallons leak away, it will take as many hours for 40 gallons to leak away as the number of times $3\frac{3}{4}$ is contained in 40.

$$3\frac{3}{4} = \frac{15}{4}. \quad \frac{40}{1} \div \frac{15}{4} = \frac{40}{1} \times \frac{4}{15} = \frac{32}{3} = 10\frac{2}{3}$$

Hence, the time required is $10\frac{2}{3}$ hours. Ans.

$$(12) \quad \frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2} = \frac{65}{100} \times \frac{14}{3} \times \frac{13}{2} = \frac{13 \times 7 \times 13}{20 \times 3} = \frac{1,183}{60} = 19\frac{43}{60}. \text{ Ans.}$$

(13) If the diameter of a wheel is $8\frac{2}{3}$ feet and the circumference is $\frac{355}{113}$ times the diameter, the circumference is

$$\frac{26}{3} \times \frac{355}{113} = \frac{9,230}{339} = 27\frac{77}{339} \text{ feet. Ans.}$$

(14) If 1 soldier consumes $1\frac{5}{8}$ pounds, as many soldiers will be required to consume $165\frac{3}{4}$ pounds as the number of times $1\frac{5}{8}$ is contained in $165\frac{3}{4}$. $165\frac{3}{4} = \frac{663}{4}$; $1\frac{5}{8} = \frac{13}{8}$. Dividing,

$$\frac{663}{4} \div \frac{13}{8} = \frac{663}{4} \times \frac{8}{13} = 102$$

Hence, there are 102 soldiers in the company. Ans.

(15) If $4\frac{4}{5}$ cwt. of steel rod cuts up into 96 pieces, 1 cwt. will cut up into $96 \div 4\frac{4}{5}$ pieces.

$$96 \div 4\frac{4}{5} = 96 \div \frac{24}{5} = \frac{96}{1} \times \frac{5}{24} = 20$$

Therefore the number of pieces to the hundredweight = 20. Ans.

$$(16) \quad 4\frac{3}{4} \text{ times } \frac{2}{3} \text{ of } 6\frac{1}{2} = \frac{19}{4} \times \frac{2}{3} \times \frac{13}{2} = \frac{19 \times 2 \times 13}{4 \times 3 \times 2} = \frac{247}{12} = 20\frac{7}{12}. \text{ Ans.}$$

(17) If one ton costs £23 $\frac{3}{4}$, 24 $\frac{3}{4}$ tons will cost £23 $\frac{3}{4} \times 24\frac{3}{4}$.

$$23\frac{3}{4} \times 24\frac{3}{4} = \frac{95}{4} \times \frac{99}{4} = \frac{9405}{16} = 587\frac{13}{16}.$$

Therefore the cost of 24 $\frac{3}{4}$ tons = £587 $\frac{13}{16}$. Ans.

- (18) The first man gets $\frac{1}{3}$ of £3,696 = £1,232. Ans.
 The amount now left = £3,696 - £1,232 = £2,464.
 The second man gets $\frac{2}{5}$ of this; $\frac{2}{5} \times £2,464 = £985\frac{3}{5}$. Ans.
 There is now left £2,464 - £985 $\frac{3}{5}$.
 Therefore the third man's share = £1,478 $\frac{2}{5}$. Ans.

- (19) $9\frac{3}{4}$ acres cost £357 $\frac{1}{2}$.
 Therefore 1 acre costs £357 $\frac{1}{2} \div 9\frac{3}{4}$.

$$357\frac{1}{2} \div 9\frac{3}{4} = \frac{715}{2} \div \frac{39}{4} = \frac{715}{2} \times \frac{4}{39} = \frac{110}{3} = 36\frac{2}{3}$$

Hence the cost per acre was £36 $\frac{2}{3}$. Ans.

- (20) If the sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$, then $296\frac{2}{3}$, or $\frac{890}{3}$, divided by $18\frac{2}{3}$, or $\frac{56}{3}$, is equal to the sum of the numbers.

$$\frac{890}{3} \div \frac{56}{3} = \frac{890}{56} \times \frac{3}{3} = \frac{445}{28} = 15\frac{5}{8}$$

$15\frac{5}{8}$ is the sum of the numbers; if one of the numbers is $12\frac{3}{4}$, then the other is $15\frac{5}{8} - 12\frac{3}{4} = 15\frac{5}{8} - 12\frac{6}{8} = 3\frac{4}{8} = 3\frac{1}{2}$. Ans.

- (21) $12\frac{7}{8}$ times the difference between the numbers = $41\frac{1}{5}$. Therefore the difference

$$= 41\frac{1}{5} \div 12\frac{7}{8} = \frac{206}{5} \div \frac{103}{8} = \frac{206}{5} \times \frac{8}{103} = \frac{16}{5} = 3\frac{1}{5}$$

One of the numbers is 21; since the other is different from it by $3\frac{1}{5}$, the other number must be either $21 + 3\frac{1}{5} = 24\frac{1}{5}$, or $21 - 3\frac{1}{5} = 17\frac{4}{5}$. } Ans.

- (22) When a wheel turns once it moves forward a distance equal to its circumference.

In turning $5\frac{3}{4}$ times, a wheel will therefore move forward a distance equal to $5\frac{3}{4}$ times its circumference.

In this case in $5\frac{3}{4}$ turns the wheel goes 108 ft.

Hence, $5\frac{3}{4}$ times the circumference = 108 ft.; and the circumference = $108 \div 5\frac{3}{4} = 108 \div \frac{23}{4} = \frac{108}{1} \times \frac{4}{23} = \frac{432}{23} = 18\frac{18}{23}$ feet. Ans.

$$(23) \quad \begin{array}{r} 67\frac{5}{8} = 67\frac{15}{24} \\ 48\frac{2}{3} = 48\frac{16}{24} \\ \hline 18\frac{23}{24} \text{ difference. Ans.} \end{array}$$

Reduce $\frac{5}{8}$ and $\frac{2}{3}$ to a common denominator, 24. $\frac{15}{24}$ is greater than $\frac{16}{24}$, so we borrow 1 from 7, calling it $\frac{24}{24}$. $\frac{24}{24} + \frac{15}{24} = \frac{39}{24}$; $\frac{39}{24} - \frac{16}{24} = \frac{23}{24}$. Then 48 from 66 leaves 18.

(24) If the man travelled $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles the next day, and $125\frac{17}{35}$ miles the third day, in the three days he would travel the sum of the three distances.

$$\begin{array}{r} 85\frac{5}{12} = 85\frac{175}{420} \\ 78\frac{9}{15} = 78\frac{252}{420} \\ 125\frac{17}{35} = 125\frac{204}{420} \end{array}$$

$$\text{sum} = 288\frac{631}{420} = 288 + 1\frac{211}{420} = 289\frac{211}{420}$$

Hence, the distance travelled is $289\frac{211}{420}$ miles. Ans.

(25) $211\frac{1}{4}$ pounds at $1\frac{7}{8}$ pence per pound cost

$$211\frac{1}{4} \times 1\frac{7}{8} = \frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32} \text{ pence.}$$

The number of pounds which must have been sold at $2\frac{1}{2}$ pence per pound to bring in $\frac{12,675}{32}$ pence is equal to the number of times $2\frac{1}{2}$ is contained in $\frac{12,675}{32}$.

Therefore the number of pounds resold

$$\begin{array}{c} 2,535 \\ = \frac{12,675}{32} \div 2\frac{1}{2} = \frac{12,675}{32} \times \frac{2}{5} = \frac{2,535}{16} = 158\frac{7}{16} \text{ pounds.} \\ 16 \end{array}$$

The weight still unsold,

$$= 211\frac{1}{4} - 158\frac{7}{16} = 211\frac{4}{16} - 158\frac{7}{16} = 52\frac{13}{16} \text{ pounds. Ans.}$$

ARITHMETIC

(PART 3)

$$(1) \quad 2\frac{1}{2} \times 3\frac{1}{3} \div 8 = \frac{5}{2} \times \frac{10}{3} \times \frac{1}{8} = \frac{25}{24} = 1.0417 \text{ —. Ans.}$$

$$24) 25.00000 (1.04166$$

$$\begin{array}{r} 24 \\ \hline 100 \\ 96 \\ \hline 40 \\ 24 \\ \hline 160 \\ 144 \\ \hline 160 \\ 144 \\ \hline \end{array}$$

Reduce $\frac{25}{24}$ to a decimal as explained in Art. 23, by dividing numerator by denominator.

Divide exactly as in whole numbers, adding to the dividend the ciphers necessary for carrying on the division, and annexing these ciphers one by one to the remainders obtained.

When the division has been carried far enough, the position of the decimal point in the quotient is found as follows:

Count the number of decimal places in the dividend and subtract from this the number of decimal places in the divisor. The number obtained gives the number of decimal places in the quotient.

In the case shown, in order to obtain five decimal figures in the quotient, it has been necessary to add ciphers to the dividend. There are no decimal places in the divisor. Therefore, 5 — 0 or five decimal places are pointed off in the quotient.

This gives 1.04166 . . ., which may be expressed correct to four decimal places as 1.0417 —, the fourth figure being increased by 1 because the fifth figure is greater than 5; to show that the fourth figure has been increased a minus sign is placed after it.

$$(2) \quad 2.0416 = 2 \frac{416}{10000} = 2 \frac{52}{1250} = 2 \frac{26}{625}. \text{ Ans.}$$

The vulgar fraction equivalent to .0416 is formed by taking the figures 416 for the numerator and 1 followed by as many ciphers as there are decimal places (four) for the denominator. See Arts. 29 and 30.

(3) Value of 128 cords of wood at £.8125 per cord

$$= \text{£}8125 \times 128 = \text{£}104.$$

$$.8125$$

$$128$$

$$\hline 65000$$

$$16250$$

Total money value given for coal

$$8125$$

$$= \text{£}104 + \text{£}218 = \text{£}322.$$

$$\hline 1040000$$

To multiply .8125 by 128 proceed as with whole numbers, afterwards pointing off from the right as many decimal places as there are in the multiplicand and multiplier taken together.

If £1·4375 is the value of 1 ton of coal, 1·4375) 322·0000 (224
 £322 must be the value of as many tons
 of coal as the number of times £1·4375 is
 contained in £322.

$$\begin{array}{r} 28750 \\ \hline 34500 \\ 28750 \\ \hline 57500 \\ 57500 \\ \hline \end{array}$$

Hence the quantity of coal received is
 $\frac{322}{1·4375} = 224$ tons. Ans.

The division of 322 by 1·4375 is carried out just as if there were no decimal point. Ciphers are added to the dividend when more figures are necessary to proceed with the division.

In this case four ciphers have to be added to allow the division to terminate. The number of decimal places in the quotient is seen to be 0, for there are four in the dividend and four in the divisor, and $4 - 4 = 0$.

(4) On 100 cases he lost £12·5.

Therefore, on one case he lost $\frac{12·5}{100} = \text{£}125$

But he paid £1·875 for each case.

Hence he must have sold at $\text{£}1·875 - \text{£}125 = \text{£}1·75$ per case. Ans.

(5) To reduce each fraction to a decimal divide the numerator by the denominator. (Arts. 23 and 24.)

4) 3·00

·75

$$\frac{3}{4} = 3 \div 4 = \cdot 75$$

8) 7·000

·875

$$\frac{7}{8} = 7 \div 8 = 875$$

16) 11·0000 (·6875

96

$$\frac{11}{16} = 11 \div 16 = \cdot 6875$$

140

128

120

112

80

$$\frac{17}{32} = 17 \div 32 = \cdot 53125$$

80

$$\text{sum} = 2·84375 \text{ Ans.}$$

32) 17·00000 (·53125

160

100

96

40

32

80

64

160

160

In adding the decimals obtained, place them with the decimal points directly below each other; the decimal point in the sum must be placed below those in the various numbers added. See Arts. 6 to 8, inclusive.

(6) If the diameter of the earth is 7,912 miles and the circumference is 3.1416 times the diameter, the circumference is $7,912 \times 3.1416 = 24,856.3392$ miles. Ans.

7,912 is multiplied by 3.1416 as if both were whole numbers: the number of decimal places in the quotient is the same as that in the multiplicand and multiplier taken together.

$$\begin{array}{r}
 7912 \\
 3.1416 \\
 \hline
 47472 \\
 7912 \\
 31648 \\
 7912 \\
 \hline
 23736 \\
 \hline
 24856.3392
 \end{array}$$

$$(7) \quad \frac{3}{5} \text{ of } \frac{2}{3} \text{ of } .0168 = \frac{3}{5} \times \frac{2}{3} \times .0168 = \frac{2}{5} \times .0168.$$

$$\frac{2}{5} = 2 \div 5 = .4, \quad .4 \times .0168 = .00672. \quad \text{Ans.}$$

Point off as many decimal places in the product as there are in multiplier and multiplicand together.

$$\begin{array}{r}
 .0168 \\
 .4 \\
 \hline
 .00672
 \end{array}$$

(8) .37 of a number plus .23 of the same number is 33.6. That is, $(.37 + .23)$ of the number is 33.6; or .6 of the number is 33.6.

$$\text{Therefore the number is } 33.6 \div .6 = 33.6 \quad .6 \overline{) 33.6} = 56. \quad \text{Ans.}$$

There is one decimal place in the dividend and one in the divisor. In the quotient the number of decimal figures is therefore $1 - 1 = 0$.

(9) .8 of a number minus $\frac{2}{3}$ of the same number is 87.8. That is, $(.8 - \frac{2}{3})$ of the number is 87.8. Putting .8 in the form of a vulgar fraction,

$$\text{And} \quad .8 = \frac{8}{10} = \frac{4}{5} \quad \frac{4}{5} - \frac{2}{3} = \frac{12 - 10}{15} = \frac{2}{15}$$

87.8 $\frac{2}{15}$ of the number is 87.8; hence the number is $87.8 \div \frac{2}{15}$

$$\begin{array}{r}
 87.8 \\
 \hline
 15 \\
 4390 \\
 \hline
 878 \\
 1317.0
 \end{array}
 \quad = 87.8 \times \frac{15}{2} = \frac{1317}{2} = 658.5. \quad \text{Ans.}$$

(10) Since the plot of land cost £3,300, and this was .165 of the cost of the house, the cost of the house must have been $£3,300 \div .165 = £20,000$. Ans.

$$3,300 \div .165 = 3,300 \div \frac{165}{1000} = \frac{20}{165} \times \frac{1000}{1} = £20,000$$

$$(11) \quad \left. \begin{array}{l}
 \text{Paid to butcher} \quad .143 \\
 \text{Paid to grocer} \quad .347 \\
 \text{Paid for clothing} \quad .256 \\
 \text{Other expenses} \quad .154 \\
 \hline
 \text{Total expenses} \quad .900
 \end{array} \right\} \text{ of annual salary.}$$

If he spent $\cdot 9$ of his salary he saved $\cdot 1$ of it. His butcher received £65.494, and this is $\cdot 143$ of his annual salary; his salary must therefore have been $\text{£}65.494 \div \cdot 143$, or £458. He saves $\cdot 1$ of his salary; hence his savings are $\cdot 1$ of £458 = £45.8 £45.8 Ans.

There are no decimal places in the quotient of $65.494 \div \cdot 143$, because there are three decimal places in both dividend and divisor, and $3 - 3 = 0$.

$$\begin{array}{r} 572 \\ 829 \\ \hline 715 \\ 1144 \\ \hline 1144 \end{array}$$

(12) The gain on 1 acre was £22.5 - £17.5 = £5. Since the gain on the whole farm was £734, the total number of acres must be equal to the number of times £5 is contained in £734.

That is, number of acres = $734 \div 5 = 146.8$.

$$\begin{array}{r} 5 \overline{) 734.0} \\ 146.8 \end{array} \quad 146.8 \text{ acres. Ans.}$$

(13) $\cdot 17$ of a certain quantity of oil is 424.5 gallons. Hence the whole quantity is $424.5 \div \cdot 17 = 2,497.06$ gal. Ans.

$\cdot 17 \overline{) 424.50000} (2,497.058 = 2,497.06$ to two places of decimals.

$$\begin{array}{r} 34 \\ \hline 84 \\ 68 \\ \hline 165 \\ 153 \\ \hline 120 \\ 119 \\ \hline 100 \\ 85 \\ \hline 150 \\ 136 \end{array}$$

To carry the figures of the quotient to the point shown in the working, four ciphers have to be added to the dividend, making five decimal places in the dividend; as there are two decimal places in the divisor there will be $5 - 2 = 3$ in the quotient. To express the quotient correct to two places, the second decimal figure must be increased by 1, because the third figure (8) is greater than 5.

(14) If one cubic inch of water weighs $\cdot 03617$ pound, 231 cubic inches will weigh

$$\cdot 03617 \times 231 = 8.355 + \text{pounds. Ans.}$$

There are five decimal places in the multiplicand and none in the multiplier; there will therefore be $5 + 0 = 5$ decimal figures in the product.

$$\begin{array}{r} .03617 \\ 231 \\ \hline 3617 \\ 10851 \\ 7234 \\ \hline 8.35527 \end{array}$$

$$\begin{aligned}
 (15) \quad \frac{3}{16} \text{ of an inch} &= \frac{3}{16} \div 12 \text{ of a foot.} \\
 &= \frac{3}{16} \times \frac{1}{12} = \frac{1}{64} = .015625 \text{ ft. Ans.}
 \end{aligned}$$

$$64) 1.0000000 (.015625)$$

$$\begin{array}{r}
 64 \\
 360 \\
 320 \\
 \hline
 400 \\
 384 \\
 \hline
 160 \\
 128 \\
 \hline
 320 \\
 320 \\
 \hline
 \end{array}$$

In dividing, add the ciphers required until the division terminates (or has been carried far enough). The digits of the quotient are 15625. There are now six decimal places in the dividend and none in the divisor. Hence, there are $6 - 0 = 6$ decimal places in the quotient. A cipher must therefore be prefixed to 15625 in order to obtain the required number of decimal places.

$$\begin{array}{rcl}
 (16) \quad 1 \text{ plus } .001 & = & 1.001000 \\
 .01 \text{ plus } .000001 & = & .010001 \\
 \text{difference} & = & .990999. \text{ Ans.}
 \end{array}$$

See Arts. 8 and 11.

$$\begin{array}{rcl}
 (17) \quad (a) \quad \left(\frac{7}{16} - .13\right) \times (.625 + \frac{5}{8}) & = & ? \\
 \text{Reduce the fractions in each bracket to decimals. } \frac{7}{16} & = & .4375; \frac{5}{8} \\
 = .625. \text{ The expression then becomes} & & \\
 (.4375 - .13) \times (.625 + .625) & & \\
 \text{Performing first the operations indicated inside the brackets, the expres-} & & \\
 \text{sion is simplified to} & & \\
 .3075 \times 1.25 & = & .384375. \text{ Ans. See Arts. 35 and 36.}
 \end{array}$$

$$\begin{array}{rcl}
 (b) \quad \left(\frac{19}{32} \times .21\right) - \left(.02 \times \frac{3}{16}\right) & & \\
 = \left(\frac{19 \times .21}{32}\right) - \left(\frac{.02 \times 3}{16}\right) & & \\
 = \frac{3.99}{32} - \frac{.03}{8} & & \\
 = .1246875 - .00375 & & \\
 = .1209375. \text{ Ans.} & &
 \end{array}$$

The operations inside the brackets must be performed first. The quantities thus obtained from each bracket are then reduced to decimals (see Arts. 23 and 24) and the operation of subtraction indicated between these decimals performed.

$$\begin{array}{rcl}
 (c) \quad \left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} & & \\
 = (3.25 + .013 - 2.17) \times (13.25 - 7.3125) & & \\
 = (3.263 - 2.17) \times 5.9375 & & \\
 = 1.093 \times 5.9375 & & \\
 = 6.4896875 \text{ Ans.} & &
 \end{array}$$

The operations inside the brackets must be performed first. The quantities thus obtained from each bracket are then reduced to decimals (see Arts. 23 and 24) and the operation of subtraction indicated between these decimals performed.

(20) Arrange the numbers to be added beneath each other as in ordinary addition, so that the decimal points are below each other. Then add in the usual way, and place the decimal point in the sum beneath those in the various numbers added. See Arts. 6 to 8.

$$\begin{array}{r}
 5.2467 \\
 17.5346 \\
 26.1347 \\
 .0328 \\
 90.0562 \\
 54.67325 \\
 \hline
 \text{sum } 112.62767
 \end{array}$$

(21) (a) $83.7621 \times 2.3542 = 197.1927$ to four places of decimals. Ans.

$$\begin{array}{r}
 83.7621 \\
 2.3542 \\
 \hline
 1675242 \\
 3350484 \\
 4188105 \\
 2512863 \\
 1675242 \\
 \hline
 19719273582
 \end{array}$$

$197.19273582 = 197.1927$ to four places.

The multiplicand contains four places of decimals and the multiplier four places; $4 + 4 = 8$; eight decimal places are therefore pointed off in the product.

(b) $76.321 \times .06572 = 5.0158$ to four places of decimals. Ans.

$$\begin{array}{r}
 76.321 \\
 .06572 \\
 \hline
 152642 \\
 534247 \\
 381605 \\
 457926 \\
 \hline
 501581612
 \end{array}$$

$5.01581612 = 5.0158$ to four places of decimals.

The multiplicand contains three places of decimals and the multiplier five places; $3 + 5 = 8$; eight decimal places are therefore pointed off in the product.

(c) $7.93016 \times 43.2013 = 342.5932$ to four places of decimals. Ans.

$$\begin{array}{r}
 7.93016 \\
 43.2013 \\
 \hline
 2379048 \\
 793016 \\
 1586032 \\
 2379048 \\
 3172064 \\
 \hline
 342593221208
 \end{array}$$

$342.593221208 = 342.5932$ to four places.

The multiplicand contains five places of decimals and the multiplier four places; $5 + 4 = 9$; nine decimal places are therefore pointed off in the product.

(22)

$$231) 17892.0000 (77.4545 +$$

$$\begin{array}{r}
 1617 \\
 \hline
 1722 \\
 1617 \\
 \hline
 1050 \\
 924 \\
 \hline
 1260 \\
 1155 \\
 \hline
 1050 \\
 924 \\
 \hline
 1260 \\
 1155 \\
 \hline
 105
 \end{array}$$

There are no decimal places in either dividend or divisor to start with. In the process of division, however, ciphers have to be added after the decimal point in the dividend; when the quotient has been carried far enough, the position of the decimal point in it is found by pointing off as many decimal places as there are ciphers after the decimal point in the dividend. As the division is shown, there are four decimal places (or four ciphers) in the dividend, and as there are none in the divisor, there will be $4 - 0$ or four in the quotient.

(23) See Arts. 32 and 33.

$$(a) \quad .7928 = .7928 \times \frac{64}{64} = 50.7392 = \frac{51}{64} \text{ approx. Ans.}$$

$$\begin{array}{r}
 64 \\
 \hline
 31712 \\
 47568 \\
 \hline
 507392
 \end{array}$$

$$(b) \quad .1416 = .1416 \times \frac{32}{32} = 4.5312 = \frac{5}{32} \text{ approx. Ans.}$$

$$\begin{array}{r}
 .1416 \\
 32 \\
 \hline
 2832 \\
 4248 \\
 \hline
 45312
 \end{array}$$

$$(c) \quad .47915 = .47915 \times \frac{16}{16} = \frac{7.66640}{16} = \frac{8}{16} \text{ approx. Ans.}$$

$$\begin{array}{r}
 .47915 \\
 16 \\
 \hline
 287490 \\
 47915 \\
 \hline
 766640
 \end{array}$$

(24) A decimal fraction has 10, 100, 1,000, etc., for its denominator, and this denominator is not written, but understood; the value of any figure in a decimal fraction is determined by its position in the number in question with respect to a point called the decimal point; figures that are one place to the right of the decimal point are tenths, those two places to the right are hundredths, and so on.

On the other hand, a vulgar fraction can have any whole number as its denominator, and the particular denominator used is written below the numerator. See Arts. 1 and 2.

(25) See Art. 3.

17 thousandths	=	.017	
2 tenths	=	.2	
47 millionths	=	<u>.000047</u>	
	<i>sum</i>	.217047	<i>Ans.</i>

ARITHMETIC

(PART 4)

(1) See Arts. 36 to 40.

(a) As there are 20 shillings in 1 pound, £8 is reduced to shillings by multiplying by 20: this gives 160s. Adding 17, the equivalent of £8 17s. is found to be 177s. This is reduced to pence by multiplying by 12; and, on adding to this product the 5d. in the original amount, 2,129 pence is obtained for the equivalent of £8 17s. 5d. in pence.

$$\begin{array}{r}
 \text{£8 17s. 5d.} \\
 \underline{20} \\
 160 \text{ s.} \\
 \underline{17} \\
 177 \text{ s.} \\
 \underline{12} \\
 2124 \text{ d.} \\
 \underline{5}
 \end{array}$$

2129 pence. Ans.

(b) £160 17s. 6d. is to be reduced to half-crowns. 17s. 6d. = $17\frac{1}{2}$ s.;

1 half-crown = 2s. 6d. = $2\frac{1}{2}$ s. Therefore, 17s. 6d. contains $\frac{17\frac{1}{2}}{2\frac{1}{2}} = \frac{35}{5} = 7$

= 7 half-crowns. Also, since £1 = 20s., there must be $\frac{20}{2\frac{1}{2}} = \frac{20}{\frac{5}{2}} = 8$ half-crowns in £1. Therefore, in £160 there are 160×8 half-crowns = 1,280 half-crowns.

In 17s. 6d. there are 7 half-crowns.

Thus, £160 17s. 6d. contains $1,280 + 7 = 1,287$ half-crowns. Ans.

(c) First multiply 7 tons by 20 obtaining 140 cwt., and add the 8 cwt. to the product; this gives 148 cwt.; multiplying by 4 to reduce hundredweights to quarters and adding the 1 qr., 593 qr. is obtained. Finally, multiplying 593 qr. by 28, since there are 28 lb. in 1 qr., and adding the 15 lb. to the product, the final result is found to be 16,619 lb.

$$\begin{array}{r}
 7 \text{ tons 8 cwt. 1 qr. 15 lb.} \\
 \underline{20} \\
 140 \text{ cwt.} \\
 \underline{8} \\
 148 \text{ cwt.} \\
 \underline{4} \\
 592 \text{ qr.} \\
 \underline{1} \\
 593 \text{ qr.} \\
 \underline{28} \\
 4744 \\
 \underline{1186} \\
 16604 \text{ lb.} \\
 \underline{15} \\
 16619 \text{ lb.} \quad \text{Ans.}
 \end{array}$$

(d) There are 4 qt. in a gallon ; to reduce 13 gal. to quarts, therefore, 13 is multiplied by 4, giving 52 qt. ; adding the 3 qt. makes this 55 qt. Since there are 2 pt. in a quart, 55×2 gives 110 pt. 3 qt. 1 pt. this in pints ; and adding the 1 pt. the final result is 111 pt.

$$\begin{array}{r}
 4 \\
 52 \text{ qt.} \\
 3 \\
 55 \text{ qt.} \\
 2 \\
 110 \text{ pt.} \\
 1 \\
 111 \text{ pt.} \quad \text{Ans.}
 \end{array}$$

(2) See Arts. 42 to 44.

(a) Reduction to a higher denomination is effected by division.

To reduce 8,262 pence to shillings, divide by 12, because there are 12 pence in a shilling. $8,262 \text{ pence} \div 12 = 688 \text{ s. and } 6 \text{ d. remainder.}$ Reduce 688s. to pounds by dividing by 20 ; $688 \text{ s.} \div 20 = £34 \text{ and } 8 \text{ s. remainder.}$ The equivalent of 8,262 pence in pounds, shillings, and pence is therefore £34 8s. 6d. Ans.

(b) First bring 81,324 oz. to pounds by dividing by 16.

$$81,324 \text{ oz.} \div 16 = 5,082 \text{ lb.} + 12 \text{ oz.}$$

Next bring 5,082 lb. to quarters by dividing by 28. $5,082 \text{ lb.} \div 28 = 181 \text{ qr.} + 14 \text{ lb.}$

In each case the actual division may be performed in full, if necessary. Finally, dividing 181 qr. by 4, the result is 45 cwt. + 1 qr. Hence, 81,324 oz. is equal to

$$45 \text{ cwt. } 1 \text{ qr. } 14 \text{ lb. } 12 \text{ oz.} \quad \text{Ans.}$$

(c) Reduce inches to feet by dividing by 12, and feet to yards by dividing the result by 3. This gives 15,762 yd. Multiply by 2 to bring this to half-yards ; since 11 half-yd. make one perch, dividing by 11 reduces the half-yards to perches, giving 2,865 per. 9 half-yd. On reducing the 9 half-yd. to yards, feet and inches, the final result is obtained.

$$\begin{array}{r}
 12 \overline{) 567432} \text{ in.} \\
 3 \overline{) 47286} \text{ ft.} + 0 \text{ in.} \\
 15762 \text{ yd.} + 0 \text{ ft.} \\
 2 \\
 11 \overline{) 31524} \text{ half yd.} \\
 2865 \text{ per.} + 9 \text{ half-yd.} \\
 = 2,865 \text{ per.} + 4\frac{1}{2} \text{ yd.} \\
 = 2,865 \text{ per. } 4 \text{ yd. } 1 \text{ ft. } 6 \text{ in.} \quad \text{Ans.}
 \end{array}$$

(d) As there are 2 pints in a quart, dividing 4,876 pt. by 2 reduces it to quarts. Since there are 4 quarts in a gallon, the 2,438 qt. thus obtained divided by 4 gives the number of gallons. $2,438 \text{ qt.} \div 4 = 609 \text{ gal.} + 2 \text{ qt.}$ Next,

$$\begin{array}{r}
 2 \overline{) 4876} \text{ pt.} \\
 4 \overline{) 2438} \text{ qt.} + 0 \text{ pt.} \\
 8 \overline{) 609} \text{ gal.} + 2 \text{ qt.} \\
 76 \text{ bush.} + 1 \text{ gal.}
 \end{array}$$

as there are 8 gal. in a bushel, divide 609 gal. by 8, obtaining 76 bush. + 1 gal. The final result is therefore 76 bush. 1 gal. 2 qt. Ans.

(3) (a) See Arts. 47 to 50.

Write each quantity in a line as shown, keeping like denominations in the same column. Then add each column in the ordinary way obtaining the sum 86 tons 58 cwt. 4 qr. 68 lb. Reducing 68 lb. to quarters gives 2 qr. 12 lb. Adding the 2 qr. to 4 qr. already obtained gives 6 qr., and 6 qr. = 1 cwt. + 2 qr. 1 cwt. added to 58 cwt. makes 59 cwt., and this reduced to tons is 2 tons 19 cwt. 86 tons + 2 tons = 88 tons.

	tons	cwt.	qr.	lb.
as shown, keeping like denominations in the same column. Then	5	16	2	
add each column in the ordinary	30	10	1	20
way obtaining the sum 86 tons	3	14		27
58 cwt. 4 qr. 68 lb. Reducing 68	48	2	1	5
lb. to quarters gives 2 qr. 12 lb.		16		16
Adding the 2 qr. to 4 qr. already	86	58	4	68
obtained gives 6 qr., and 6 qr.	88 tons	19 cwt.	2 qr.	12 lb.
= 1 cwt. + 2 qr. 1 cwt. added to 58 cwt. makes 59 cwt., and this				
reduced to tons is 2 tons 19 cwt. 86 tons + 2 tons = 88 tons.				

(b) Add the column containing halfpence and farthings; $\frac{1}{2}d. + \frac{1}{4}d. + \frac{3}{4}d. + \frac{1}{2}d. = 2 \text{ far.} + 1 \text{ far.}$ 208 15 6 + 3 far. + 2 far. = 8 far. = 2d. This is added to the sum of the pence column; 42d. + 2d. = 44d. = 3s. + 8d. Add the shillings column, obtaining 47s.; this, with the 3s. from the pence column makes 50s. or £2 10s. The sum of the pounds column is £372, which, on adding the £2 from the shillings column, becomes £374. The whole sum is therefore £374 10s. 8d. Ans.

	£	s.	d.
farthings; $\frac{1}{2}d. + \frac{1}{4}d. + \frac{3}{4}d. + \frac{1}{2}d. = 2 \text{ far.} + 1 \text{ far.}$	208	15	6
+ 3 far. + 2 far. = 8 far. = 2d. This is added to	27	13	$7\frac{1}{2}$
the sum of the pence column; 42d. + 2d. = 44d.	16	14	$9\frac{3}{4}$
= 3s. + 8d. Add the shillings column, obtaining 47s.;	86	3	$10\frac{1}{4}$
this, with the 3s. from the pence column makes 50s. or	35	2	$10\frac{1}{2}$
£2 10s. The sum of the pounds column is £372, which,	374	10	8

(4) Arrange the sums of money which are to be added in rows, keeping pounds, shillings, and pence in different columns.

Adding the pence column, there is first $\frac{1}{2}d.$ to bring down; the sum of the pence is 21d. = 1s. 9d. Carry the 1s. to add in the shillings column, making it 19s. The sum of the pounds column is £49. The whole sum is therefore £49 19s. $9\frac{1}{2}d.$

	£	s.	d.
	23	4	10
	11	13	$9\frac{1}{2}$
	15	1	2
sum	49	19	$9\frac{1}{2}$
	£	s.	d.
	114	0	$2\frac{1}{4}$
	49	19	$9\frac{1}{2}$
difference	64	0	$4\frac{3}{4}$ Ans.

This has to be subtracted from £114 0s. $2\frac{1}{4}d.$, and is therefore written underneath it.

$\frac{1}{2}d.$ or 2 far. cannot be subtracted from 1 far., so 1d. or 4 far. is taken from the pence in the minuend, and added to the far., giving 5 far. 5 far. - 2 far. = $\frac{3}{4}d.$, which is written in the remainder. There is now 1d. left in the pence column of the minuend, and 9d. cannot be subtracted from this. There are no shillings in the minuend from which 1s. can be taken to increase the pence, so £1 is taken from the pounds of the minuend, giving 20s. It is now possible to take 1s. or 12d. from this and

add it to the 1d. in the pence of the minuend, making 13d.; 13d. — 9d. = 4d.

The shillings column now has 19s. in both minuend and subtrahend. 19s. — 19s. = 0s. Lastly, the pounds of the minuend have been decreased by £1, leaving £113, and £113 — £49 = £64. The whole difference is therefore £64 0s. 4 $\frac{3}{4}$ d.

(5) (a) To multiply 3 yd. 2 ft. 10 in. by 26, first reduce this length to inches, multiplying the yards by 3 and adding the 2 feet, and the feet thus obtained by 12, and adding the 10 in.

This gives 142 in., which, when multiplied as required by 26, becomes 3,692 in.

Reducing this to higher denominations by dividing by 12 and 3, the resulting product, expressed in yards, feet, and inches, is found to be 102 yd., 1 ft. 8 in. Ans.

$$\begin{array}{r}
 3 \text{ yd. } 2 \text{ ft. } 10 \text{ in.} \\
 \underline{3} \\
 9 \text{ ft.} \\
 \underline{2} \\
 11 \text{ ft.} \\
 \underline{12} \\
 132 \text{ in.} \\
 \underline{10} \\
 142 \text{ in.} \\
 \underline{26} \\
 852 \\
 \underline{284} \\
 12) \underline{3692} \text{ in., product} \\
 3) \underline{307} \text{ ft. } + 8 \text{ in.} \\
 102 \text{ yd. } + 1 \text{ ft.} \\
 \underline{\text{£}6 \text{ } 3\text{s. } 2\frac{1}{2}\text{d.}} \\
 \underline{20} \\
 120 \text{ s.} \\
 \underline{3} \\
 123 \text{ s.} \\
 \underline{12} \\
 1476 \text{ d.} \\
 \underline{2\frac{1}{2}} \\
 1478\frac{1}{2} \text{ d.} \\
 \underline{2} \\
 2957 \text{ halfpence} \\
 \underline{29} \\
 26613 \\
 \underline{5914} \\
 2) \underline{85753} \text{ halfpence} \\
 12) \underline{42876}\frac{1}{2} \text{ d.} \\
 20) \underline{3573\text{s. } + 0\frac{1}{2} \text{ d.}} \\
 \underline{\text{£}178 + 13 \text{ s.}}
 \end{array}$$

(c) 7 tons 8 cwt. 1 qr. 11 lb.

$$\begin{array}{r}
 63 \\
 \hline
 467 \text{ tons } 5 \text{ cwt. } 3 \text{ qr. } 21 \text{ lb. } \text{Ans.}
 \end{array}$$

Multiplying 11 lb. by 63, $11 \times 63 = 693$ lb. Since there are 28 lb. in a quarter, $693 \div 28 = 24$ qr. + 21 lb. Write the 21 lb. in the product, and reserve the 24 qr. to add to the product of the quarters. 1 qr. $\times 63 = 63$ qr.; 63 qr. + 24 qr. = 87 qr. There are 4 qr. in a hundredweight, therefore 87 qr. = 21 cwt. 3 qr. Writing the 3 qr. in the product, add 21 cwt. to the product of the hundredweights. $(8 \text{ cwt.} \times 63) + 21 \text{ cwt.} = 504 \text{ cwt.} + 21 \text{ cwt.} = 525 \text{ cwt.}$ Since there are 20 cwt. in one ton, $525 \text{ cwt.} = 525 \div 20 = 26$ tons 5 cwt.

Finally multiplying 7 tons by 63 obtain 441 tons; and adding to this the 26 tons already found, gives 467 tons.

The whole product is therefore 467 tons 5 cwt. 3 qr. 21 lb.

This example may also be solved by reducing the whole quantity to pounds, multiplying by 63, and reducing the product obtained back to higher denominations.

(6) (a)

	£	s.	d.
12)	690	4	2.5
	57	10	4.208

See Art. 61.

First divide £690 by 12, obtaining £57 and £6 or $6 \times 20 = 120$ shillings remainder. Adding to this the 4s. in the dividend, 124s. is obtained, and this, divided by 12, gives 10s. and 4s. or 48d. remainder. Add this remainder to the 2.5d. in the dividend, obtaining 50.5d.; this, divided by 12, becomes 4.208d. The whole quotient, when the decimals are limited to two places, is therefore £57 4s. 4.21d., nearly. Ans.

Divide by long division as explained in 42) £690 4s. $2\frac{1}{2}$ d. (£16 Arts. 62 and 63.

First divide 42 into £690, getting a quotient of £16 and a remainder of £18. Reduce this remainder to shillings, adding the 4s. in the dividend.

Next divide the 364s. thus obtained by 42; the quotient is 8s. and the remainder 28s. The remainder is reduced to pence, and at the same time the $2\frac{1}{2}$ d. or 2.5d. from the dividend is added, giving 338.5d. Divide this 338.5d. by 42, obtaining 8.059 or 8.06 pence for the quotient.

The whole quotient is found by taking the 3 quotients already obtained, and is therefore £16 8s. 8.06d., nearly. Ans.

	42	
	270	
	252	
£	18	remainder
	20	
42)	364	s. (8 s.
	336	
	28	s. remainder
	12	
42)	338.500	d. (8.059 d.
	336	
	250	
	210	
	400	
	378	

$$\begin{array}{r}
 (b) \quad 330 \overline{) 3000} \text{ tons } 15 \text{ cwt. } 2 \text{ qr. } 16 \text{ lb. (9 tons} \\
 \underline{2970} \\
 30 \text{ tons} \\
 20 \\
 \hline
 330 \overline{) 615} \text{ cwt. (1 cwt.} \\
 \underline{330} \\
 285 \text{ cwt.} \\
 4 \\
 \hline
 330 \overline{) 1142} \text{ qr. (3 qr.} \\
 \underline{990} \\
 152 \text{ qr.} \\
 28 \\
 \hline
 1216 \\
 304 \\
 \hline
 16 \\
 330 \overline{) 4272} \text{ lb. (12 lb.} \\
 \underline{330} \\
 972 \\
 \underline{660} \\
 312 \text{ remainder}
 \end{array}$$

First divide 3,000 tons by 330, obtaining 9 tons as quotient and 30 tons as remainder. Reduce the 30 tons to hundredweights, adding the 15 cwt. in the dividend. Divide 615 cwt. thus obtained by 330; the quotient is 1 cwt., and there is a remainder of 285 cwt. Reduce this remainder to quarters, adding the 2 qr. in the dividend, and obtaining 1,142 qr. Divide this by 330, and obtain 3 qr. for the quotient and a remainder of 152 qr. Reduce this to pounds, and add the 16 lb. in the dividend, the result being 4,272 lb. Divide 4,272 lb. by 330, obtaining a quotient of 12 lb. with 312 lb. remainder; express this remainder as a fraction of a pound, thus obtaining a quotient of $12\frac{312}{330}$ or $12\frac{156}{165}$ lb.

The whole quotient is obtained by taking the separate quotients already found, and is therefore 9 tons 1 cwt. 3 qr. $12\frac{156}{108}$ lb. Ans.

$$108 \overline{) 3000 \text{ tons } 15 \text{ cwt. } 2 \text{ qr. } 16 \text{ lb. } (27 \text{ tons}$$

$$\begin{array}{r} 216 \\ \hline 840 \\ 756 \\ \hline 84 \text{ tons} \\ 20 \end{array}$$

$$108 \overline{) 1695 \text{ cwt. } (15 \text{ cwt.}$$

$$\begin{array}{r} 108 \\ \hline 615 \\ 540 \\ \hline 75 \text{ cwt.} \\ 4 \end{array}$$

$$108 \overline{) 302 \text{ qr. } (2 \text{ qr.}$$

$$\begin{array}{r} 216 \\ \hline 86 \text{ qr.} \\ 28 \\ \hline 688 \\ 172 \\ 16 \end{array}$$

$$108 \overline{) 2424 \text{ lb. } (22\frac{48}{108} = 22\frac{4}{9} \text{ lb.}$$

$$\begin{array}{r} 216 \\ \hline 264 \\ 216 \\ \hline 48 \text{ lb. remainder} \end{array}$$

Divide 3,000 tons by 108, and reduce the remainder to hundredweights, adding the 15 cwt. in the dividend and obtaining 1,695 cwt. Divide this by 108, and reduce the remainder to quarters, adding the 2 qr. in the dividend. This gives 302 qr. Dividing again by 108, obtain a remainder of 86 qr., which on reducing to pounds, and adding 16 lb. in the dividend, becomes 2,424 lb.

Lastly, divide this by 108, obtaining 48 lb. remainder. This remainder expressed as a fraction is $\frac{48}{108}$ or $\frac{4}{9}$ lb.

Take the quotients found in each division and get for the whole quotient 27 tons 15 cwt. 2 qr. $22\frac{4}{9}$ lb. Ans.

(7) The area of the second passage, which is 28 yd. long and $\frac{3}{4}$ yd. wide, is $28 \times \frac{3}{4} = 21$ sq. yd. The area of the first passage is the same as this, and its width is $\frac{7}{8}$ yd.; therefore, the length of the first passage must be $21 \div \frac{7}{8} = 21 \times \frac{8}{7} = 24$ yd. Ans.

(8) The contents of the block is the product of its length, breadth, and thickness, thus :

$$6 \text{ ft.} \times 4\frac{1}{2} \text{ ft.} \times 3 \text{ ft.} = 3 \times 4\frac{1}{2} \times 6 = 3 \times \frac{9}{2} \times 6 = 81 \text{ cu. ft.}$$

1 cu. ft. costs 4s. $7\frac{1}{2}$ d.

Therefore, 81 cu. ft. costs 4s. $7\frac{1}{2}$ d. $\times 81 = 55\cdot5$ d. $\times 81 = 4495\cdot5$ d.

$$4 \text{ s. } 7\frac{1}{2} \text{ d.} \qquad = \text{£}18 \text{ 14s. } 7\frac{1}{2} \text{ d.} \quad \text{Ans.}$$

$$\begin{array}{r} 12 \\ 48 \text{ d.} \\ \hline 7\cdot5 \\ 55\cdot5 \text{ d.} \\ \hline 81 \\ 555 \\ \hline 4440 \\ 12 \overline{) 4495\cdot5 \text{ d.}} \\ 20 \overline{) 374 \text{ s.} + 7\frac{1}{2} \text{ d.}} \\ \hline \text{£}18 + 14 \text{ s.} \end{array}$$

Reduce 4s. $7\frac{1}{2}$ d. to pence before multiplying, obtaining 55·5d. Multiply this by 81, getting 4495·5d.

Reduce this to higher denominations by dividing by 12 and 20. The final quotient and the two remainders give the result, £18 14s. $7\frac{1}{2}$ d. Ans.

(9) The circumference of a wheel is 3·1416 times its diameter. Hence the circumference of a wheel whose diameter is 12 ft. is $12 \times 3\cdot1416$, or 37·6992.

2 ft. 8 in. is equal to $2\frac{8}{12}$ or $2\frac{2}{3}$ ft., and this must be divided into 37·6992 ft.

$$\begin{aligned} & 37\cdot6992 \div 2\frac{2}{3} \\ &= 37\cdot6992 \div \frac{8}{3} = 37\cdot6992 \times \frac{3}{8} \\ &= \frac{113\cdot0976}{8} = 14\cdot1372 \end{aligned}$$

Therefore, 2 ft. 8 in. must be repeated 14·1372 times. Ans.

(10) Contents of cubical cistern of 8 ft. 6 in., or 102 in., side

$$= 102 \times 102 \times 102 \text{ cu. in.}$$

$$= 1,061,208 \text{ cu. in.}$$

277·463 cu. in. is equivalent to 1 gal.

Therefore, 1,061,208 cu. in. is equivalent to $\frac{1061208}{277\cdot463} = 3,824\cdot68$ gal., nearly
Ans.

$$\begin{array}{r} (11) \quad 8\cdot75 \\ 80 \\ \hline 700\cdot00 \end{array}$$

$$\begin{array}{r} 16 \overline{) 700} (43\frac{3}{4} \\ 64 \\ \hline 60 \\ 48 \\ \hline 12 \\ 16 \\ \hline \end{array} = 43\frac{3}{4}$$

If they used $8\frac{3}{4}$ oz. per day, in 80 days they would use $80 \times 8\cdot75 = 700$ oz. In 1 lb. there are 16 oz., therefore in 700 oz. there are as many pounds as the number of times 16 is contained in 700, or $43\frac{3}{4}$ lb. Ans.

(12) 25 ft. square is an area 25 ft. long and 25 ft. wide, or $(25 \times 25) = 625$ sq. ft. $625 - 75 = 550$ sq. ft. Ans.

(13) Since the volume is the product of the length, width, and thickness, it follows that the width is equal to the volume divided by the product of the length and thickness. $10 \text{ in.} = \frac{5}{8} \text{ ft.}$ $40 \div (36 \times \frac{5}{8}) = 40 \div 30 = \frac{4}{3}$. The width is therefore $\frac{4}{3} \text{ ft.} = \frac{4}{3} \times 12 \text{ in.} = 16 \text{ in.}$ Ans.

(14) (a) The area of the passage-way is the product of its length and width.

$$\text{Length} = 18.75 \text{ ft.} = 18\frac{75}{100} \text{ ft.} = 18\frac{3}{4} \text{ ft.}$$

$$\text{Width} = 2 \text{ ft. } 10 \text{ in.} = 2\frac{10}{12} \text{ ft.} = 2\frac{5}{6} \text{ ft.}$$

$$\text{Area} = 18\frac{3}{4} \text{ ft.} \times 2\frac{5}{6} \text{ ft.} = \frac{25}{4} \times \frac{17}{6} \text{ sq. ft.}$$

$$= \frac{425}{8} = 53.125 \text{ sq. ft.} \quad \text{Ans.}$$

(b) If 1 sq. ft. costs 4d., 53.125 sq. ft. will cost $53.125 \times 4 = 212.5\text{d.}$

$$12 \overline{) 212.5 \text{ d.}}$$

$$17\text{s. } 8\frac{1}{2} \text{ d.}$$

$$= 17\text{s. } 8\frac{1}{2} \text{ d.} \quad \text{Ans.}$$

(15) Contents of bin = 8 ft. $\times 6\frac{1}{2}$ ft. $\times 6$ ft.

$$= 8 \times \frac{13}{2} \times 6 = 312 \text{ cu. ft.}$$

$$1728$$

$$\underline{312}$$

$$3456$$

$$1728$$

$$\underline{5184}$$

$$539136$$

There are 1,728 cu. in. in 1 cu. ft., therefore 312 cu. ft. = $312 \times 1,728 \text{ cu. in.} = 539,136 \text{ cu. in.}$ Again, 1 bush. = 8 gal., and 1 gal. = 277.463 cu. in. Hence, 1 bush. = 277.463 $\times 8 = 2,219.704 \text{ cu. in.}$ The bin contains 539,136 cu. in., and 1 bush. contains 2,219.704 cu. in. Hence the bin must contain $539,136 \div 2,219.704 = 242.88 + = 242.9$ bush., nearly.

Ans.

(16) 63 gal.

$$\underline{4}$$

$$252 \text{ qt.}$$

$$\underline{2}$$

$$504 \text{ pt.}$$

18 gal. 3 qt. 1 pt.

$$\underline{4}$$

$$75 \text{ qt.}$$

$$\swarrow \underline{2}$$

$$151 \text{ pt.}$$

Reducing each quantity to pints, 63 gal. = 504 pt., and 18 gal. 3 qt. 1 pt. = 151 pt. Art. 39.

(a) The fractional part that 18 gal. 3 qt. 1 pt. is of 63 gal. is therefore $\frac{151}{504}$. Ans.

(b) The decimal part that 18 gal. 3 qt. 1 pt. is of 63 gal. is $151 \div 504 = .2996 +$. Ans.

(17) If the tank is 3 m. $\times$ 3 m. $\times$ 2 m., its volume is 18 cu. m. 1 cu. m. = 1,000 cu. dm.; therefore, 18 cu. m. = $18 \times 1,000 \text{ cu. dm.} = 18,000 \text{ cu. dm.}$ (Art. 80.) Since 1 cu. dm. of water weighs 1 Kg. (see Art. 86), 18,000 cu. dm. weighs 18,000 Kg. Ans.

(18) Since 1 m. = 3.281 ft. (see Art. 67), $56.8 \text{ m.} = 56.8 \times 3.281$
 $= 186.36 \text{ ft.}; \frac{186.36}{3} = 62.1 \text{ yd.}$ Ans.

(19) Contents of the block is $(3.75 \times 2.8 \times 2.5) \text{ cu. m.} = 26.25$
 cu. m. One cubic metre = 35.3148 cu. ft. (see Art. 80); then, 26.25 cu. m.
 $= 26.25 \times 35.3148 = 927.014 \text{ cu. ft.}$ If 1 cu. ft. weighs 168.75 lb., 927.014
 cu. ft. weighs $927.014 \times 168.75 = 156,433.6$, or say 156,434 lb. Ans.

(20) Write the numbers as for addition, with units of like denom-
 ination under each other. Since 6d. cannot be subtracted from
 3d., 1s. (= 12d.) is borrowed from the column of
 £ s. d. shillings. $12 + 3 = 15\text{d.}$ Then, $15\text{d.} - 6\text{d.} = 9\text{d.}$

£	s.	d.	shillings.
21	5	3	Since 1 shilling was borrowed from 5s., only 4s.
13	9	6	remain; and as 9s. cannot be subtracted from 4s.,
£7	15 s.	9 d.	£1 (= 20s.) is borrowed from the £ column, reduced

 to shillings and added to 4s., giving 24s.; then,
 $24\text{s.} - 9\text{s.} = 15\text{s.}$ Finally, $£20 - £13 = £7$; and the final remainder
 is £7 15s. 9d. Ans.

(21) There are 25 li. in a rod (see Art. 13); therefore, to reduce links to
 rods, 4,763,254 li. is divided by 25,
 the result being 190,530 rd. and 4 li.
 remainder.

25)	4763254	links
4)	190530	rd. + 4 li.
80)	47632	ch. + 2 rd.
	595	mi. + 32 ch.

To reduce rods to chains, divide by 4; as there are 4 rods in a chain;
 and obtain 47,632 ch. and 2 rd. remainder. Lastly, to reduce chains to
 miles, since there are 80 ch. in a mile, divide 47,632 ch. by 80, obtaining
 a quotient of 595 mi. and a remainder of 32 ch.

The whole result is found by taking the last quotient (595 mi.), and
 all the remainders, and it is therefore 595 mi. 32 ch. 2 rd. 4 li. Ans.

(22) 1 rail is 17 ft. 3 in. long; therefore, the length of 51 rails would
 be $51 \times (17 \text{ ft. } 3 \text{ in.})$. $17 \text{ ft. } 3 \text{ in.} = 204 \text{ in.} + 3 \text{ in.} = 207 \text{ in.}$ 207 in.
 $\times 51 = 10,557 \text{ in.} = \text{length of 51 rails.}$ Reduce this length in inches
 to higher denominations. First divide 10,557 in. by 12, because there
 are 12 in. in a foot; the result is 879 ft. 12 (10557 in.
 and 9 in. remainder. There are 3 ft. in a
 yard; therefore, to reduce 879 ft. to
 yards, divide by 3, obtaining 293 yd. and
 no remainder. To reduce yards to rods:
 since there are $5\frac{1}{2}$ yd. in a rod, there are
 11 half-yards in a rod. First multiply
 293 yd. by 2 to reduce it to half-yards,
 and then divide this by 11 to reduce half-yards to rods. The result is
 53 rd. and 3 half-yd., or $1\frac{1}{2}$ yd. remainder. Reducing $\frac{1}{2}$ yd. to feet and
 inches, the total length is 53 rd. 1 yd. 2 ft. 3 in. Ans.

3)	879	ft. + 9 in.
	293	yd. + 0 ft.
11)	586	half-yd.
	53	rd. + 3 half-yd.
	= 53	rd. + $1\frac{1}{2}$ yd.

$$(23) \quad \begin{array}{r} 7 \text{ tons} \quad 15 \text{ cwt.} \quad 10\cdot5 \text{ lb.} \\ 20 \\ \hline 155 \text{ cwt.} \\ 112 \\ \hline 310 \\ 155 \\ \hline 155 \\ 10\cdot5 \\ \hline 17370\cdot5 \text{ lb.} \\ 1\cdot7 \\ \hline 1215935 \\ 173705 \\ \hline 112 \cdot 29529\cdot85 \text{ lb. (263 cwt.} \end{array}$$

$$\begin{array}{r} 20 \\ \hline 155 \text{ cwt.} \\ 112 \\ \hline 310 \\ 155 \\ \hline 155 \\ 10\cdot5 \\ \hline 17370\cdot5 \text{ lb.} \end{array}$$

$$112$$

$$310$$

$$155$$

$$155$$

$$10\cdot5$$

$$17370\cdot5 \text{ lb.}$$

$$1\cdot7$$

$$1215935$$

$$173705$$

$$112 \cdot 29529\cdot85 \text{ lb. (263 cwt.}$$

$$224$$

$$712$$

$$672$$

$$409$$

$$336$$

$$73\cdot85 \text{ lb. remainder}$$

$$20 \cdot 263 \text{ cwt.}$$

$$13 \text{ tons 3 cwt.}$$

Before multiplying by 1·7, reduce 7 tons 15 cwt. 10·5 lb. to pounds. There are 20 cwt. in a ton; therefore multiply 7 tons by 20 and at the same time add 15 cwt. to the product.

The result is 155 cwt. Next, since there are 112 lb. in 1 cwt., multiply 155 cwt. by 112, adding 10·5 lb. This gives 17,370·5 lb. for the original weight expressed in pounds.

Now multiply by 1·7, obtaining 29,529·85 lb. To express this in tons, cwt., etc., perform ascending reduction, dividing first by 112 and obtaining a quotient of 263 cwt. and a remainder of 73·85 lb. Reduce 263 cwt. to 13 tons 3 cwt. by dividing by 20. The final result is 13 tons 3 cwt. 73·85 lb. Ans.

$$\begin{array}{r} \text{ft. in.} \\ (24) \quad 20 \text{ rd.} = 20 \times 5\frac{1}{2} \text{ yd.} = 20 \times \frac{11}{2} \times 3 \text{ ft.,} \quad 362 \quad 9 \\ \text{since there are } 5\frac{1}{2} \text{ yd. in 1 rd., and 3 ft. in 1 yd.} \quad 300 \quad 0 \\ \hline 62 \quad 9 \end{array}$$

$$10 \quad 20 \times \frac{11}{2} \times 3 = 330 \text{ ft. } 330 \text{ ft.} + 32 \text{ ft.} = 362 \text{ ft.}$$

The whole minuend is therefore 362 ft. 9 in. Subtracting 300 ft. from this, the remainder is 62 ft. 9 in. Reduce 62 ft. 9 in. to higher denominations. First divide by 3, obtaining 20 yd. and 2 ft. remainder. Reduce 20 yd. to half-yards by multiplying by 2, and divide the 40 half-yd., thus obtained, by 11, as there are 11 half-yd. in a rod.

The result of this is 3 rd. + 7 half-yd. or 3 rd. $3\frac{1}{2}$ yd.

Reserving the 3 rd., $3\frac{1}{2}$ yd. = $\frac{7}{2} \times 3$ ft. = $10\frac{1}{2}$ ft. = 10 ft. 6 in. Add to this the remainders, 2 ft. 9 in., already obtained, the result being 13 ft. 3 in. This, with the 3 rd. reserved, gives the final result 3 rd. 13 ft. 3 in.

$$3 \overline{) 62 \text{ ft. } 9 \text{ in.}}$$

$$20 \text{ yd. } + 2 \text{ ft.}$$

$$2$$

$$11 \overline{) 40 \text{ half-yd.}}$$

$$3 \text{ rd. } + 7 \text{ half-yd.}$$

$$= 3 \text{ rd. } + 3\frac{1}{2} \text{ yd.}$$

Ans.

$$(25) \quad 7 \overline{) 358 \text{ ac. } 57 \text{ sq. rd. } 6 \text{ sq. yd. } 2 \text{ sq. ft.}}$$

$$51 \text{ ac. } 31 \text{ sq. rd. } 0 \text{ sq. yd. } 8 \text{ sq. ft.}$$

Begin with the highest denomination and divide each term in succession by 7.

Dividing 358 ac. by 7, the quotient is 51 ac. and the remainder 1 ac. 1 ac. = 160 sq. rd. Adding to this the 57 sq. rd. in the dividend, gives 217 sq. rd. Divide 217 sq. rd. by 7; the result is 31 sq. rd. and there is no remainder. Next, as 6 sq. yd. cannot be divided by 7, reduce it to square feet. There are 9 sq. ft. in a square yard; therefore, 6 sq. yd. = $9 \times 6 = 54$ sq. ft.; this, when the 2 sq. ft. in the dividend is added, becomes 56 sq. ft. Lastly, dividing the 56 sq. ft. by 7, 8 sq. ft. is obtained.

The whole quotient consists of all the separate quotients obtained, and is therefore 51 ac. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

ARITHMETIC

(PART 5)

(1) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power, or square, of 108 is $108 \times 108 = 11,664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 1080 \\ \hline 11664 \end{array}$$

(2) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59,049$. Ans.

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 \\ 9 \\ \hline 729 \\ 9 \\ \hline 6561 \\ 9 \\ \hline 59049 \end{array}$$

(3) $\cdot 0133^3 = \cdot 0133 \times \cdot 0133 \times \cdot 0133 = \cdot 000002352637$. Ans.

Since there are four decimal places in the multiplicand and four in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product ;

but as there are only five figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

$$\begin{array}{r} \cdot 0133 \\ \cdot 0133 \\ \hline 399 \\ 399 \\ \hline 133 \\ \cdot 00017689 \\ \cdot 0133 \\ \hline 53067 \\ 53067 \\ \hline 17689 \\ \hline \cdot 000002352637 \end{array}$$

Since there are eight decimal places in the multiplicand and four in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product ; but as there are only seven figures in the product, we prefix five ciphers to make the twelve necessary decimal places in the final product.

(4) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(5) See page 9 of the table.

$$\begin{array}{rcl}
 9 \cdot 49^2 & = & 90 \cdot 0601 \\
 9 \cdot 48^2 & = & 89 \cdot 8704 \\
 \text{first difference} & = & 1897 \\
 1897) 1296000 & (& 683 \text{ or } 68 \\
 \underline{11382} & & \\
 15780 & & \\
 \underline{15176} & & \\
 6040 & & \\
 \underline{5691} & & \\
 349 & &
 \end{array}
 \qquad
 \begin{array}{rcl}
 \text{given number} & = & 90 \cdot 0000 \\
 9 \cdot 48^2 & = & 89 \cdot 8704 \\
 \text{second difference} & = & 1296
 \end{array}$$

The first three figures of the root are found opposite the smaller number, and are 948; the next two figures are those just obtained by division. Since there is only one integral period in the original number, there will be only one figure preceding the decimal point in the root.

Therefore, $\sqrt{90} = 9 \cdot 4868$. Ans.

(6) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$(3\frac{3}{4})^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3375}{64} = 52\frac{47}{64} = 52 \cdot 734375. \quad \text{Ans.}$$

$$\begin{array}{rcl}
 3\frac{3}{4} & = & \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4} \\
 \begin{array}{r}
 15 \\
 \underline{15} \\
 75 \\
 \underline{15} \\
 225 \\
 \underline{15} \\
 1125 \\
 \underline{225} \\
 3375
 \end{array} & & \begin{array}{r}
 64) 3375 (52\frac{47}{64} \\
 \underline{320} \\
 175 \\
 \underline{128} \\
 47
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 64 \overline{) 47.000000} \{ .734375 \\
 \underline{448} \\
 220 \\
 \underline{192} \\
 280 \\
 \underline{256} \\
 240 \\
 \underline{192} \\
 480 \\
 \underline{448} \\
 320 \\
 \underline{320} \\
 0
 \end{array}$$

Since six ciphers were annexed to the dividend, and there are no ciphers in the divisor, six decimal places must be pointed off in the quotient.

(7) Given number = 92,416 = 92'416.

Altered number = 92.416.

See page 4 of the table.

$$\begin{array}{rcl}
 4.53^3 = 92.9597 & & \text{altered number} = 92.4160 \\
 4.52^3 = 92.3454 & & 4.52^3 = 92.3454 \\
 \text{first difference} = \underline{6143} & & \text{second difference} = \underline{706} \\
 6143 \overline{) 706.0000} \{ .114 \text{ or } .11 \\
 \underline{6143} \\
 9170 \\
 \underline{6143} \\
 30270 \\
 \underline{24572} \\
 5698
 \end{array}$$

Therefore, $\sqrt[3]{92.416} = 4.5211$, and $\sqrt[3]{92,416} = 45.211$. Ans.

(8) Given number = 502,681 = 50'2681.

Altered number = 50.2681.

Referring to page 7 of the table, $7.09^2 = 50.2681$; hence, $\sqrt{50.2681} = 7.09$. Since there are three periods in the integral part of the given number, there are three figures in the integral part of the root; therefore, $\sqrt{502,681} = 709$. Ans.

$$(9) \quad \sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}. \quad \text{Ans.}$$

$$(10) \quad 4^3 = 4 \times 4 \times 4 = 64.$$

$$\sqrt[3]{8} = 2.$$

$$4^3 - \sqrt[3]{8} = 64 - 2 = 62. \quad \text{Ans.}$$

(11) Since $\frac{3}{8} = .375$, $\sqrt[3]{\frac{3}{8}} = \sqrt[3]{.375}$.

Given number = .375.

Altered number = 375.000.

See page 7 of the table.

7.22 ³ = 3 7 6.3 6 7	altered number = 3 7 5.0 0 0
7.21 ³ = 3 7 4.8 0 5	" 7.21 ³ = 3 7 4.8 0 5
first difference = 1 5 6 2	second difference = 1 9 5
1 5 6 2) 1 9 5.0 0 0 (.1 2 4 or .1 2	
1 5 6 2 3 8 8 0 3 1 2 4 7 5 6 0 6 2 4 8 1 3 1 2 3 8 8 0 3 1 2 4 7 5 6 0 6 2 4 8 1 3 1 2	

Therefore, $\sqrt[3]{375} = 7.2112$, and $\sqrt[3]{.375} = \sqrt[3]{\frac{3}{8}} = .72112$. Ans.

(12) Given number = .3364 = .33'64.

Altered number = 33.6400.

Referring to page 5 of the table, $5.80^2 = 33.6400$; therefore, $\sqrt{33.6400} = 5.80$; and since in the original number there are no cipher periods following the decimal point, the decimal point will precede the first figure of the root. Therefore, $\sqrt{.3364} = .58$. Ans.

(13) Given number = .7854 = .78'54.

Altered number = 78.5400.

See page 8 of the table.

8.87 ² = 7 8.6 7 6 9	altered number = 7 8.5 4 0 0
8.86 ² = 7 8.4 9 9 6	8.86 ² = 7 8.4 9 9 6
first difference = 1 7 7 3	second difference = 4 0 4
1 7 7 3) 4 0 4.0 0 0 (.2 2 7 or .2 3	
3 5 4 6 4 9 4 0 3 5 4 6 1 3 9 4 0 1 2 4 1 1 1 5 2 9 3 5 4 6 4 9 4 0 3 5 4 6 1 3 9 4 0 1 2 4 1 1 1 5 2 9	

Therefore, $\sqrt{78.54} = 8.8623$, and $\sqrt{.7854} = .88623$. Ans.

(14) The number is evidently the square root of 114.9184.

Given number = 114.9184 = 1'14'.91'84.

Altered number = 1.149184 = 1.1492.

See page 1 of the table.

1.08 ² = 1.1 6 6 4	altered number = 1.1 4 9 2
1.07 ² = 1.1 4 4 9	1.07 ² = 1.1 4 4 9
first difference = 2 1 5	second difference = 4 3

$$\begin{array}{r} 215 \overline{) 430} \cdot 2 \\ 430 \end{array}$$

Therefore, $\sqrt{1.1492} = 1.072$, and $\sqrt{114.9184} = 10.72$. Ans.

(15) Given number = 3,486,784 = 3'48'67'84.

Altered number = 3.486784 = 3.4868.

See page 1 of the table.

$$\begin{array}{rcl} 1.87^2 & = & 3.4969 \\ 1.86^2 & = & 3.4596 \\ \text{first difference} & = & 373 \\ 373 \overline{) 272.000} & (& .729 \text{ or } .73 \\ 2611 & & \\ 1090 & & \\ 746 & & \\ 3440 & & \\ 3357 & & \\ \hline & & 83 \end{array} \quad \begin{array}{l} \text{altered number} = 3.4868 \\ 1.86^2 = 3.4596 \\ \text{second difference} = 272 \end{array}$$

Therefore, $\sqrt{3.4868} = 1.8673$, and $\sqrt{3,486,784} = 1,867.3$. Ans.

(16) Given number = .00041209 = .00'04'12'09.

Altered number = 4.1209.

See page 2 of the table.

$2.03^2 = 4.1209$. Therefore, $\sqrt{4.1209} = 2.03$; and as there is one complete cipher period following the decimal point in the given number, there is one cipher after the decimal point in the root. Hence, $\sqrt{.00041209} = .0203$. Ans.

(17) Given number = 2,490.31 = 2490'3100.

Altered number = given number = 2490.31.

See page 7 of the table.

$$\begin{array}{rcl} 7.07^4 & = & 2498.49 \\ 7.06^4 & = & 2484.38 \\ \text{first difference} & = & 1411 \\ 1411 \overline{) 593.000} & (& .420 \text{ or } .42 \\ 5644 & & \\ 2860 & & \\ 2822 & & \\ \hline & & 380 \end{array} \quad \begin{array}{l} \text{given number} = 2490.31 \\ 7.06^4 = 2484.38 \\ \text{second difference} = 593 \end{array}$$

Therefore, $\sqrt[4]{2490.31} = 7.0642$. Ans.

(18) Given number = 6,039,065,434 = '60390'65134'.

Altered number = 60390.65434 = 60390.7.

See page 9 of the table.

$$\begin{array}{rcl} 9.05^5 & = & 60707.6 \\ 9.04^5 & = & 60372.9 \\ \text{first difference} & = & 3347 \\ 3347 \overline{) 178} & & \end{array} \quad \begin{array}{l} \text{altered number} = 60390.7 \\ 9.04^5 = 60372.9 \\ \text{second difference} = 178 \end{array}$$

$$\begin{array}{r}
 3347 \overline{) 178000} \text{ (} \cdot 053 \text{ or } \cdot 05 \\
 \underline{16735} \\
 10650 \\
 \underline{10041} \\
 609
 \end{array}$$

Therefore, $\sqrt[5]{60390 \cdot 7} = 9 \cdot 0405$, and $\sqrt[5]{6,039,065 \cdot 434} = 90 \cdot 405$. Ans.

(19) Given number = $\cdot 127 = \cdot 12700$.

Altered number = 12700.

See page 6 of the table.

$$\begin{array}{rcl}
 6 \cdot 62^5 & = & 12714 \cdot 2 \\
 6 \cdot 61^5 & = & 12618 \cdot 5 \\
 \text{first difference} & = & 957 \\
 957 \overline{) 815000} \text{ (} \cdot 851 \text{ or } \cdot 85 \\
 \underline{7656} \\
 4940 \\
 \underline{4785} \\
 1556 \\
 \underline{957} \\
 593
 \end{array}
 \qquad
 \begin{array}{rcl}
 \text{altered number} & = & 12700 \cdot 0 \\
 6 \cdot 61^5 & = & 12618 \cdot 5 \\
 \text{second difference} & = & 815
 \end{array}$$

Therefore, $\sqrt[5]{12700} = 6 \cdot 6185$, and $\sqrt[5]{\cdot 127} = \cdot 66185$. Ans.

(20) Given number = $72 \cdot 415 = 72' \cdot 41500'$.

Altered number = 724150.

See page 2 of the table.

$$\begin{array}{rcl}
 2 \cdot 36^5 & = & 73208 \cdot 2 \\
 2 \cdot 35^5 & = & 71670 \cdot 3 \\
 \text{first difference} & = & 15379 \\
 15379 \overline{) 7447000} \text{ (} \cdot 484 \text{ or } \cdot 48 \\
 \underline{61516} \\
 129540 \\
 \underline{123032} \\
 65080 \\
 \underline{61516} \\
 3564
 \end{array}
 \qquad
 \begin{array}{rcl}
 \text{alterea number} & = & 72415 \cdot 0 \\
 2 \cdot 35^5 & = & 71670 \cdot 3 \\
 \text{second difference} & = & 7447
 \end{array}$$

Therefore, $\sqrt[5]{72415} = 2 \cdot 3548$. Ans.

ARITHMETIC

(PART 6)

(1) (a) The value of the ratio of $\frac{3}{4} : \frac{4}{5} = \frac{3}{4} \div \frac{4}{5}$ (See Art. 2) $= \frac{3}{4}$
 $\times \frac{5}{4} = \frac{15}{16}$.

Dividing this by 5, $\frac{15}{16} \div 5 = \frac{15}{16} \times \frac{1}{5} = \frac{3}{16}$. Ans.

(b) The value of the ratio of $.2 : .05 = .2 \div .05 = 4$. This value, divided by 4, becomes $4 \div 4 = 1$. Ans.

(c) The value of the inverse ratio of 12.5 to 125 is $125 : 12.5 = \frac{125}{12.5} = 10$. (See Art. 29.)

Dividing this by $\frac{3}{2}$, $10 \div \frac{3}{2} = 10 \times \frac{2}{3} = \frac{20}{3} = 6\frac{2}{3}$. Ans.

(2) $2 \text{ lb. } 3 \text{ oz.} = 35 \text{ oz.}$
 $5 \text{ lb. } 1\frac{1}{2} \text{ oz.} = 81.5 \text{ oz.}$

Hence the required ratio $= 35 : 81.5 = 70 : 163$. Ans.

(3) (a) The ratio of $\frac{4}{3}$ to $\frac{3}{4} = \frac{4}{3} \div \frac{3}{4} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$.

Multiplying this by 3, the value of the ratio becomes

$$\frac{16}{9} \times 3 = \frac{16}{3} = 5\frac{1}{3}. \text{ Ans.}$$

(b) The volume of a gallon is 277.463 cu. in. 1 cu. ft. = 1,728 cu. in. Therefore, the ratio of the volume of a gallon to a cubic foot

$$= 277.463 : 1,728 = \frac{277.463}{1,728}$$

Multiplying by 27, the value of the ratio is

$$\frac{277.463}{64} \times 27 = \frac{277.463}{64} = 4.335+. \text{ Ans.}$$

(c) The inverse ratio of $33\frac{1}{3}$ to $187\frac{1}{2}$

$$= 187\frac{1}{2} : 33\frac{1}{3} = \frac{375}{2} \div \frac{100}{3} = \frac{375}{2} \times \frac{3}{100} = \frac{45}{8}$$

Multiplying this ratio by 4 its value becomes

$$\frac{45}{8} \times 4 = \frac{45}{2} = 22\frac{1}{2}. \text{ Ans.}$$

(4)

$$3 : 8 = \frac{3}{8} = \frac{9}{24}$$

$$5 : 12 = \frac{5}{12} = \frac{10}{24}$$

$\frac{10}{24}$ is greater than $\frac{9}{24}$ by $\frac{1}{24}$; therefore the ratio of 5 : 12 is greater than that of 3 : 8 by $\frac{1}{24}$. Ans.

(5) (a) $20 + 7 : 10 + 8 = 3 : x$

Performing the indicated additions,

$$27 : 18 = 3 : x$$

Whence,
$$x = \frac{18 \times 3}{27} = 2. \text{ Ans.}$$

(b)

$$12^2 : 100^2 :: 4 : x$$

Squaring 12 and 100, $144 : 10,000 :: 4 : x$, whence

$$x = \frac{10,000 \times 4}{144} = 277.8-. \text{ Ans.}$$

(6) The two given distances are 444 mi. and 1,060 mi., and it is required to find the length of time which bears the same ratio to 8 hr. 40 min. that 444 mi. bears to 1,060 mi. Calling the unknown time x , the proportion is written

$$444 : 1,060 = 8 \text{ hr. } 40 \text{ min.} : x$$

For 8 hr. 40 min. write 520 min.

$$\text{Then, } 444 : 1,060 = 520 : x, \text{ and hence, by Art. 25, } x = \frac{1060 \times 520}{444}$$

$$= 1241.44 \text{ min.} = 20 \text{ hr. } 41\frac{1}{2} \text{ min., nearly. Ans.}$$

(7) (a) $x : 35 :: 4 : 7$. As x is an extreme, divide the product of the means by the other extreme. (Art. 25.) Then $x = \frac{35 \times 4}{7} = 20.$
Ans.

$$(b) \sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x. \quad \sqrt[3]{1,000} = 10; \sqrt[3]{1,331} = 11.$$

Substituting these values in the proportion, it becomes $10 : 11 = 27 : x$.

$$\text{Whence,} \quad x = \frac{11 \times 27}{10} = \frac{297}{10} = 29.7. \quad \text{Ans.}$$

(c) $64 : 81 = 21^2 : x^2$. To obtain the value of x^2 , since it is an extreme, divide the product of the means by the other extreme.

$$\text{Then } x^2 = \frac{81 \times 21^2}{64} = \frac{81 \times 21 \times 21}{64} = \frac{35,721}{64} = 558.1406$$

$$\text{And} \quad x = \sqrt{558.1406} = 23.62+. \quad \text{Ans.}$$

(8) The ratio of 100 gal. to the unknown number of gallons must be equal to the ratio of the time in which 100 gal. runs over to the time in which the unknown quantity runs over. Putting x for the unknown number of gallons, the proportion obtained is

$$100 : x = 2 \text{ hr.} : 14 \text{ hr. } 28 \text{ min.}$$

Reducing the time to minutes

$$100 : x = 120 : 868$$

$$\text{Whence,} \quad x = \frac{100 \times 868}{120} = \frac{8680}{12} = 723\frac{1}{3} \text{ gal.} \quad \text{Ans.}$$

(9) (a) If a proportion is true, the product of the means is equal to the product of the extremes (Art. 23). Thus, if the proportion $20 : 23 = 39 : 45$ is correct, 23×39 (or 897) should be equal to 20×45 (or 900). Since these products are not equal, the proportion is, therefore, not a true one.

(b) To find what the correct number is which should replace 23, the second term, write x for the second term. (Art. 24.)

$$\begin{array}{rcl} 20 : x = 39 : 45 & & \\ & 300 & \\ x = \frac{20 \times 45}{39} = \frac{900}{39} = 23\frac{1}{13} & & \end{array}$$

The original second term, 23, must, therefore, be increased by $\frac{1}{13}$. Ans.

(10) Since 5 men do something, 5 men is a cause, and since they take 9 days of 10 hr. each to do the work, 9 days and 10 hr. are each an element of the first cause. For the same reason 8 men working x days of 9 hr. each constitute a second cause. The first effect, that which is done, is a wall 100 ft. long, 6 ft. high, and 18 in. thick, and the second effect is a wall 120 ft. long, 8 ft. high, and 20 in. thick. Hence, we write the following proportion and cancel where possible :

$$\begin{array}{ccc|ccc} & & 10 & 20 & & \\ 5 & \cancel{9} & \cancel{100} & \cancel{120} & & \\ \cancel{9} & x = & \cancel{6} & \cancel{8} & , \text{ or } x = & \frac{20 \times 5}{9} = \frac{100}{9} = 11\frac{1}{9} \\ & & 9 & 2 & & \\ 10 & \cancel{8} & \cancel{18} & \cancel{20} & & \end{array}$$

The required time is $11\frac{1}{9}$ days, and since a day's work consists of 9 hr., $\frac{1}{9}$ day = 1 hr.; hence, $11\frac{1}{9}$ days = 11 days 1 hr. Ans.

$$(11) \quad \begin{array}{r|l|l} 4 & & \\ \frac{12}{15} & \frac{20}{25} & \\ 3 & & \\ \frac{2}{3} & 5 = x & 2 \\ 7 & 10 & 20 \end{array}$$

First cancel factors inside the lines with factors outside the lines, taking care to cancel only numerators with numerators and denominators with denominators. The value of x is then obtained by dividing the product of the extremes by that of the means.

$$x = \frac{2 \times 4 \times 2 \times 7}{5} = \frac{112}{5} = 22.4. \text{ Ans.}$$

(12) If the volume of the air varied directly as the pressure the proportion would be written

$$17.8 : 67.2 = 10 : x,$$

where x is the required volume in cubic feet.

As, however, the proportion is inverse,

$$17.8 : 67.2 = x : 10$$

$$\text{Whence, } x = \frac{17.8 \times 10}{67.2} = 2.65 \text{ cu. ft., nearly. Ans.}$$

(13) The incorrect proportion given is

$$3 : 7 = 8 : 12$$

To correct this by altering the first term, 3, write x instead of 3, when the proportion becomes $x : 7 = 8 : 12$, and, therefore, x ,

$$= \frac{8 \times 7}{12} = \frac{14}{3} = 4\frac{2}{3}. \text{ The first term has therefore to be increased by } 2$$

$$4\frac{2}{3} - 3 = 1\frac{2}{3}. \text{ Ans.}$$

To correct this by altering the second term, 7, write x instead of 7. The proportion then becomes $3 : x = 8 : 12$, and, therefore,

$$x = \frac{12 \times 3}{8} = \frac{9}{2} = 4\frac{1}{2}.$$

The original second term was 7, so that it must be decreased by $7 - 4\frac{1}{2}$ or $2\frac{1}{2}$. Ans.

(14) Calling depths and lengths the causes, and safe loads the effects, the direct proportion is written thus, the depths being squared because the safe load varies as the square of the depth :

$$\begin{array}{c|c} 9^2 & 12^2 \\ 15 & 20 \end{array} = 14 \cdot 2 \quad \left| \quad x, \right.$$

where x is the required safe load in tons.

As, however, the safe load varies inversely as the length, the terms 15 and 20 are inverted (Art. 29), when the proportion becomes

$$\begin{array}{c|c} 9^2 & 12^2 \\ 20 & 15 \end{array} = 14 \cdot 2 \quad \left| \quad x \right.$$

Whence,

$$x = \frac{12^2 \times 15 \times 14 \cdot 2}{9^2 \times 20} = \frac{\overset{16}{144} \times \overset{3}{15} \times 14 \cdot 2}{\underset{3}{81} \times \underset{4}{20}} = \frac{56 \cdot 8}{3} = 18 \cdot 9 \text{ tons, nearly.} \quad \text{Ans.}$$

(15) (a) 25% of 8,428 lb. is some number of pounds which bears the same ratio to 8,428 lb. that 25 bears to 100. Putting x for the unknown weight, the following proportion can be written :

$$25 : 100 = x : 8,428$$

$$\text{Therefore, } x = \frac{25 \times 8,428}{100} = 2,107 \text{ lb.} \quad \text{Ans.}$$

This may also be solved thus: $25\% = \frac{25}{100} = \frac{1}{4}$, $\frac{1}{4}$ of 8,428 lb. = 2,107 lb. Ans.

(b) $\frac{1}{2}\%$ of £35,000 is a certain sum of money which bears the same ratio to £35,000 that $\frac{1}{2}$ does to 100. Hence the following proportion may

$$\text{be written : } \frac{1}{2} : 100 = x : 35,000. \text{ From this, } x = \frac{\frac{1}{2} \times 35,000}{100} = 175 = \text{£175.} \quad \text{Ans.}$$

Or, the following solution can be obtained :

$$\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{200}. \quad \frac{1}{200} \times \text{£35,000} = \text{£175.} \quad \text{Ans.}$$

(c) It is required to find a number which bears the same ratio to 100 that £176 8s. does to £2,520. £176 8s. = £176.4.

The proportion obtained is, therefore,

$$\begin{array}{l} x : 100 = \text{£176} \cdot 4 : \text{£2,520} \\ 44 \cdot 1 \\ x = \frac{176 \cdot 4 \times 100}{2520} = \frac{441}{63} = 7 \end{array}$$

The ratio of £176 8s. to £2,520 is, therefore, 7%. Ans.

(16) The ratio of the increase of speed to the speed guaranteed is $7\frac{1}{2}\%$, or the ratio of $7\frac{1}{2} : 100$. Thus the following proportion can be written, x being the increase of speed in knots,

$$x : 32 = 7.5 : 100$$

Whence, x , the increase of speed, $= \frac{32 \times 7.5}{100} = 2.4$ knots.

The increased speed obtained on the trials was, therefore,

$$32 + 2.4 = 34.4 \text{ knots. Ans.}$$

Or the problem may be solved thus :

If the speed is increased by $7\frac{1}{2}\%$, it is increased in the ratio of $100 : 107\frac{1}{2}$.

Writing x for the increased speed on trials, $32 : x = 100 : 107\frac{1}{2}$.

Therefore, the increased speed $x = \frac{32 \times 107.5}{100} = 34.4$ knots. Ans.

$$(17) \text{ Spent on board } 24\% \text{ of } £250 = \frac{£250 \times 24}{100} = £60$$

$$\text{Spent on clothing } 12\frac{1}{2}\% \text{ of } £250 = \frac{£250 \times 12.5}{100} = £31\frac{1}{4}$$

$$\text{Other expenses } 17\% \text{ of } £250 = \frac{£250 \times 17}{100} = £42\frac{1}{2}$$

$$\text{Total amount spent} = £133\frac{3}{4}$$

Therefore, amount saved $= £250 - £133\frac{3}{4} = £116\frac{1}{4} = £116 \text{ 5s.}$ Ans.

Or the problem may be solved as follows :

$$\left. \begin{array}{l} \text{Spent on board, } 24 \% \\ \text{Spent on clothing } 12\frac{1}{2}\% \\ \text{Other expenses } 17 \% \end{array} \right\} \text{ of } £250$$

$$\text{Total amount spent } 53\frac{1}{2}\% \text{ of } £250$$

Therefore, amount saved $= (100 - 53\frac{1}{2})\% \text{ of } £250 = 46\frac{1}{2}\% \text{ of } £250$
 $= \frac{£250 \times 46.5}{100} = £116\frac{1}{4} = £116 \text{ 5s.}$ Ans.

(18) The speed has increased by 4% of the original speed (which is to be found); that is, the ratio of the original speed to the increased speed is equal to the ratio of $100 : 104$.

Hence, $100 : 104 = x : 140$, where x is the original speed.

Therefore, $x = \frac{100 \times 140}{104} = 134.6$ revolutions per minute. Ans.

(19) If the sum was diminished by 35% of itself it must have reached $(100 - 35)$ or 65% of its former value. Therefore, the ratio of $65 : 100$ is equal to that of $£4,810$, the diminished amount, to the unknown original sum x . The proportion is

$$65 : 100 = 4,810 : x$$

Whence, $x = \frac{100 \times 4,810}{65} = £7,400.$ Ans.

(20) The time saved = 3 hr. 35 min. — 3 hr. 25 min. = 10 min.

It is required to find what per cent. this time saved is of the original time taken, and to find what number bears the same ratio to 100 that 10 min. does to 3 hr. 35 min. or 215 min. The proportion obtained is, therefore,

$$x : 100 = 10 : 215$$

And $x = \frac{1,000}{215} = 4.65\%$, nearly. Ans.

APPLICATION OF FORMULÆ

- (1) Substituting for A, h, D, x , and B their values,

$$\begin{aligned}
 T &= \sqrt{\frac{A^2 \left[490 + \frac{(hx)^2}{D^2} \right]}{h + \frac{x}{D} (A^2 - B)^2}} = \sqrt{\frac{5^2 \left[490 + \frac{(200 \times 12)^2}{120^2} \right]}{200 + \frac{12}{120} (5^2 - 10)^2}} \\
 &= \sqrt{\frac{25 (490 + 400)}{200 + \frac{1}{10} (225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22,250}{222.5}} \\
 &= \sqrt{100} = 10. \quad \text{Ans.}
 \end{aligned}$$

- (2) Substituting in formula, $A = \frac{39}{2} \sqrt{28^2 - \left(\frac{28^2 + 39^2 - 37^2}{2 \times 39} \right)^2}$

$$\begin{aligned}
 &= 19.5 \sqrt{784 - \left(\frac{784 + 1,521 - 1,369}{78} \right)^2} = 19.5 \sqrt{784 - 12^2} \\
 &= 19.5 \sqrt{640} = 19.5 \times 25.298 = 493.311 \text{ sq. ft.} \quad \text{Ans.}
 \end{aligned}$$

- (3) Substituting in the formula, $l = \sqrt{3.1416^2 \times 5^2 + 2^2}$

$$= \sqrt{9.87 \times 25 + 4} = \sqrt{246.75 + 4} = \sqrt{250.75} = 15.84. \quad \text{Ans.}$$

- (4) Substituting in the formula, $P = \frac{SA}{1 + .00004 \frac{l^2}{G^2}}$

$$\begin{aligned}
 &= \frac{70,000 \times .7854 (10^2 - 7^2)}{1 + .00004 \times \frac{180^2}{10^2 + 7^2}} = \frac{70,000 \times .7854 \times 51}{1 + .00004 \times \frac{34,596}{9.3125}} \\
 &= \frac{70,000 \times 40.0554}{1 + .1486} = \frac{2,803,878}{1.1486} = 2,441,127. \quad \text{Ans.}
 \end{aligned}$$

- (5) Substituting in the formula,

$$\begin{aligned}
 25 &= \sqrt{3.1416^2 \times 6^2 + t^2} = \sqrt{9.87 \times 36 + t^2} \\
 25 &= \sqrt{355.32 + t^2}
 \end{aligned}$$

From Arts. 26 and 28, $25^2 = 355 \cdot 32 + t^2$

$$625 = 355 \cdot 32 + t^2$$

Transposing, $t^2 = 625 - 355 \cdot 32$

$$t^2 = 269 \cdot 68$$

$$t = \sqrt{269 \cdot 68} = 16 \cdot 42, \text{ Ans.}$$

$$(6) \quad c = \frac{18 \times 12}{10} = 21 \cdot 6. \text{ Substituting in formula,}$$

$$\begin{aligned} P &= 5,000 \times \frac{700 + 15 \times 21 \cdot 6}{700 + 15 \times 21 \cdot 6 + 21 \cdot 6^2} = 5,000 \times \frac{700 + 324}{700 + 324 + 466 \cdot 56} \\ &= 5,000 \times \cdot 687 = 3,435. \text{ Ans.} \end{aligned}$$

(7) Substituting the numerical values given in the formula,

$$8 \cdot 9 = \frac{6 \cdot 9 \, d^2}{6 \cdot 66}$$

Clearing of fractions, $8 \cdot 9 \times 6 \cdot 66 = 6 \cdot 9 d^2 = 59 \cdot 274 = 6 \cdot 9 \, d^2$

Transposing, $6 \cdot 9 \, d^2 = 59 \cdot 274$

Dividing both sides by $6 \cdot 9$, $d^2 = \frac{59 \cdot 274}{6 \cdot 9} = 8 \cdot 59$,

$$d = \sqrt{8 \cdot 59} = 2 \cdot 93. \text{ Ans.}$$

(8) $d = D \sqrt{\frac{P}{n S}}$. Squaring the whole equation (Art. 26).

$$d^2 = D^2 \times \frac{P}{n S} = \frac{D^2 P}{n S}$$

Clearing of fractions, $d^2 n S = D^2 P$

Dividing both sides of the equation by $d^2 n S = \frac{D^2 P}{d^2 n}$

Substituting the numerical values given,

$$S = \frac{17^2 \times 165}{\cdot 85^2 \times 15} = \frac{47,685}{10 \cdot 8375} = 4,400. \text{ Ans.}$$

(9) Substituting in formula,

$$\begin{aligned} A &= \frac{3 \cdot 1416 \times 12^2 \times 150}{360} - \frac{23 \cdot 182}{2} (12 - 8 \cdot 894) = 188 \cdot 496 - 11 \cdot 591 \\ &\times 3 \cdot 106 = 188 \cdot 496 - 36 \cdot 0016 = 152 \cdot 4944. \text{ Ans.} \end{aligned}$$

(10) Substituting the numerical values,

$$\begin{aligned} d_1 &= 1 \cdot 625 \left(1 + \frac{\cdot 08}{3} + \frac{\cdot 4}{\sqrt{3}} \right) = 1 \cdot 625 (1 + \cdot 02667 + \cdot 23094) \\ &= 1 \cdot 625 \times 1 \cdot 2576 = 2 \cdot 0436. \text{ Ans.} \end{aligned}$$

GEOMETRY AND MENSURATION

(PART 1)

(1) See Art. 106. Area of parallelogram = base $\times$ altitude. Base = 129 in. = 10.75 ft.; altitude = 7 ft. $10.75 \times 7 = 75.25$ sq. ft. Ans.

(2) See Art. 107. If a and b are the lengths of the parallel sides, h is the altitude of the trapezoid. Then, area = $\frac{1}{2}(a + b) \times h$. $a = 15$ ft. 7 in.; $b = 21$ ft. 11 in.; $h = 7$ ft. 8 in. = 7.67 ft. $a + b = 37$ ft. 6 in. = 37.5 ft. Then area = $\frac{1}{2} \times 37.5 \times 7.67 = 143.81$ sq. ft. Ans.

(3) Angle $EOA = 360^\circ - (\text{angle } EOD + \text{angle } DOC + \text{angle } COB + \text{angle } BOA) = 360^\circ - (72^\circ + 63^\circ + 56^\circ + 46^\circ) = 360^\circ - 237^\circ = 123^\circ$. Ans.

(4) If one-third of the angle A is $14^\circ 47' 10''$, then the angle A must be $3 \times 14^\circ 47' 10'' = 44^\circ 21' 30''$. The angle $B = 2\frac{1}{2} \times 44^\circ 21' 30'' = 110^\circ 53' 45''$. The angle $C = 180^\circ - (A + B) = 180^\circ - (44^\circ 21' 30'' + 110^\circ 53' 45'') = 24^\circ 44' 45''$.

$$\left. \begin{array}{l} A = 44^\circ 21' 30'' \\ B = 110^\circ 53' 45'' \\ C = 24^\circ 44' 45'' \end{array} \right\} \text{Ans.}$$

(5) (a) Applying the formula of Art. 119, A being equal to 89.42 sq. in., $d = \sqrt{\frac{89.42}{.7854}} = 10.67$ in. Ans.

(b) Circumference = $\pi d = 3.1416 \times 10.67 = 33.52$ in. Ans.

(6) (a) $90^\circ - 37^\circ = 53^\circ$. Ans.

(b) $180^\circ - 37^\circ = 143^\circ$. Ans.

(7) (a) $CD = OD - OC$. $OC = \sqrt{(AO)^2 - (AC)^2} = \sqrt{3^2 - (2.25)^2} = 1.98$ ft. $OD = 3$ ft.; hence, $CD = 3 - 1.98 = 1.02$ ft. = 1 ft. $\frac{1}{4}$ in. Ans.

(b) $OC = OD - CD$; $AB = 2AC = 2\sqrt{(OA)^2 - (OC)^2}$. Ans.

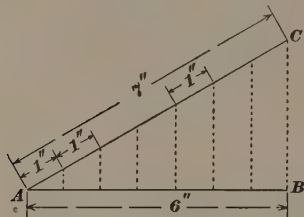
(8) Using the formula of Art. 111, $s = \frac{48.5 + 52.5 + 59}{2} = 80$.

Therefore, $A = \sqrt{80 \times 21 \times 27.5 \times 31.5} = 1,206.36$ sq. ft. = area of one floor. Area of five floors = 6,032 sq. ft. Ans.

(9) Rise for 8 feet = 12.5 ft. Rise for 1 foot = $\frac{12.5}{8} = 1.5625$ ft.
= $18\frac{3}{4}$ in. Ans.

(10) See Art. 63.

(11) $FB = \sqrt{24^2 + 18^2} = 30$ ft.; $FD = \sqrt{30^2 + 14^2} = 33.11$ ft.;
 $EA = \sqrt{24^2 + 30^2} = 38.42$ ft.; $EC = \sqrt{12^2 + 14^2} = 18.44$ ft. Ans.



(12) Let AB (see figure) be the line 6 in. long. Draw $AC = 7$ in., and connect C and B . Draw parallels to CB through each inch mark on AC . These lines will divide AB as required. Ans.

(13) Since the angles DCA and CDB are each 45° and AE is perpendicular to CD , $CE = FD$ = $AE = 10$ ft.; therefore, $AB = 36 - (10 \times 2) = 16$ ft. Ans.

(14) (a) Since the side slopes are half pitch, $FG = FH = \frac{1}{2} AB$, or 11 ft. Then, $GH = \sqrt{(FH)^2 + (FG)^2} = \sqrt{11^2 + 11^2} = 15.56$ ft., or 15 ft. $6\frac{3}{4}$ in., the length of a common rafter. Ans.

(b) $BC = \sqrt{(BD)^2 + (DC)^2}$, but $BD^2 = ED^2 + BE^2 = 11^2 + 11^2$; $DC = 11$ ft.; therefore, $BC = \sqrt{11^2 + 11^2 + 11^2} = 19.05$ ft., or 19 ft. $\frac{5}{8}$ in. Ans.

(15) (a) MD, DL, KB, BI, AH , and AG are all equal, and their true length is the hypotenuse, as $M'D'$, of a right triangle, the sides of which, as $M'D''$ and $D'D''$, are each 10 ft.; hence, $M'D'$, etc. = $\sqrt{10^2 + 10^2} = 14.14$ ft.

$$\text{Area, } ABIIH = H'A' \times HI = 14.14 \times 36 = 509.04 \text{ sq. ft.}$$

$$\begin{aligned} \text{Area, } GFC A &= \frac{1}{2} (GF + AC) \times A'G' = \frac{12 + 22}{2} \\ &\times 14.14 = 240.38 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} \text{Area, } EKBC &= \frac{1}{2} (EK + CB) \times K'B' = \frac{4 + 14}{2} \\ &\times 14.14 = 127.26 \text{ sq. ft.} \end{aligned}$$

$$\begin{aligned} \text{Area, } MDCF \text{ and } DLEC &= 2 \left[\frac{1}{2} (MF + DC) \right. \\ &\times M'D'] = 2 \left(\frac{10 + 20}{2} \times 14.14 \right) = 424.20 \text{ sq. ft.} \end{aligned}$$

$$\text{Total, } 1,300.88 \text{ sq. ft.}$$

Ans.

(b) The length of the valley FC is the hypotenuse of a right triangle on the roof surface of which the real length of FN (which is $M'D'$) and NC (= $AG = 10$ ft.) are the sides; hence, $FC = \sqrt{(M'D')^2 + (NC)^2}$ = $\sqrt{(14.14)^2 + 10^2} = \sqrt{300} = 17.32$ ft. = 17 ft. $3\frac{7}{8}$ in. Ans.

(16) In the formula of Art. 116,

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}, \quad c = 3\frac{1}{2} \text{ ft.} = 42 \text{ in.}; \quad h = 3 \text{ in.} \quad \text{Substituting,}$$

$$l = \frac{4\sqrt{42^2 + 3^2 \times 4} - 42}{3} = \frac{4\sqrt{1,800} - 42}{3} = 42.57 \text{ in.} = 3 \text{ ft. } 6\frac{9}{16} \text{ in.} \quad \text{Ans.}$$

(17) The angle included between two radii drawn to the ends of a side of a regular nonagon is, since the sides are equal, $360^\circ \div 9 = 40^\circ$. In any triangle, the sum of the angles is 180° ; hence, the sum of the other two angles is $180^\circ - 40^\circ = 140^\circ$; the angles being equal, each is $\frac{1}{2}$ of 140° , or 70° . As a radius drawn to the angle of a regular polygon bisects it, then the angle between two sides of the polygon is twice 70° , or 140° .

Ans.

(18) Applying the principle stated in Art. 83, $FB^2 = AB \times BC$; or, $FB = \sqrt{AB \times BC} = \sqrt{4 \times 12} = 6.93 \text{ ft.}$; or, thus, $OB = AO - AB = 4 \text{ ft.}$, and $FB = \sqrt{(OF)^2 - (OB)^2} = \sqrt{8^2 - 4^2} = 6.93 \text{ ft.}$ Hence, $DF = 11 \text{ ft.} - 6.93 \text{ ft.} = 4.07 \text{ ft.}$ Ans.

(19) (a) In the formula of Art. 85,

$$r = \frac{c^2 + 4h^2}{8h}, \quad c = 6 \text{ ft. } 8 \text{ in.} = 80 \text{ in.}, \text{ and } h = 8 \text{ in.} \quad \text{Substituting,}$$

$$r = \frac{80^2 + 4 \times 8^2}{8 \times 8} = 104 \text{ in.} = 8 \text{ ft. } 8 \text{ in.} \quad \text{Ans.}$$

(b) In the formula of Art. 116,

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}, \quad c = 80 \text{ in.}, \quad h = 8 \text{ in.} \quad \text{Substituting,}$$

$$l = \frac{4\sqrt{6,400 + 256} - 80}{3} = \frac{4 \times 81.58 - 80}{3} = 82.11 \text{ in.},$$

the length of arc; as there are 13 stones, the length of each is $82.11 \div 13 = 6.32 \text{ in.}$ Ans.

(20) The required area is $\frac{27500}{10000} = 2.75 \text{ sq. in.}$ The diameter

$$= \sqrt{\frac{2.75}{.7854}} = 1.87 \text{ in.} = 1\frac{7}{8} \text{ in., nearly.} \quad \text{Ans.}$$

GEOMETRY AND MENSURATION

(PART 2)

(1) The monument consists of a frustum of a pyramid and a pyramid. For the former, the area of the lower base is $3 \times 3 = 9$ sq. ft.; that of the upper base is $1 \times 1 = 1$ sq. ft. Using the formula

$$V = \frac{h}{3}(A + a + \sqrt{A \times a}) = \frac{18}{3}(9 + 1 + \sqrt{9 \times 1}) = 78 \text{ cu. ft.}$$

The volume of the pyramid is $\frac{a h}{3}$; $a = 1$ sq. ft.; $h = 2$ ft.; therefore, $\frac{a h}{3} = \frac{1 \times 2}{3} = \frac{2}{3}$ cu. ft. The total is $78 + \frac{2}{3} = 78\frac{2}{3}$ cu. ft., and the weight, at 170 lb. per cu. ft., is $78\frac{2}{3} \times 170 = 13,373\frac{1}{3}$ lb. Ans.

(2) The quantity discharged is the volume of a cylinder whose length is $5 \times 60 = 300$ ft.; the area of a 4-inch circle is 12.57 sq. in. = .09 sq. ft.; hence, the volume is $.09 \times 300 = 27$ cu. ft., which, reduced to gallons, gives $27 \times 6.24 = 168.48$ gal. Ans.

(3) The volume of the metal will be $3 \div .41 = 7.32$ cu. in. The area of a $1\frac{3}{4}$ -inch circle is 2.4 sq. in.; that of a $1\frac{1}{2}$ -inch circle = 1.77 sq. in.; the area of the ring is, therefore, $2.40 - 1.77 = .63$ sq. in.; hence, the length required is $7.32 \div .63 = 11.6$ in. Ans.

(4) The area required is the difference in areas of a 12-inch circle, and the segment $A D B$. The area of the circle is $.7854 \times 12^2 = 113.10$ sq. in. The length $A C = \sqrt{(A O)^2 - (O C)^2} = \sqrt{6^2 - 3^2} = 5.2$ in., and $A B = 5.2 \times 2 = 10.4$ in. The length of the arc $A D B$ is found by formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$$

$c = A B = 10.4$ in.; $h = C D = 3$ in.; hence,

$$l = \frac{4\sqrt{108.16 + 36} - 10.4}{3} = \frac{4 \times 12.0 - 10.4}{3} = 12.53 \text{ in.}$$

The area of sector

$$O A D B = \frac{l r}{2} = \frac{12.53 \times 6}{2} = 37.59 \text{ sq. in.};$$

the area of triangle is $\frac{10.4 \times 3}{2} = 15.60$ sq. in., whence, area of segment

$ADB = 37.59 - 15.60 = 21.99$ sq. in. Deducting 21.99 sq. in. from 113.10 sq. in., the area of the circle, the remainder is 91 sq. in., nearly.
Ans.

(5) Since the height of the gables is $\frac{5}{8}$ the width, 24 ft., it is $\frac{5}{8} \times 24 = 15$ ft. The length of the roof slope is the hypotenuse of a right triangle, the sides of which are 15 ft. and 12 ft. (one-half the width); it is, therefore, $\sqrt{15^2 + 12^2} = 19.21$ ft. Adding 15 in. = 1.25 ft., the total width of each half of the roof is $19.21 + 1.25 = 20.46$ ft. The length is 32 ft. + (15 in. $\times$ 2) = 34.5 ft. The area of each half is $34.5 \times 20.46 = 705.87$ sq. ft.; the area of the whole is, therefore, 705.87 sq. ft. $\times$ 2 = 1,411.74 sq. ft., or 1,412 sq. ft., nearly. Ans:

(6) In the formula for surface of a sphere, $S = \pi d^2$, d is 3 ft.; the surface of the hemisphere is, therefore, $\frac{1}{2}\pi d^2 = \frac{1}{2} \times 3.1416 \times 9 = 14.14$ sq. ft. Deducting 5 sq. ft. for openings, the net surface is 9.14 sq. ft. Ans.

(7) The outer diameter is 10 in. + ($\frac{1}{2}$ in. $\times$ 2) = 11 in. The area of an 11-inch circle is 95.03 sq. in.; that of a 10-inch circle is 78.54 sq. in.; the area of the ring is, therefore, $95.03 - 78.54 = 16.49$ sq. in. The length is 12 ft. = 144 in.; multiplying 16.49 by 144, the contents are 2,374.56 cu. in. = $2,374.56 \div 1,728 = 1.37$ cu. ft.; 1.37 cu. ft., at 450 lb. per cu. ft., weighs $1.37 \times 450 = 616.5$ lb. Ans.

(8) (a) The circumference of the tower = $\pi d = 3.1416 \times 8$ ft. = 25.13 ft. The area of the surface is 25.13×28 (the height) = 703.64 sq. ft., or 704 sq. ft.; nearly. Ans.

(b) The exposed area of one slate is $6 \times 5 = 30$ sq. in. Since the total area of slating is 704 sq. ft., or 101,376 sq. in., the number of slates required is $101,376 \div 30 = 3,380$, nearly. Ans.

(9) The area of the base plate is $15 \times 15 = 225$ sq. in., less the area of a 7-inch circle and the portion cut out at the corners, which is equal (for four corners) to the difference between a 4-inch square and a circle 4 inches in diameter; or, 16 sq. in. $- 12.57$ sq. in. = 3.43 sq. in. The area of a 7-inch circle is 38.48 sq. in. Adding 38.48 and 3.43, the total deduction is 41.91 sq. in., which subtracted from 225 sq. in. leaves 183.09 sq. in. The thickness being $1\frac{1}{2}$ in. = 1.5 in., the contents are $183.09 \times 1.5 = 274.63$ cu. in. The height of the cylindrical part is 2 in.; its area is the difference between the areas of an 11-inch and a 10-inch circle, which is $95.03 - 78.54 = 16.49$ sq. in.; hence, the contents are $16.49 \times 2 = 32.98$ cu. in. The triangular side area of the ribs is $\frac{2+0}{2} \times 4\frac{1}{2}$ in. = 4.5 sq. in.; the thickness being $\frac{1}{2}$ in., the contents of each are $4.5 \times .5 = 2.25$ cu. in.; and for four the contents are $2.25 \times 4 = 9$ cu. in. Adding the several portions, the total contents = $274.63 + 32.98 + 9 = 316.61$ cu. in. The weight is, therefore, $316.61 \times .26 = 82.32$ lb.

Ans.

(10) The area of the lower base is $12 \times 30 = 360$ sq. ft.; that of the upper base is $7 \times 22 = 154$ sq. ft. The area of a mid-section is

$$\frac{12+7}{2} \times \frac{30+22}{2} = 9.5 \times 26 = 247 \text{ sq. ft.}$$

By the prismoidal formula, the cubical contents are

$$\frac{2.8}{6} (360 + 154 + 247 \times 4) = \frac{2.8}{6} \times 1,502 = 7,009.33 \text{ cu. ft.}$$

$$= 7,009.33 \div 27 = 259.6 \text{ cu. yd.}$$

At £5 per cu. yd., the cost is $259.6 \times 5 = \text{£}1,298$. Ans.

(11) The volume of the ball is $25 \div .26 = 96.15$ cu. in. The diameter

$$= \sqrt[3]{\frac{96.15}{.5236}} = \sqrt[3]{183.63} = 5.68 \text{ in.} = 5\frac{1}{16} \text{ in., nearly. Ans.}$$

(12) The area of the two plates is $2(12 \times \frac{3}{8}) = 9$ sq. in. Each channel consists of a rectangle which has an area of $10 \times \frac{1}{4} = 2\frac{1}{2}$ sq. in., and 2 trapezoids each $(2\frac{1}{2} \text{ in.} - \frac{1}{4} \text{ in.})$ long, and with an average width of $\frac{\frac{5}{8} + \frac{3}{8}}{2} = \frac{1}{2}$ in., and an area of $2 \times (2\frac{1}{4} \times \frac{1}{2}) = 2\frac{1}{4}$ sq. in., making a total for each channel of $2\frac{1}{2} + 2\frac{1}{4} = 4\frac{3}{4}$ sq. in.; for two channels the area is $4\frac{3}{4} \times 2 = 9\frac{1}{2}$ sq. in. Adding the area of the plates, the total is $9\frac{1}{2} + 9 = 18\frac{1}{2}$ sq. in. Multiplying by 12 (inches in a foot), the contents per foot of length are $18\frac{1}{2} \times 12 = 222$ cu. in., and the weight at .283 lb. per cu. in. is $222 \times .283 = 62.83$ lb. Ans.

(13) (a) The area of the footing is $\frac{107,520}{2,240 \times 3} = 16$ sq. ft. Each side is therefore $\sqrt{16} = 4$ ft. Ans.

(b) The area of the base plate is $\frac{107,520}{2,240 \times 8} = 6$ sq. ft.; and each side is $\sqrt{6}$ ft. = 2.45 ft. Ans.

(14) (a) The area of the ring is the difference between a 7-inch and a 6-inch circle, or $\frac{\pi}{4}(7^2 - 6^2) = 10.21$ sq. in.; that of the flanges $\left| = \frac{3}{4} \text{ in.} \times 1 \text{ in.} \times 4 = 3 \text{ sq. in., making a total of } 13.21 \text{ sq. in. Ans.} \right.$

(b) Since a strip 1 foot long and 1 inch square weighs 3.33 lb., 1 foot of the column will weigh $13.21 \times 3.33 = 43.99$ lb. Increasing this by 2 per cent., the weight per foot is $43.99 \times 1.02 = 44.87$ lb. Ans.

(15) Perimeter of the house is $12 + 10 + 12 + 14 + 24 + 24 = 96$ ft. The height of it being 22 ft., the area is $96 \times 22 = 2,112$ sq. ft. Adding for gables, one $\frac{14 \times 7}{2} = 49$ sq. ft., and one $\frac{12 \times 7}{2} = 42$ sq. ft., the total area is $2,112 + 49 + 42 = 2,203$ sq. ft. Deduct 10 windows, 3 ft. $\times$ 7 ft. = 210 sq. ft.; 10 windows, 3 ft. $\times$ 6 ft. = 180 sq. ft.; and 4 doors, 3 ft. 6 in. $\times$ 8 ft. = 112 sq. ft. The total deduction is $210 + 180 + 112 = 502$ sq. ft.; therefore, the net surface is $2,203 - 502$

= 1,701 sq. ft., requiring, at 7 bricks per sq. ft., $1,701 \times 7 = 11,907$ bricks. Ans.

(16) The major axis of the glass is 7 ft. 4 in. = 7.33 ft.; the minor axis is 4 ft. 4 in. = 4.33 ft. Then the area is, by the formula $A = .7854 d D$
 $= .7854 \times 4.33 \times 7.33 = 24.93$ sq. ft. Ans.

(17) The area of the semi-elliptical arch is one-half of $.7854 d D$;
 $d = 4 \times 2 = 8$ ft.; $D = 10$ ft.; hence, the area is $\frac{1}{2} (.7854 \times 8 \times 10)$
 $= 31.42$ sq. ft. Adding this to the rectangular area, 6 ft. $\times$ 10 ft.
 $= 60$ sq. ft., the total area is 91.42 sq. ft., which exceeds the required
 area, 88 sq. ft., by 3.42 sq. ft. Ans.

(18) (a) Find the area of cross-section, beginning at the bottom:

$$7.5 \text{ ft.} \times 1 \text{ ft.} = 7.50 \text{ sq. ft.}$$

$$6.5 \text{ ft.} \times 1 \text{ ft.} = 6.50 \text{ sq. ft.}$$

$$\begin{array}{r} \text{Triangular portion,} \quad 1.25 \text{ ft.} \times 15 \text{ ft.} \\ \hline \quad \quad \quad \quad \quad \quad 2 \end{array} = 9.37 \text{ sq. ft.}$$

$$\text{Centre part,} \quad 2 \text{ ft.} \times 15 \text{ ft.} = 30.00 \text{ sq. ft.}$$

Under steps—

$$\begin{array}{r} 1 \text{ strip,} \quad \quad \quad 3.75 \\ 1 \text{ strip } (3.75 \times 2), \quad 7.50 \\ 1 \text{ strip } (3.75 \times 3), \quad 11.25 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} .75 \text{ ft.} \times 22.5 = 16.87 \text{ sq. ft.}$$

$$\text{Total,} \quad \quad \quad 22.50$$

$$\text{Coping,} \quad \quad \quad 2.5 \text{ ft.} \times .5 \text{ ft.} = 1.25 \text{ sq. ft.}$$

$$\text{Total area of cross-sections,} \quad \quad \quad 71.49 \text{ sq. ft.} \quad \text{Ans.}$$

(b) The length being 16 ft., the contents are $71.49 \times 16 = 1,143.84$ cu. ft.
 $= 1,143.84 \div 27 = 42.36$ cu. yd. Ans.

(19) (a) The angles of the triangles being equal, the bevel angle is 60° .
 Ans.

$$(b) CE = 2.5 \text{ ft.} \quad OG = \sqrt{(OC)^2 - (GC)^2} = \sqrt{(2.5)^2 - (1.25)^2}$$

$$= \sqrt{4.69} = 2.16 \text{ ft.} \quad DF : CE = OH : OG; \text{ or, } DF = \frac{CE \times OH}{OG}$$

$$= \frac{2.5 \times 2.5}{2.16} = 2.89 \text{ ft.} = 2 \text{ ft. } 10.7 \text{ in.} \quad \text{Ans.}$$

$$(c) \text{ Since the width, as } IH, \text{ is } 8 \text{ in.} = .67 \text{ ft., } OI = 2.5 - .67$$

$$= 1.83 \text{ ft.} \quad \text{Then, } AB : DF = OI : OH; \text{ or, } AB = \frac{DF \times OI}{OH}$$

$$= \frac{2.89 \times 1.83}{2.5} = 2.12 \text{ ft.} = 2 \text{ ft. } 1\frac{1}{2} \text{ in., nearly.} \quad \text{Ans.}$$

(20) Using the formula of Art. 9, the length of 8 spaces = 20 ft.,
 and the length of one space = 2.5 ft.

$$A = \left(\frac{a+k}{2} + b +, \text{ etc.} \right) x = \left(\frac{0+0}{2} + 9.3 + 12.6 + 14.2 + 14.8 + 14.2 \right.$$

$$\left. + 12.6 + 9.3 \right) 2.5 = 87 \times 2.5 \text{ sq. ft.} = 217.5 \text{ sq. ft.} \quad \text{Ans.}$$

PRELIMINARY BUILDING OPERATIONS

- (1) See Art. 9.
- (2) See Art. 25.
- (3) See Art. 62.
- (4) See Art. 6.
- (5) See Art. 10.
- (6) See Art. 28.
- (7) See Art. 55.
- (8) See Art. 8.
- (9) See Art. 15.
- (10) See Art. 36.
- (11) See Art. 42.
- (12) See Art. 10.
- (13) See Art. 15.
- (14) See Art. 37.
- (15) See Art. 65.
- (16) See Art. 20.
- (17) See Arts. 67 and 68.
- (18) See Art. 8.
- (19) See Art. 22.
- (20) See Art. 18.

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- (1) See Art. 44.
- (2) See Art. 33.
- (3) See Art. 67.
- (4) See Art. 18.
- (5) See Art. 79.
- (6) See Arts. 8 and 52.
- (7) See Arts. 24, 25, and 27.
- (8) See Art. 45.
- (9) See Art. 34.
- (10) See Art. 4.
- (11) See Art. 54.
- (12) See Arts. 60 and 61.
- (13) See Art. 39.
- (14) See Art. 48.
- (15) See Art. 62.
- (16) See Art. 30.
- (17) See Art. 75.
- (18) According to Art. 27, $L = \frac{Wh}{8d}$; therefore,

$$L = \frac{13 \times 144}{8 \times \frac{1}{2}} = 468 \text{ cwt.} = 23.4 \text{ tons. Ans.}$$
- (19) See Arts. 70 to 72.
- (20) See Art. 50.

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- (1) See Art. 5.
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- (1) See Art. 9.
- (2) See Art. 14.
- (3) See Art. 4.
- (4) See Art. 45.
- (5) See Arts. 7 and 8.
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- (7) See Art. 1.
- (8) See Art. 61.
- (9) See Art. 53.
- (10) See Arts. 33 and 41.
- (11) See Art. 35.
- (12) See Arts. 22 to 24.
- (13) (a) and (b) See Art. 1.
- (14) See Art. 26.
- (15) See Art. 14.
- (16) See Arts. 36 and 37.
- (17) See Art. 10.
- (18) (a) and (b) See Art. 29.
- (19) See Art. 4.
- (20) See Art. 9.

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- (1) See Art. 35.
- (2) See Arts. 6, 8, and 9.
- (3) See Arts. 45 and 46.
- (4) See Art. 24.
- (5) See Art. 49.
- (6) See Art. 2.
- (7) See Art. 38.
- (8) See Art. 12 or 13.
- (9) See Art. 25.
- (10) See Art. 12.

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- (1) See Art. 46.
- (2) See Art. 48.
- (3) See Art. 40.
- (4) See Art. 59.
- (5) See Art. 18.
- (6) See Art. 97.
- (7) See Art. 11.
- (8) See Arts. 56 and 57.
- (9) See Art. 5.
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- (12) See Art. 32.
- (13) See Art. 13.
- (14) See Art. 69.
- (15) See Art. 88.
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- (17) See Arts. 71 to 73.
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- (19) See Art. 104.
- (20) See Art. 27.

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